PRINCETON UNIVERSITY

IAS INSTITUTE FOR ADVANCED STUDY

UF UNIVERSITY of FLORIDA




Cosmology from the Galaxy 4-Point Correlation Function

Oliver Philcox (Princeton / IAS)

Cosmology from Home Conference, 2021

***Based on**: 2105.08722, 2106.10278*

Hou et al. (prep.), Philcox et al. (prep.)

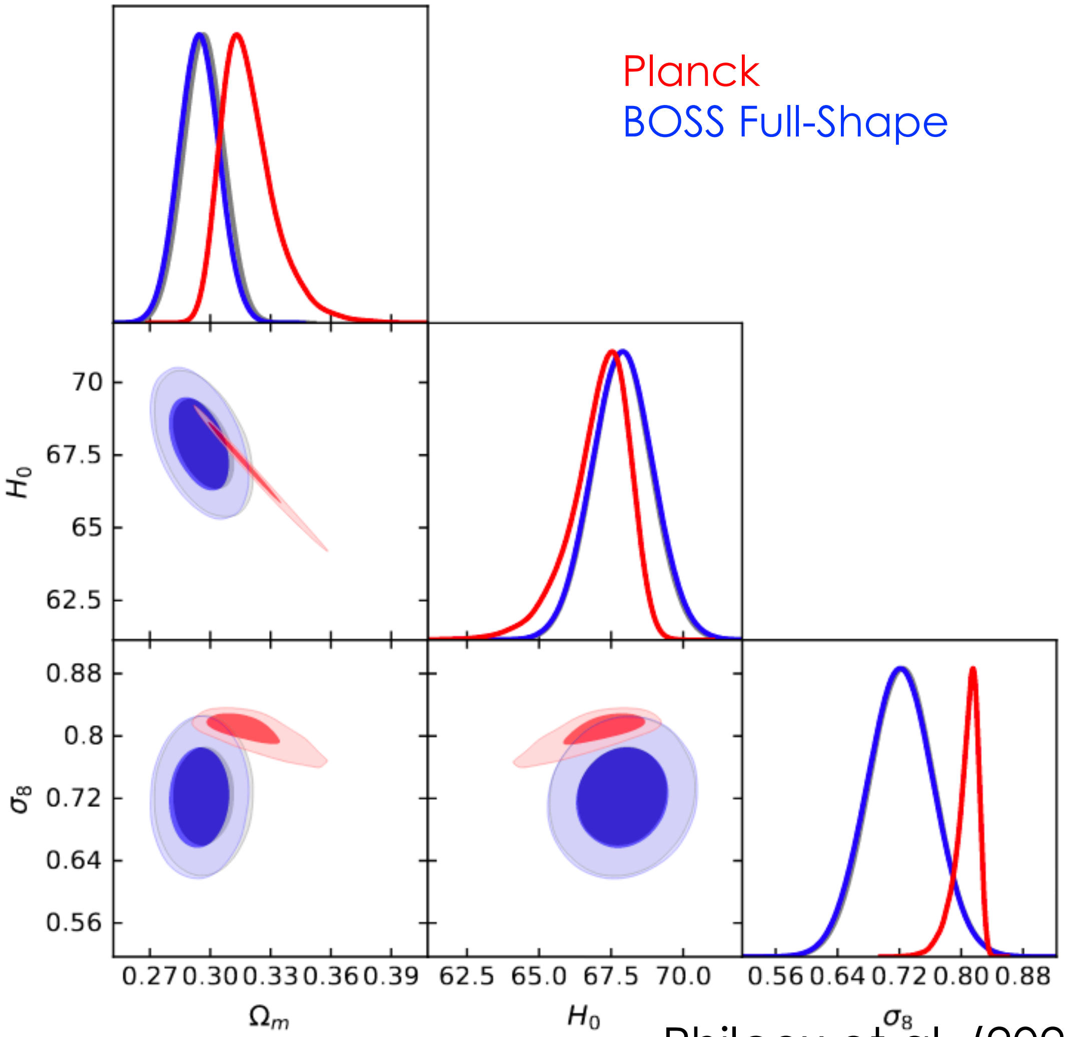
Cosmology from Galaxy Surveys

- Constraints from **spectroscopic surveys** rival those of the CMB

$\boxed{\text{Planck: } H_0 = 67.1^{+1.3}_{-0.7}\text{ , BOSS: } H_0 = 67.9 \pm 1.1}$

- DESI, Euclid, Roman & Rubin will be **much** stronger

- Almost all analyses comes from **two-point** statistics:
    - Power Spectrum
    - 2-Point Correlation Function (2PCF)



Philcox et al. (2020)

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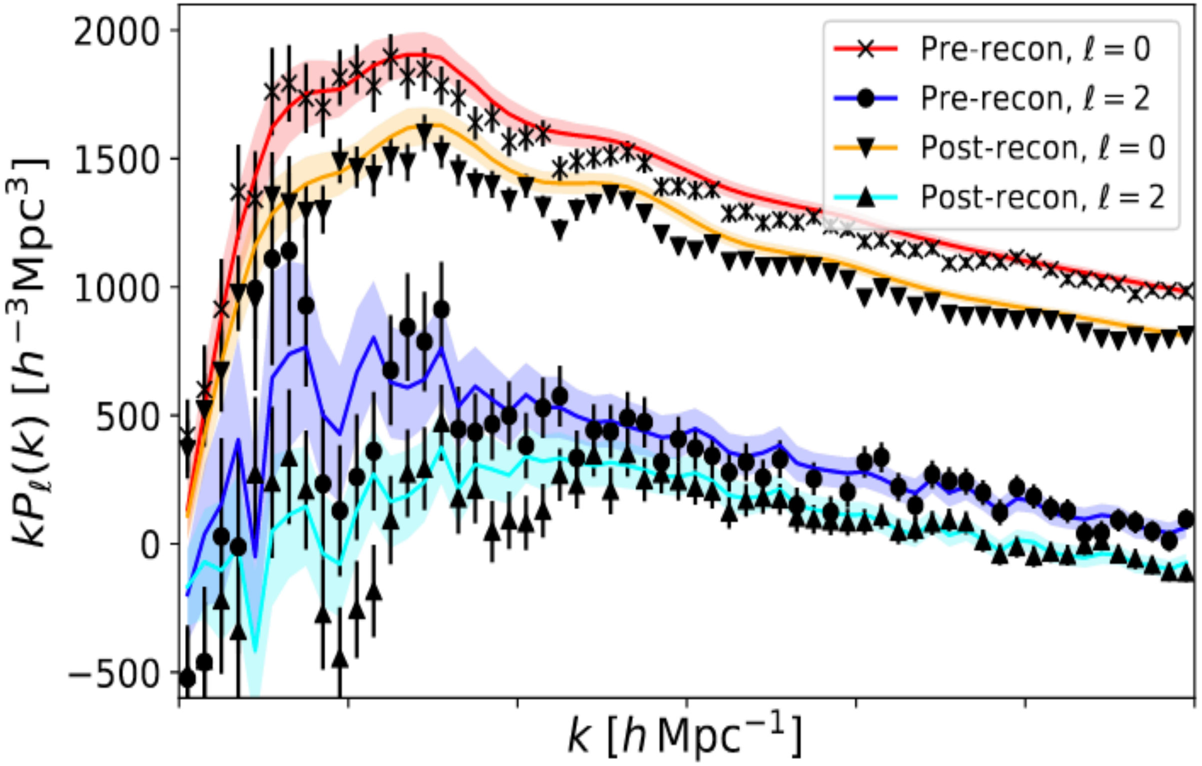
BOSS DR12



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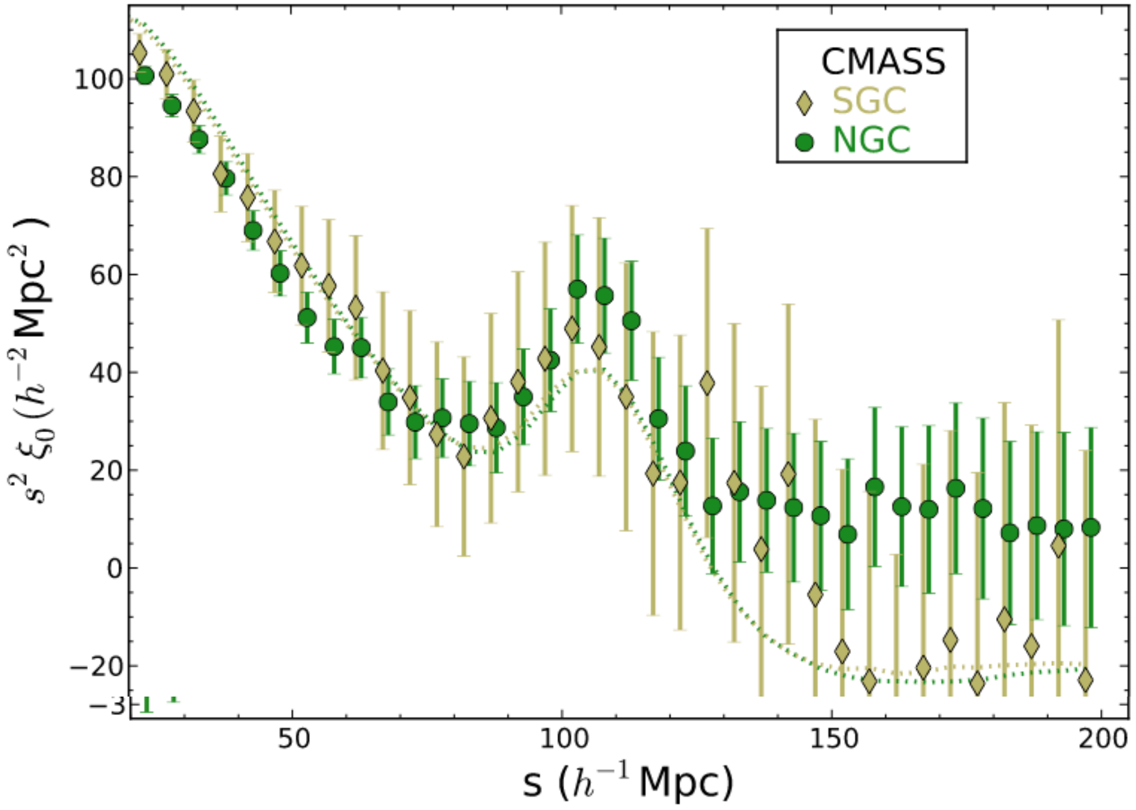


