How to Model a Galaxy Survey

The whys, hows and woes of Effective Field Theory

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