Ross et al. (2016)

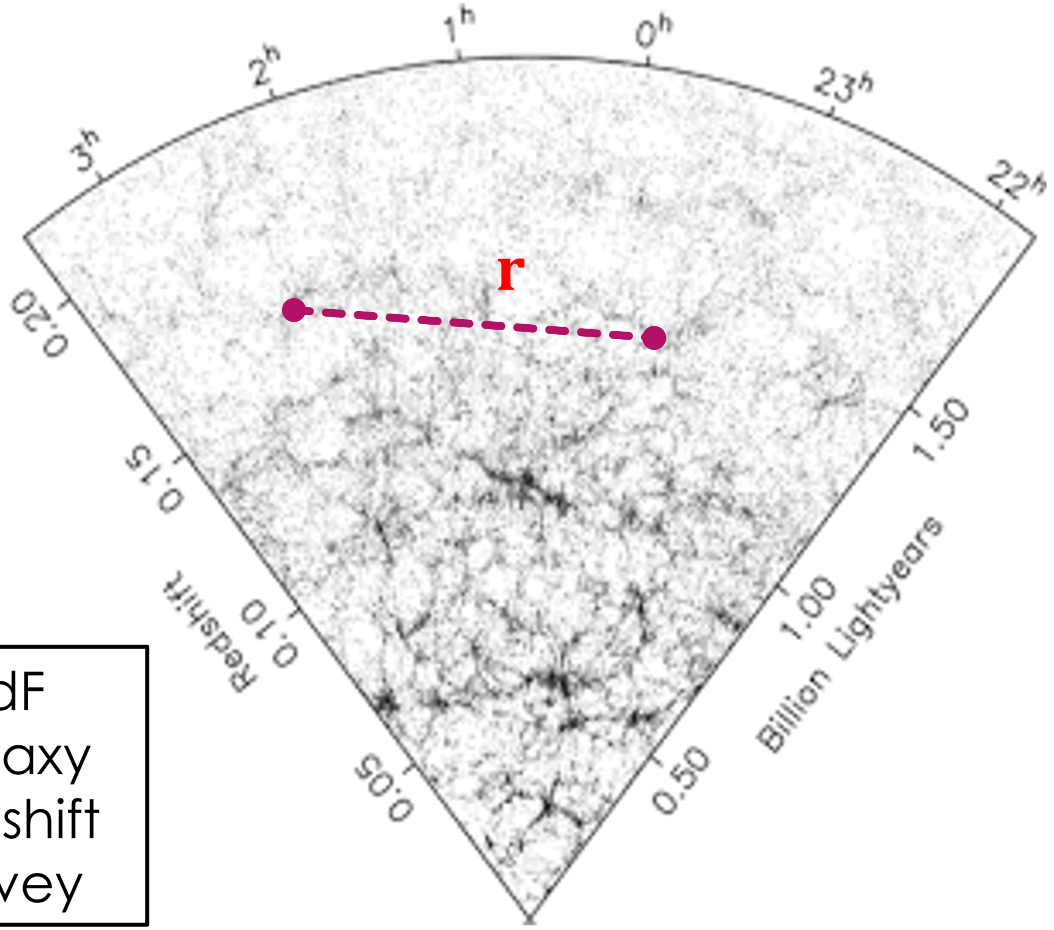
N-Point Correlation Functions

2-point function:

$$\boxed{\xi(\mathbf{r}) = \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}) \rangle}$$

Excess probability of finding two galaxies separated by **r**

Contains **all** information if field is **Gaussian**



2dF Galaxy Redshift Survey

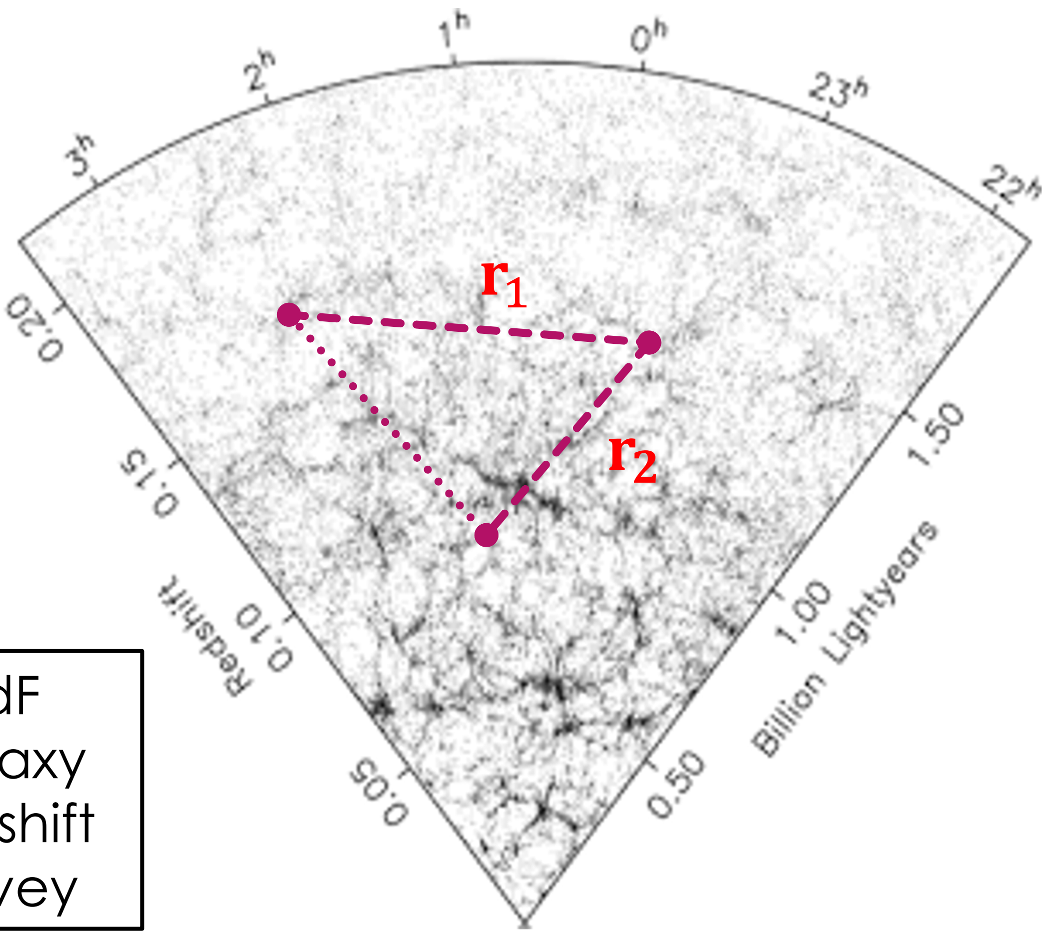
N-Point Correlation Functions

3-point function:

$$\boxed{\zeta(\mathbf{r}_1, \mathbf{r}_2) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}_1) \delta(\mathbf{x} + \mathbf{r}_2) \rangle}$$

Excess probability of finding three galaxies separated by $\mathbf{r}_1$ and $\mathbf{r}_2$

Only useful for **non-Gaussian** fields



2dF Galaxy Redshift Survey

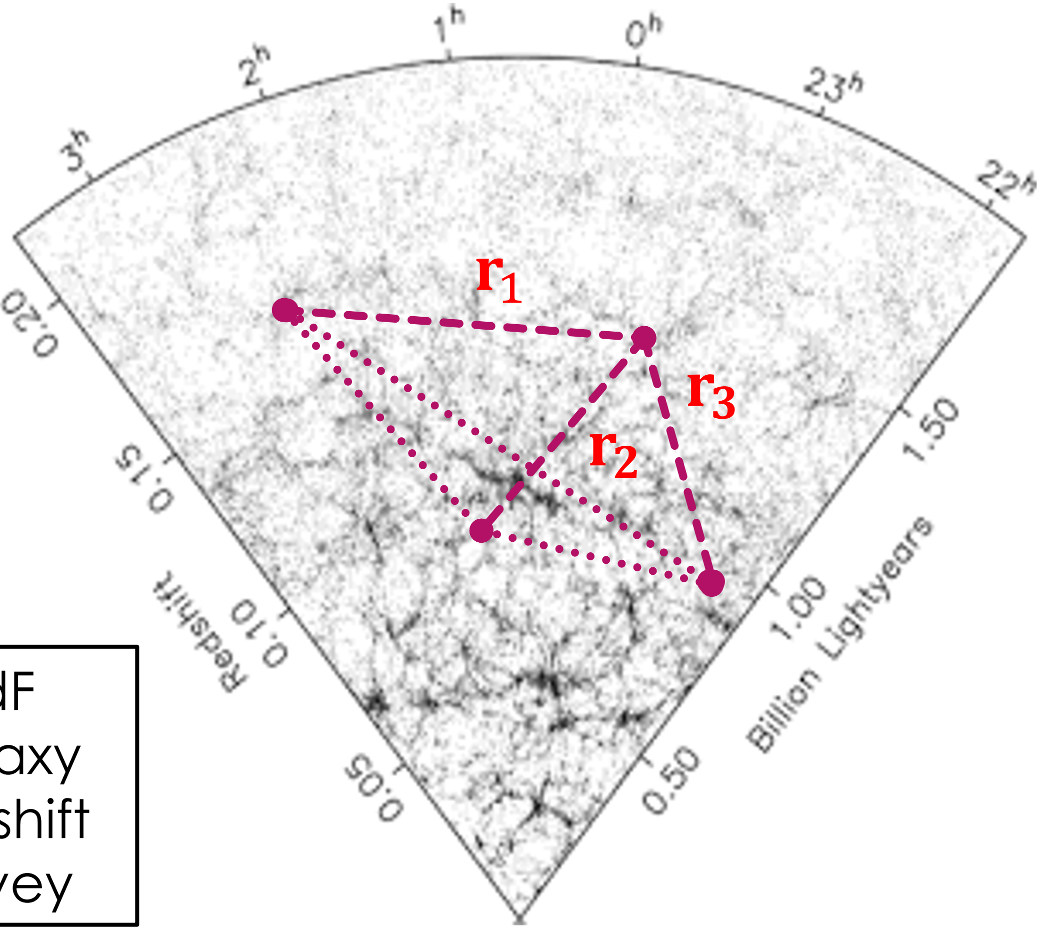
N-Point Correlation Functions

4-point function:

$$\boxed{\begin{aligned}\zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = {} & \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}_1)\delta(\mathbf{x}+\mathbf{r}_2)\delta(\mathbf{x}+\mathbf{r}_3)\rangle \\ & -\langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}_1)\rangle\langle\delta(\mathbf{x}+\mathbf{r}_2)\delta(\mathbf{x}+\mathbf{r}_3)\rangle \\ & -2 \text{ perms.}\end{aligned}}$$

Excess probability of finding four galaxies separated by $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, removing **disconnected** piece

Only useful for **non-Gaussian** fields



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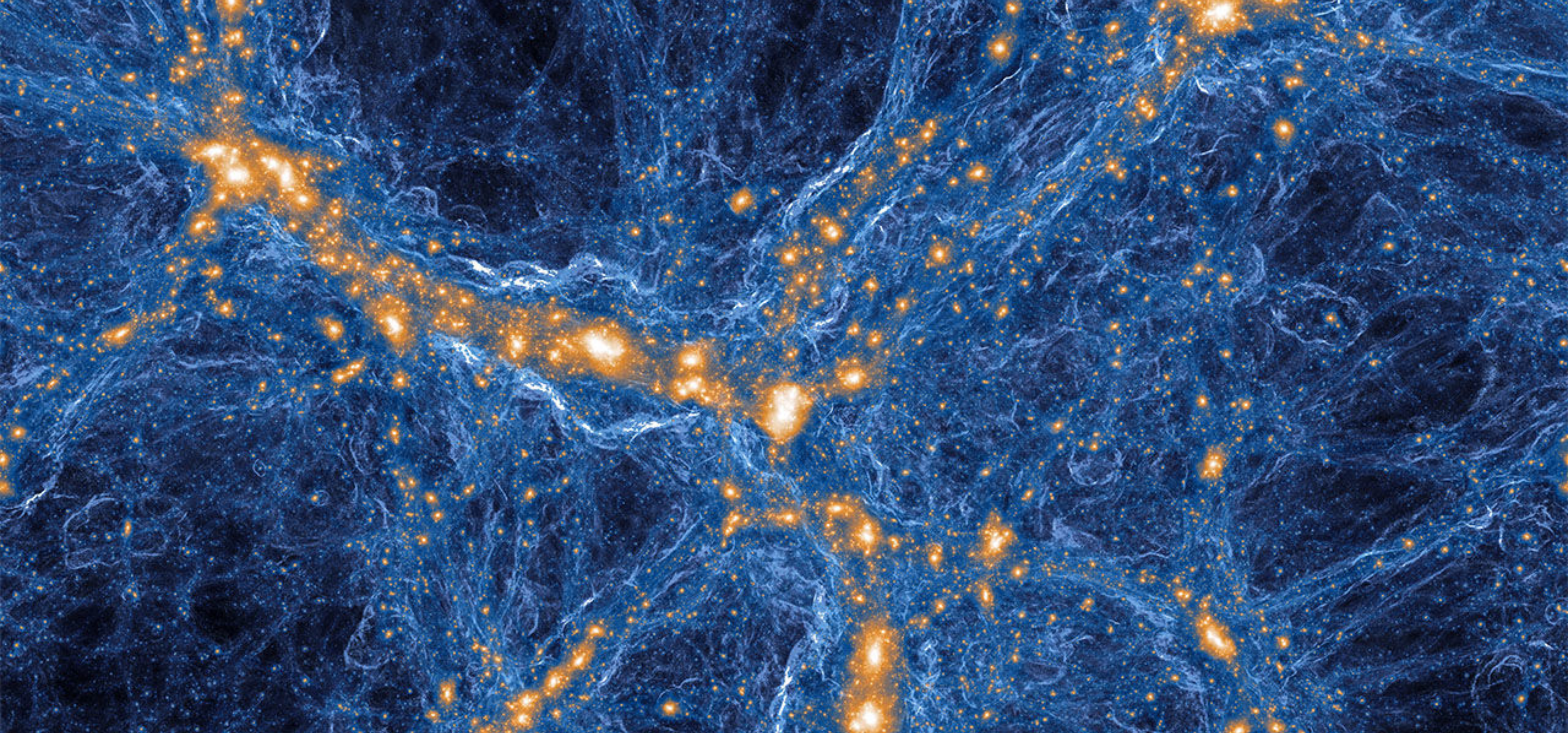


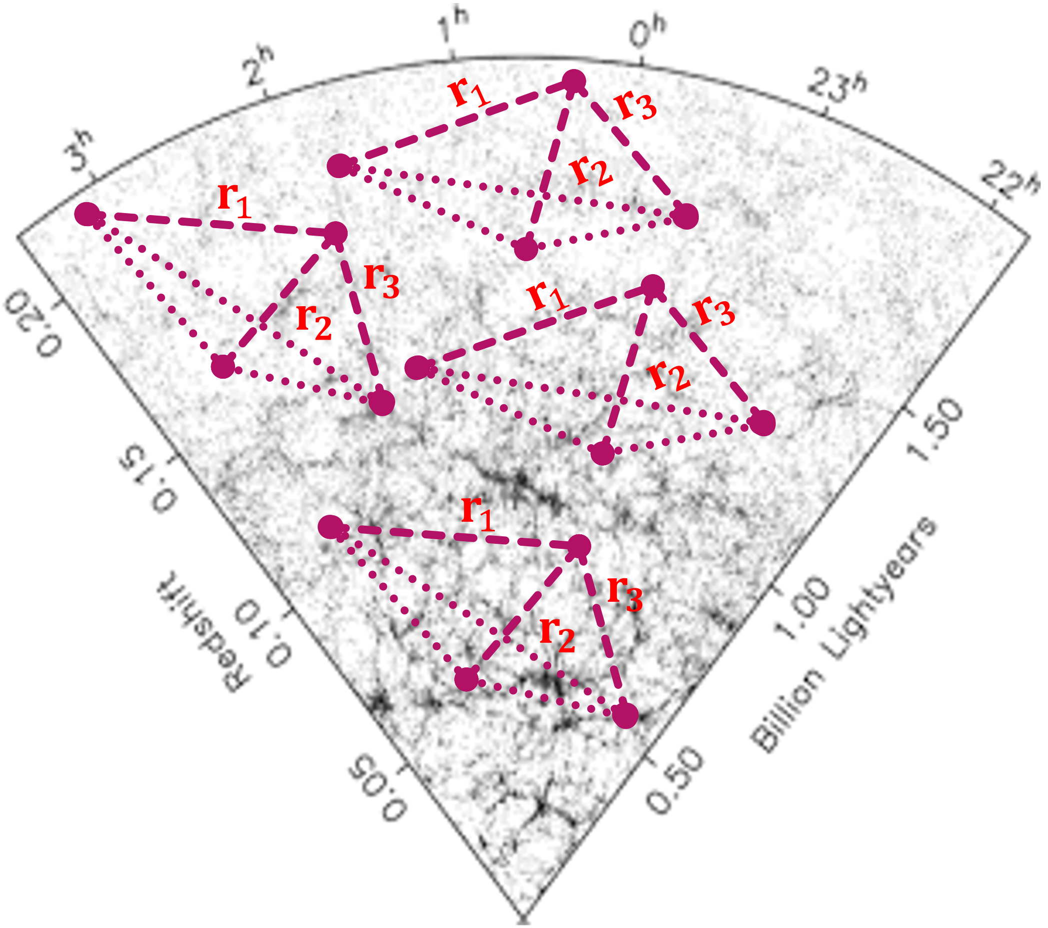
1. How to Compute an N-point Function

NPCF Estimation

NPCF: Excess probability of finding N galaxies separated by $\mathbf{r}_1, \dots, \mathbf{r}_N$

Naïve Estimator:

- Count all N-tuplets of galaxies and place them into bins
- Scales as N_{gal}^N
- **Prohibitively slow** for $N > 2$



See also Sabiu et al. (2019)

NPCF Estimation

Better Approach:

- Decompose NPCF into a **separable angular basis**

Basis Functions

$$\zeta(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3) = \sum_{\ell_1\ell_2\ell_3} \zeta_{\ell_1\ell_2\ell_3}(r_1,r_2,r_3)\mathcal{P}_{\ell_1\ell_2\ell_3}(\hat{\boldsymbol{r}}_1,\hat{\boldsymbol{r}}_2,\hat{\boldsymbol{r}}_3)$$

Coefficients

- This involves the **spherical harmonics**

$$\mathcal{P}_{\ell_1\ell_2\ell_3}(\hat{\boldsymbol{r}}_1,\hat{\boldsymbol{r}}_2,\hat{\boldsymbol{r}}_3) = (-1)^{\ell_1+\ell_2+\ell_3} \sum_{m_1=-\ell_1}^{\ell_1} \sum_{m_2=-\ell_2}^{\ell_2} \sum_{m_3=-\ell_3}^{\ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y_{\ell_1}^{m_1}(\hat{\boldsymbol{r}}_1) Y_{\ell_2}^{m_2}(\hat{\boldsymbol{r}}_2) Y_{\ell_3}^{m_3}(\hat{\boldsymbol{r}}_3),$$

Cahn & Slepian (2020), Philcox et al. (2021)

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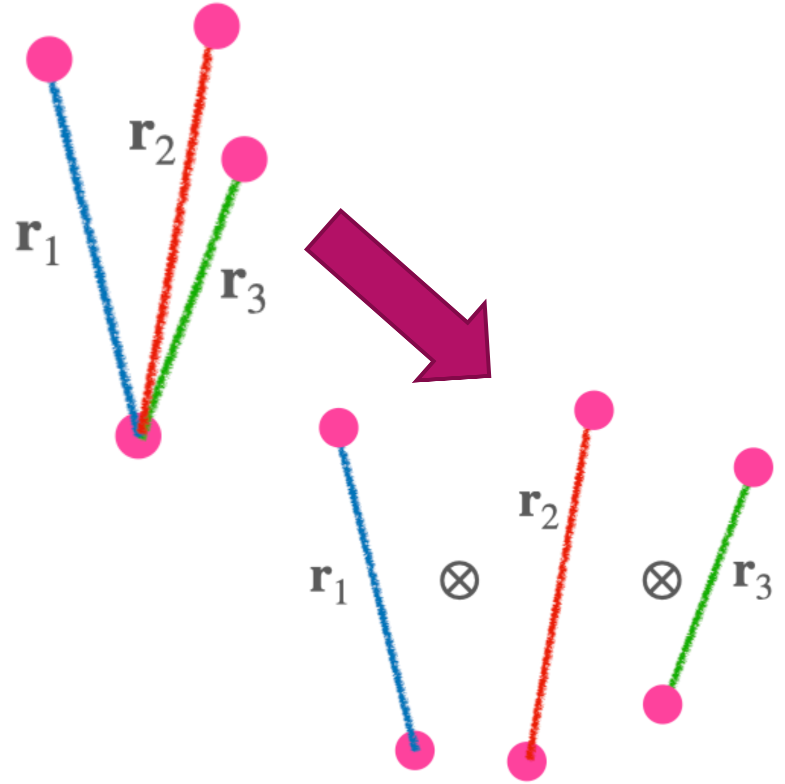

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Cahn & Slepian (2020), Philcox et al. (2021)

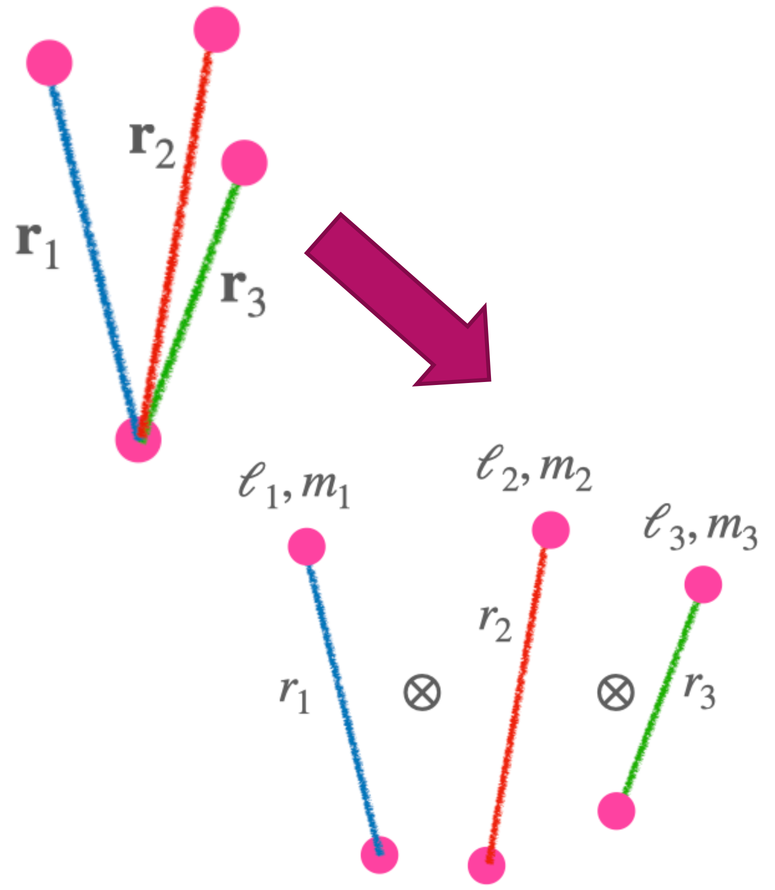
NPCF Estimation

Better Approach:

- NPCF becomes a sum over **pairs** of galaxies

$$\zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \sim \sum_{\text{galaxies}} \sum_{\ell_i, m_i} a_{\ell_1}^{m_1}(r_1) a_{\ell_2}^{m_2}(r_2) a_{\ell_3}^{m_3}(r_3) \times Y_{\ell_1}^{m_1}(\hat{r}_1) Y_{\ell_2}^{m_2}(\hat{r}_2) Y_{\ell_3}^{m_3}(\hat{r}_3)$$

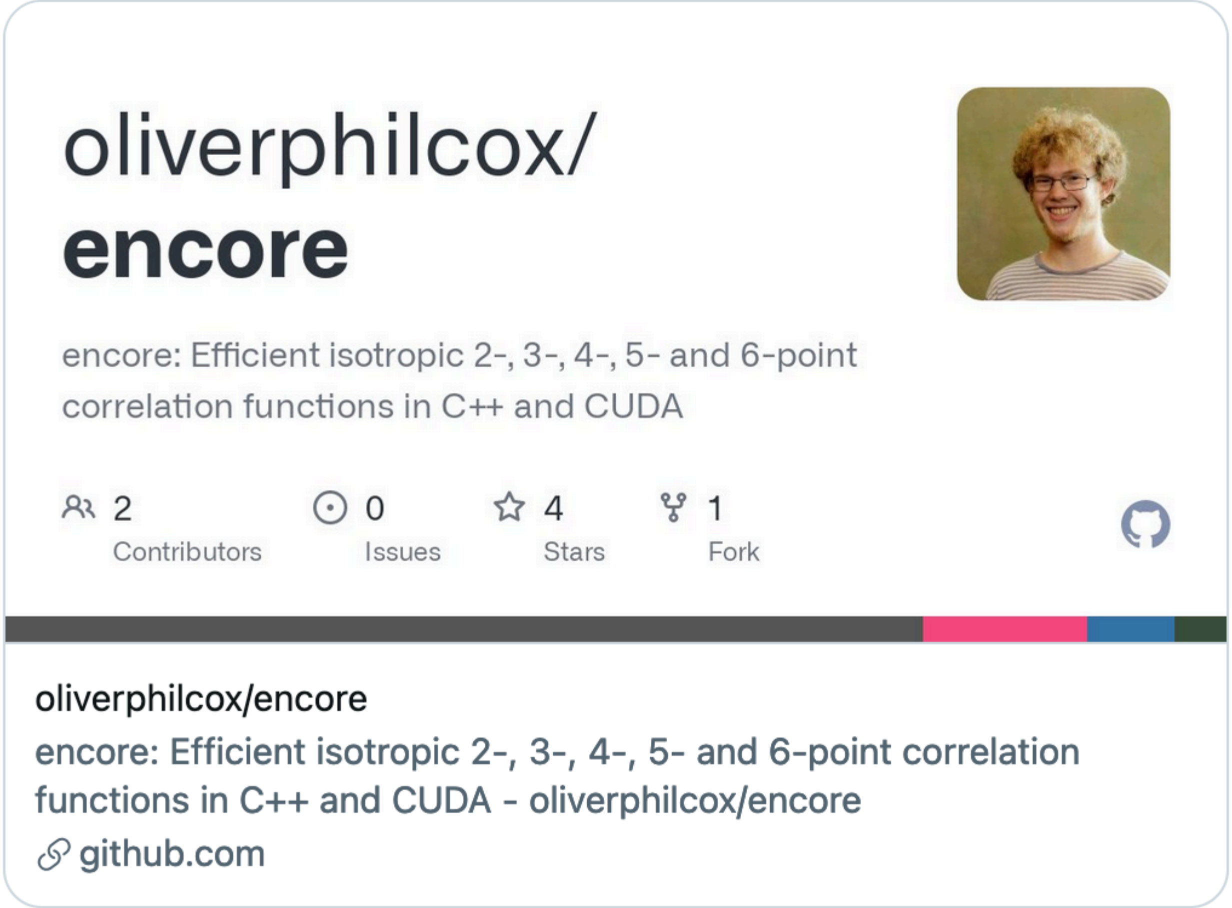
- Scales as N_{gal}^2
- Extendable to **higher dimensions**, **anisotropy** and **curvature**



Philcox et al. (2021), Philcox & Slepian (2021)

encore: Ultra-fast N-point functions

- **Public** C++ code
- Computes isotropic 2-, 3-, 4-, 5- and 6-point correlation functions
- Includes **survey geometry** correction
- Fully parallelized, including GPU support
- BOSS 4PCF computed in ~ 40 CPU-hours



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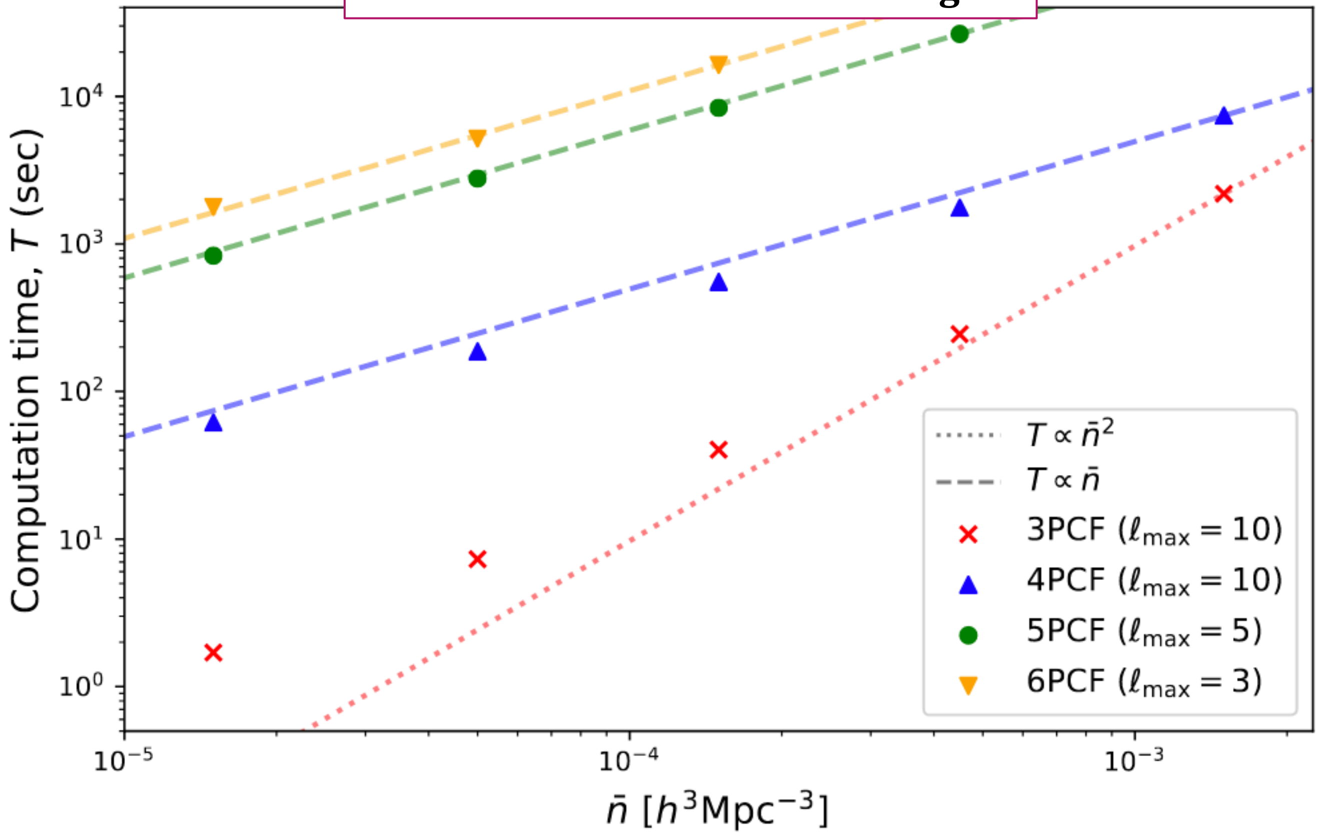
Computation time $\propto N_{\mathbf{gal}}$



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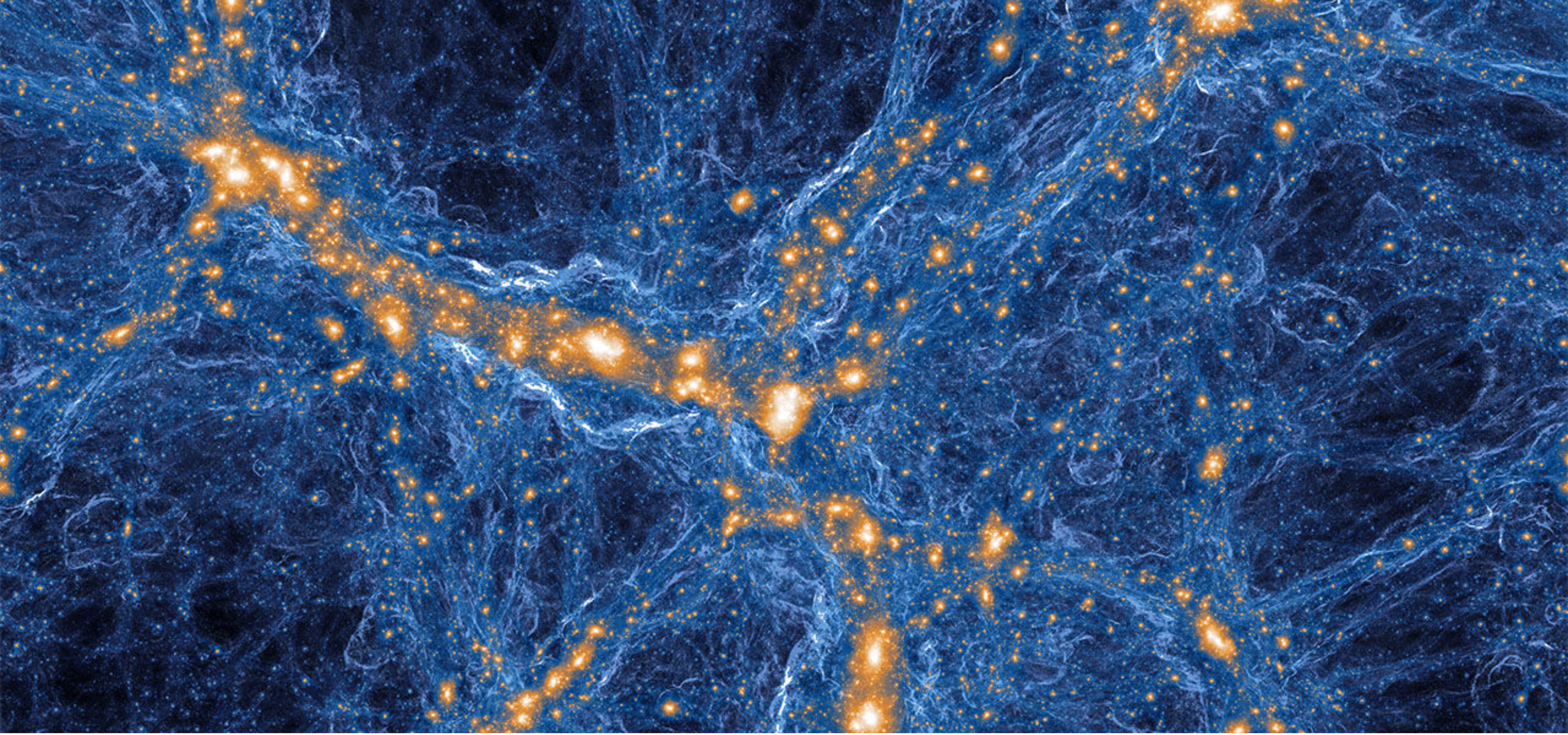
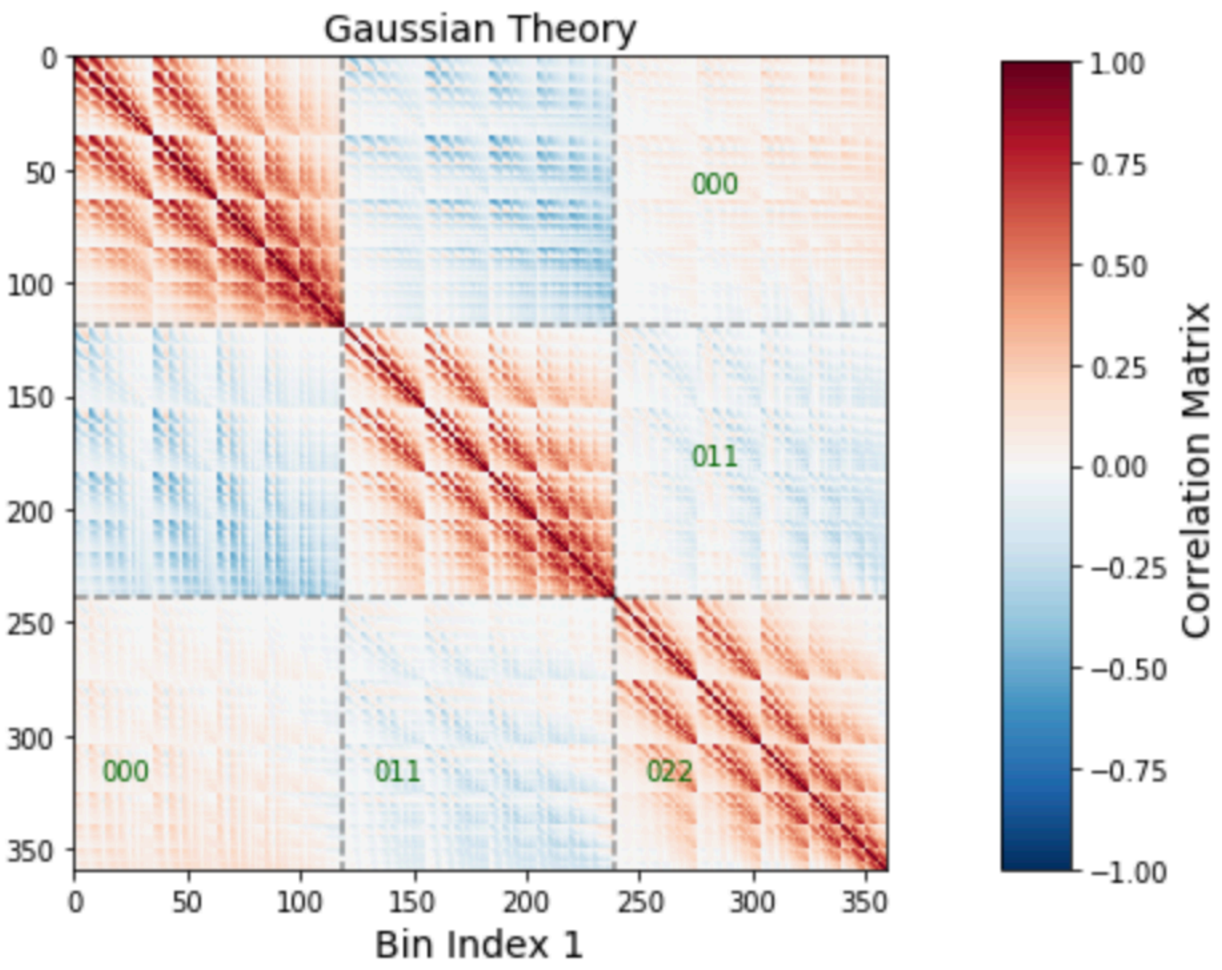


2. What to Do with an N-point Function

NPCF Covariances

- Large dimension makes **sample** covariances difficult
- Construct **analytic** covariances, assuming:
  1. Gaussianity
  2. Isotropy
  3. Idealized Geometry
- Not exactly correct, but useful for **compression** and **forecasting**



Hou et al. (in prep.)

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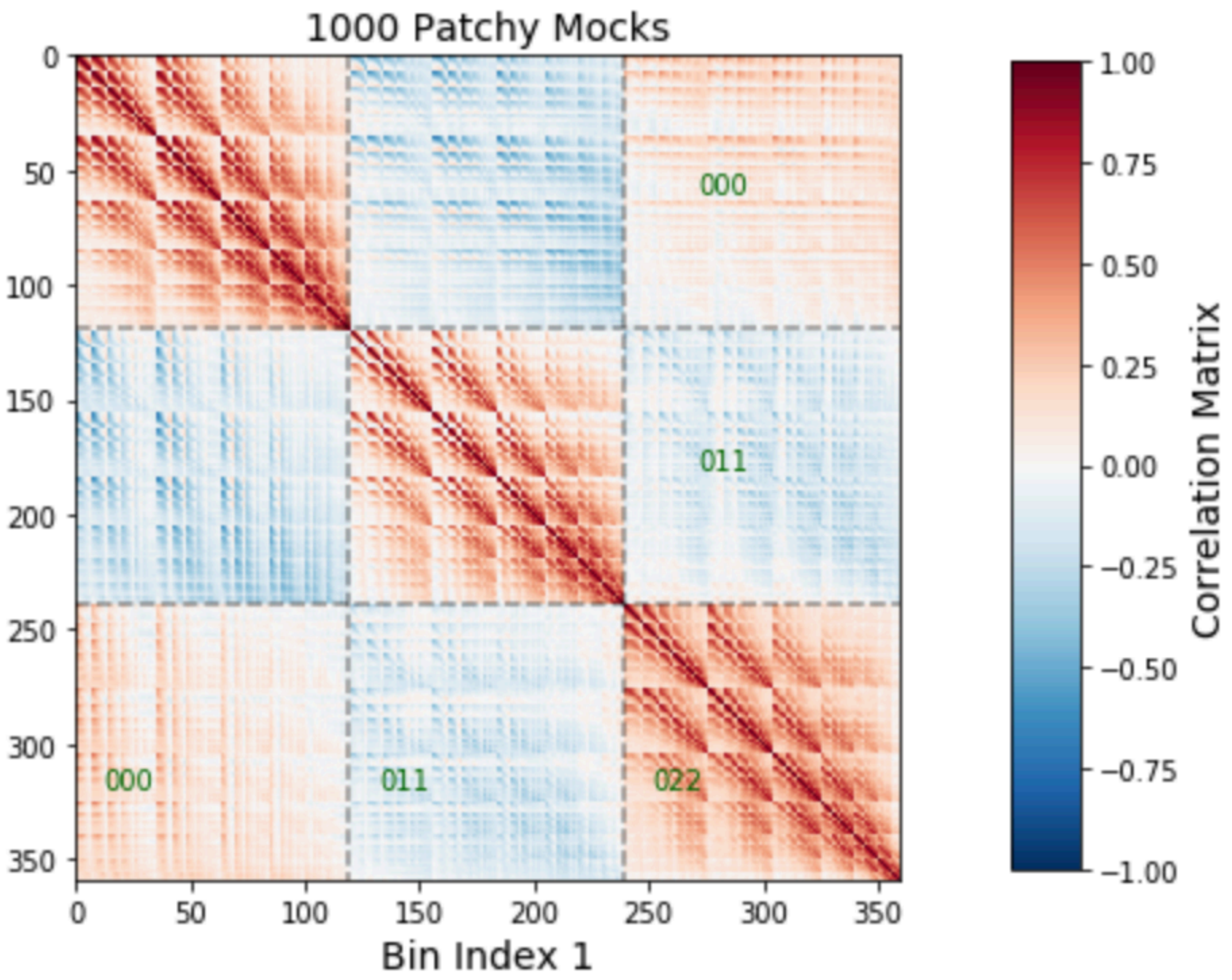

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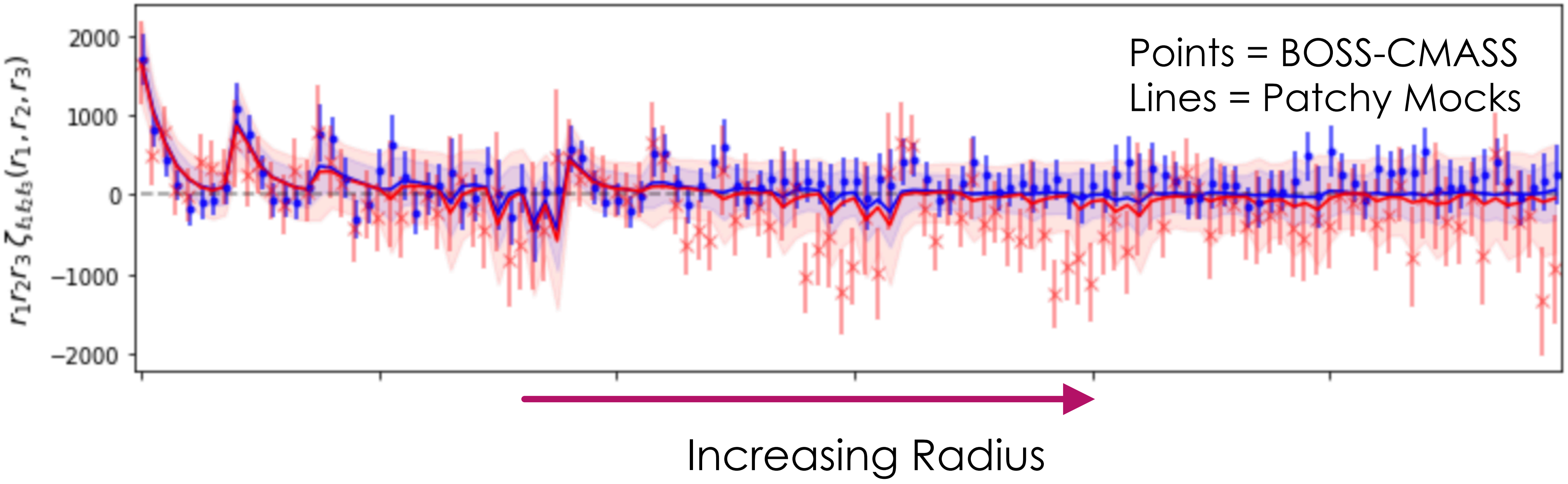
Measuring the BOSS 4PCF using encore

- Compute the (connected) 4PCF from ~700k **BOSS-CMASS** galaxies
- **Null hypothesis**: zero 4PCF

4PCF (one of 42 components)



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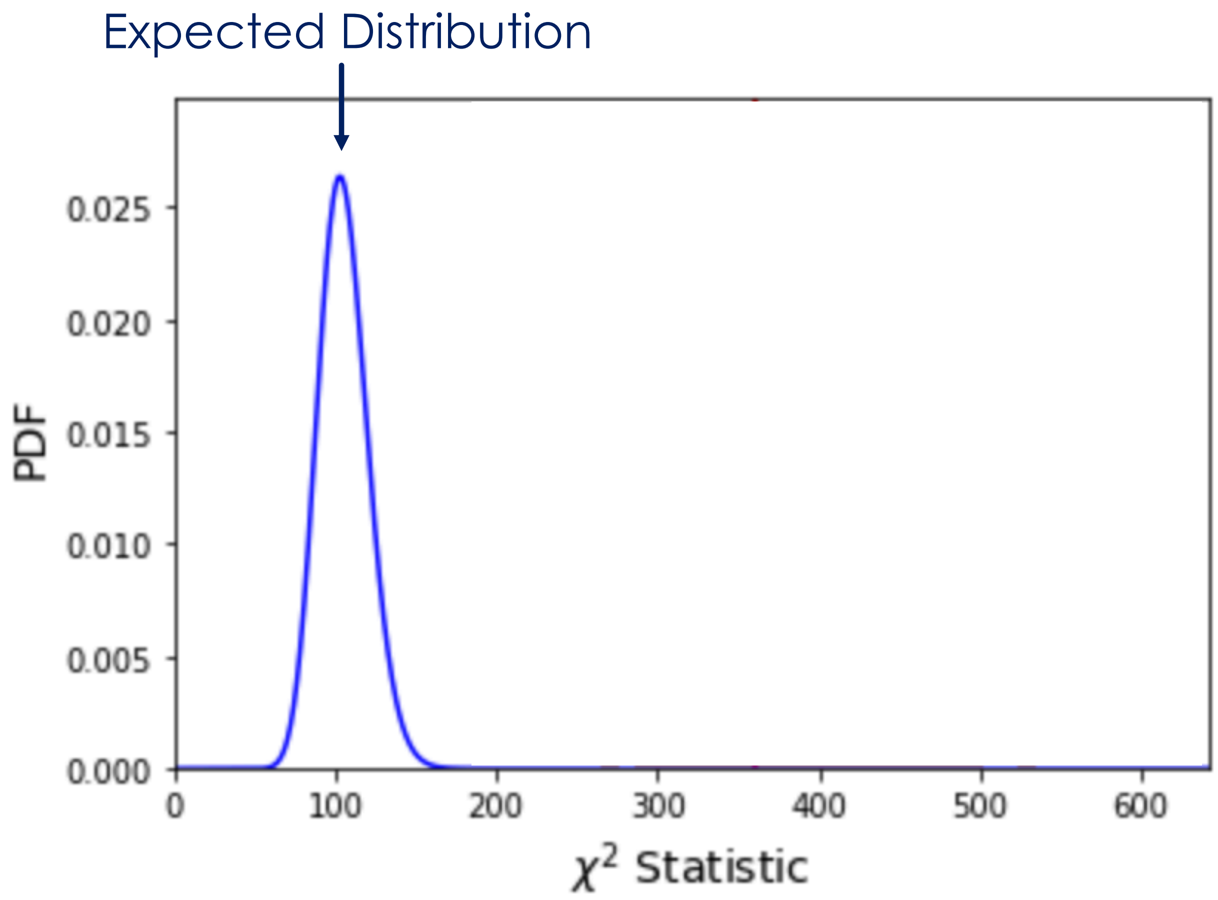

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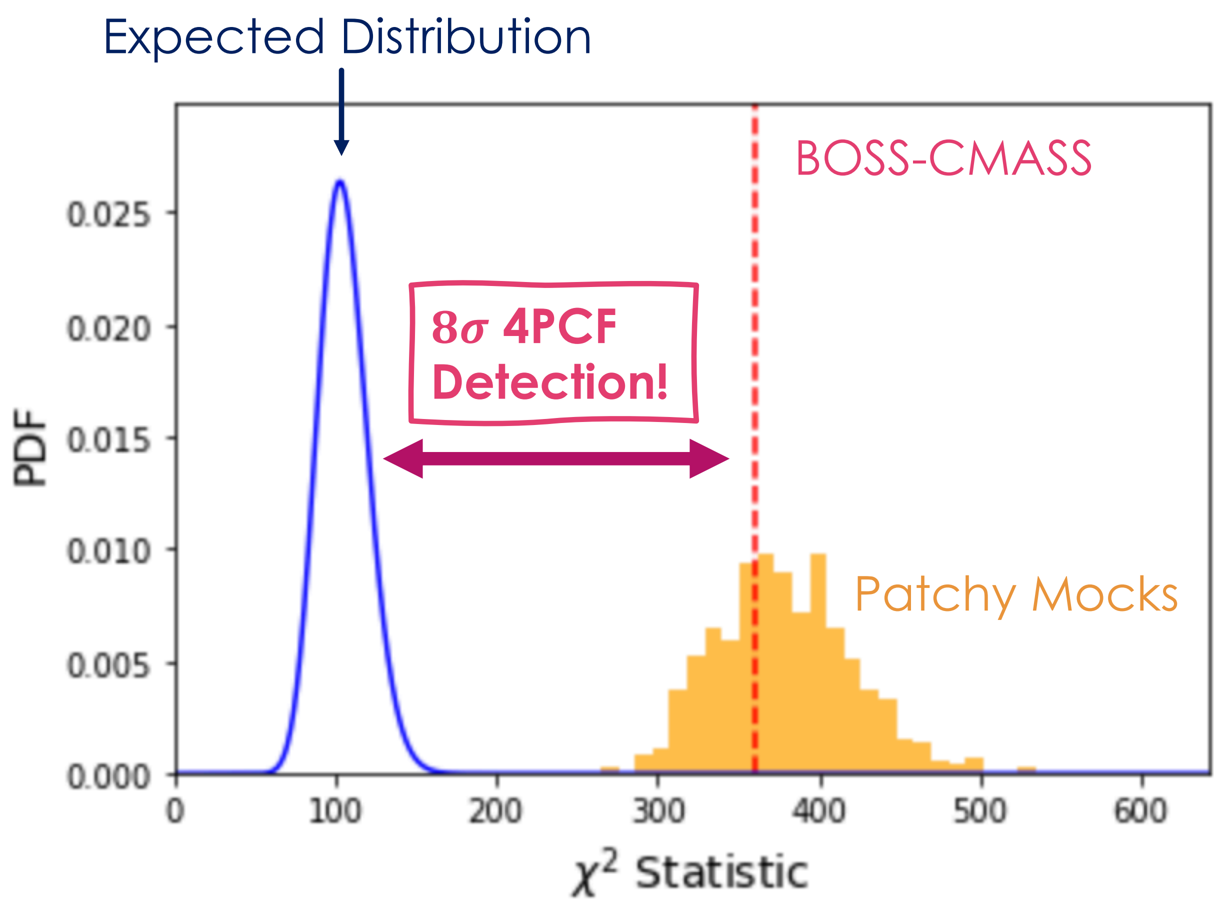
Strong detection of non-Gaussian 4PCF!



Philcox et al. (in prep.)

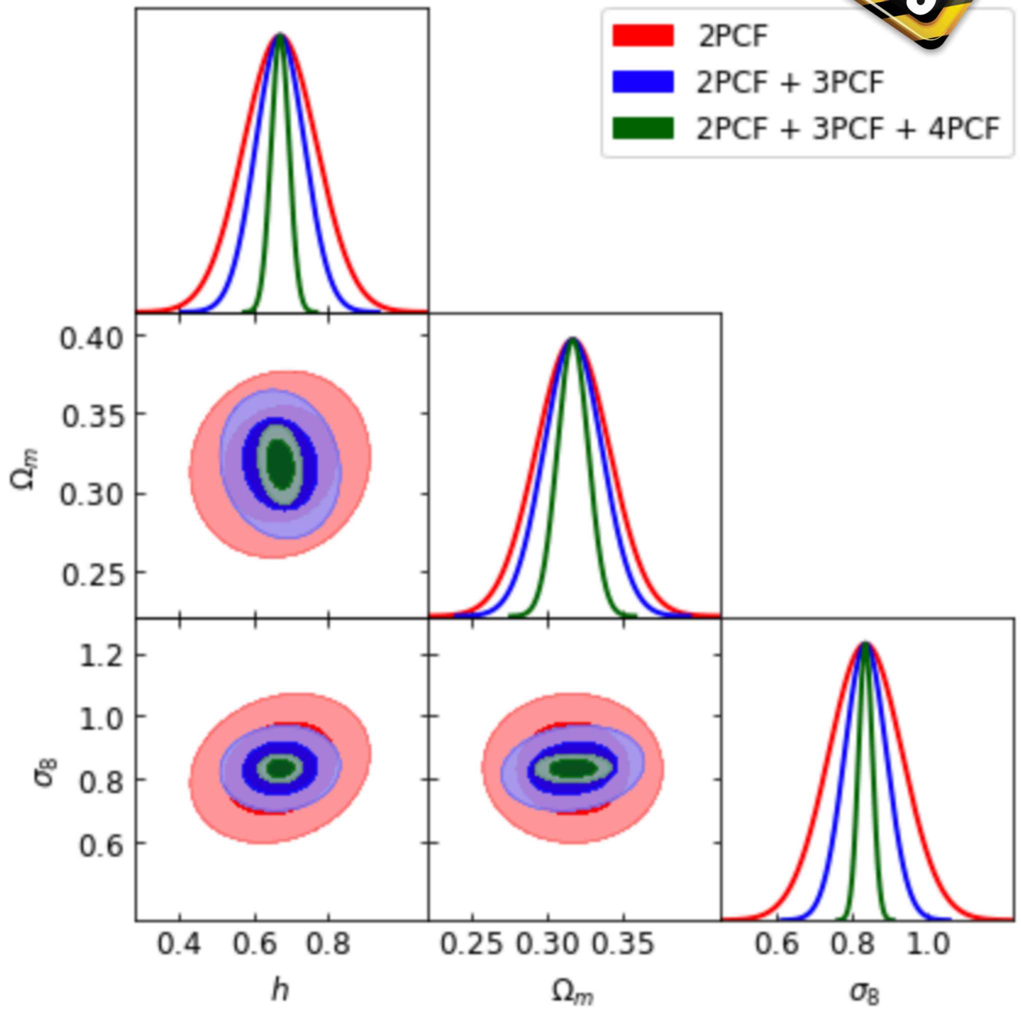
Fisher Forecasts

- Assess **information content** with **Fisher matrices**

$$\mathrm{F}_{ij} \equiv \frac{d\zeta^T}{d\theta_i} \mathrm{C}^{-1} \frac{d\zeta}{d\theta_j}$$

- Use the **Molino** simulation suite:
  - 6 cosmological parameters
  - 5 HOD parameters

- 4PCF seems to **significantly** tighten constraints on all cosmological parameters



***Disclaimer**: This is overoptimistic for several reasons, including bias from noise.*

Philcox et al. (in prep.), Hahn & Villaescusa-Navarro (2020)

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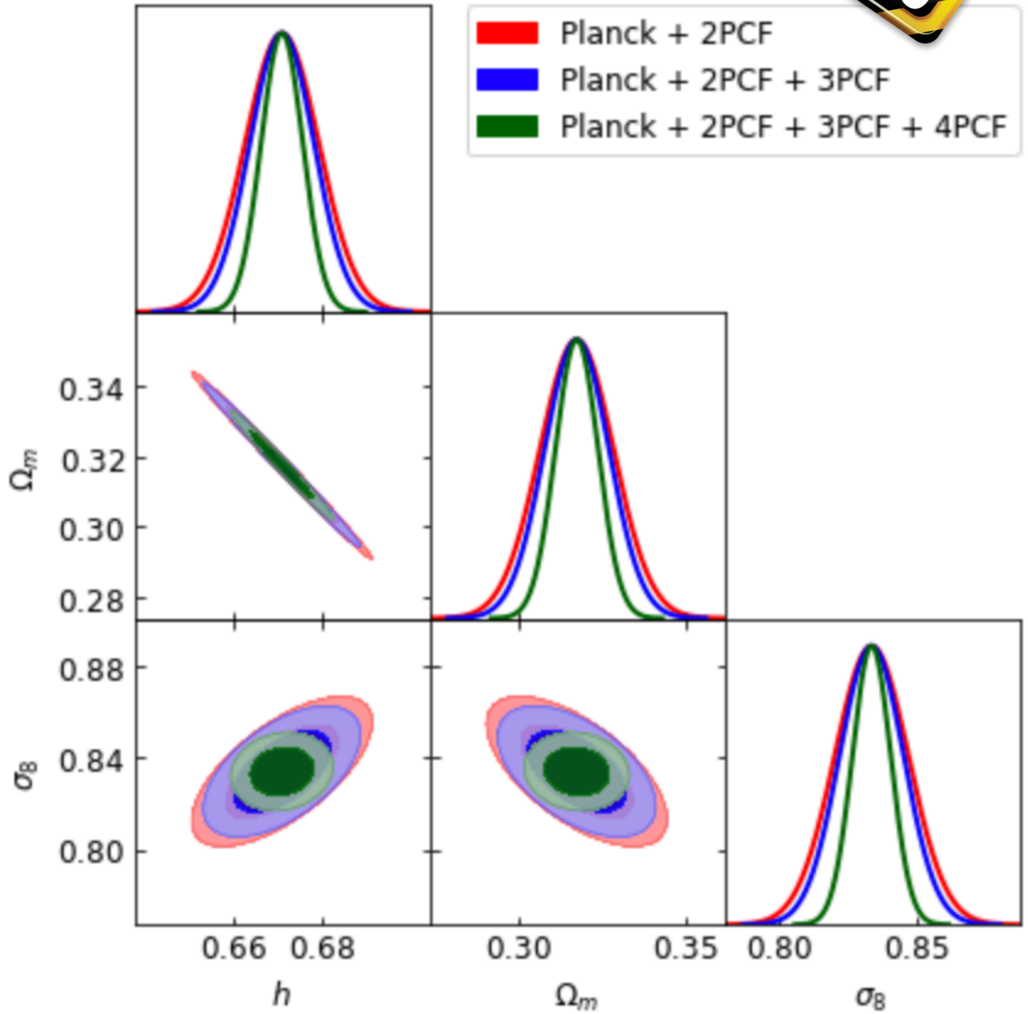

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Philcox et al. (in prep.), Hahn & Villaescusa-Navarro (2020)

Testing Parity Invariance

In 2D

- For the 2PCF:

  Parity inversion = Rotation

- For the 3PCF and beyond:

  Parity inversion ≠ Rotation

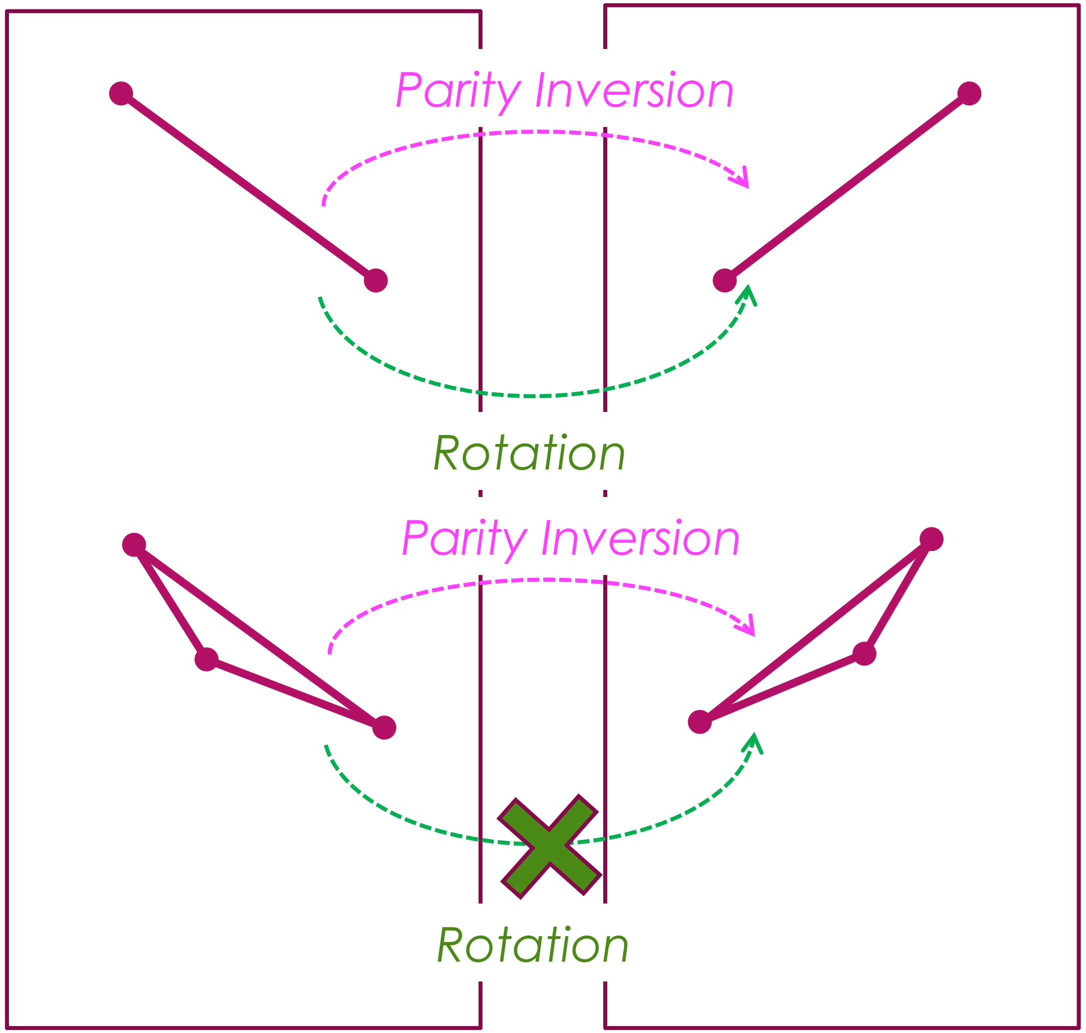
In 3D

- For the 2PCF **and** 3PCF:

  Parity inversion = Rotation

- For the 4PCF and beyond:

  Parity inversion ≠ Rotation



Philcox et al. (in prep.)

Testing Parity Invariance

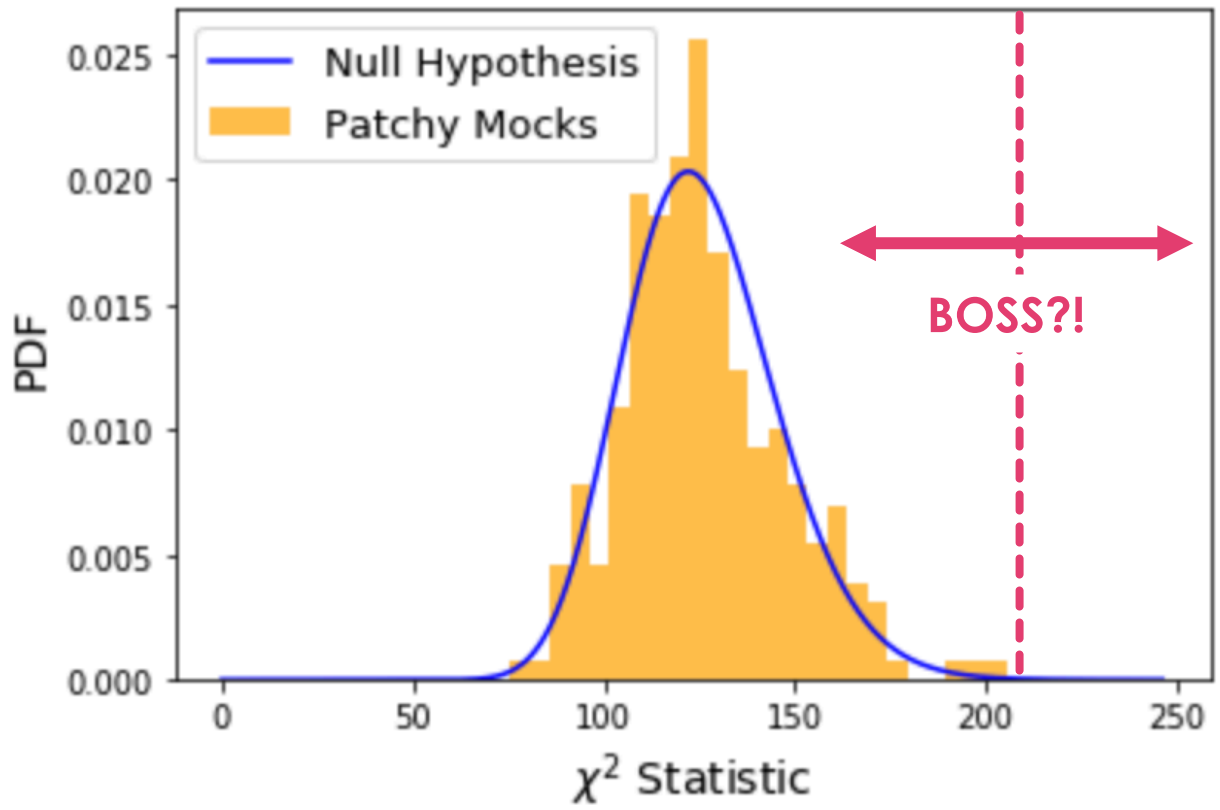
- Odd-Parity 4PCF probes **parity-violation**

$$\mathbb{P}\left[\zeta_{-}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\right] = -\zeta_{-}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$$

- Should be **zero** from gravitational evolution

- Possible sources:
  - Primordial magnetic fields?
  - Inflation?
  - Systematics?

Stay tuned for unblinding…



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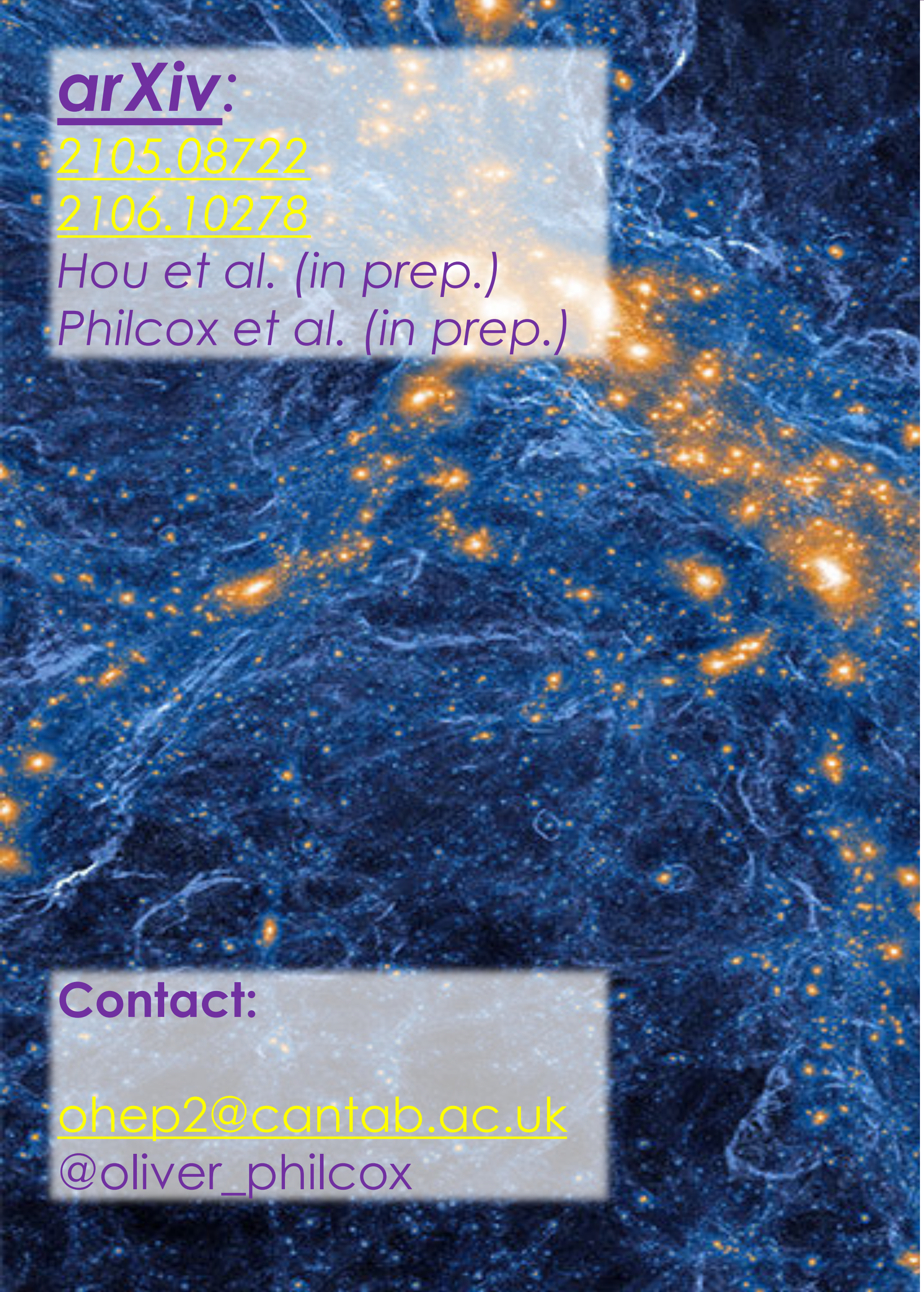

arXiv:

2105.08722

2106.10278

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Conclusions

- New estimators enable **fast** measurement of galaxy N-Point Functions
- Connected 4PCF of BOSS-CMASS **detected** at 8σ
- Using NPCFs can give:
  - **Tighter** parameter constraints
  - Tests of **parity-violation** (for N>3)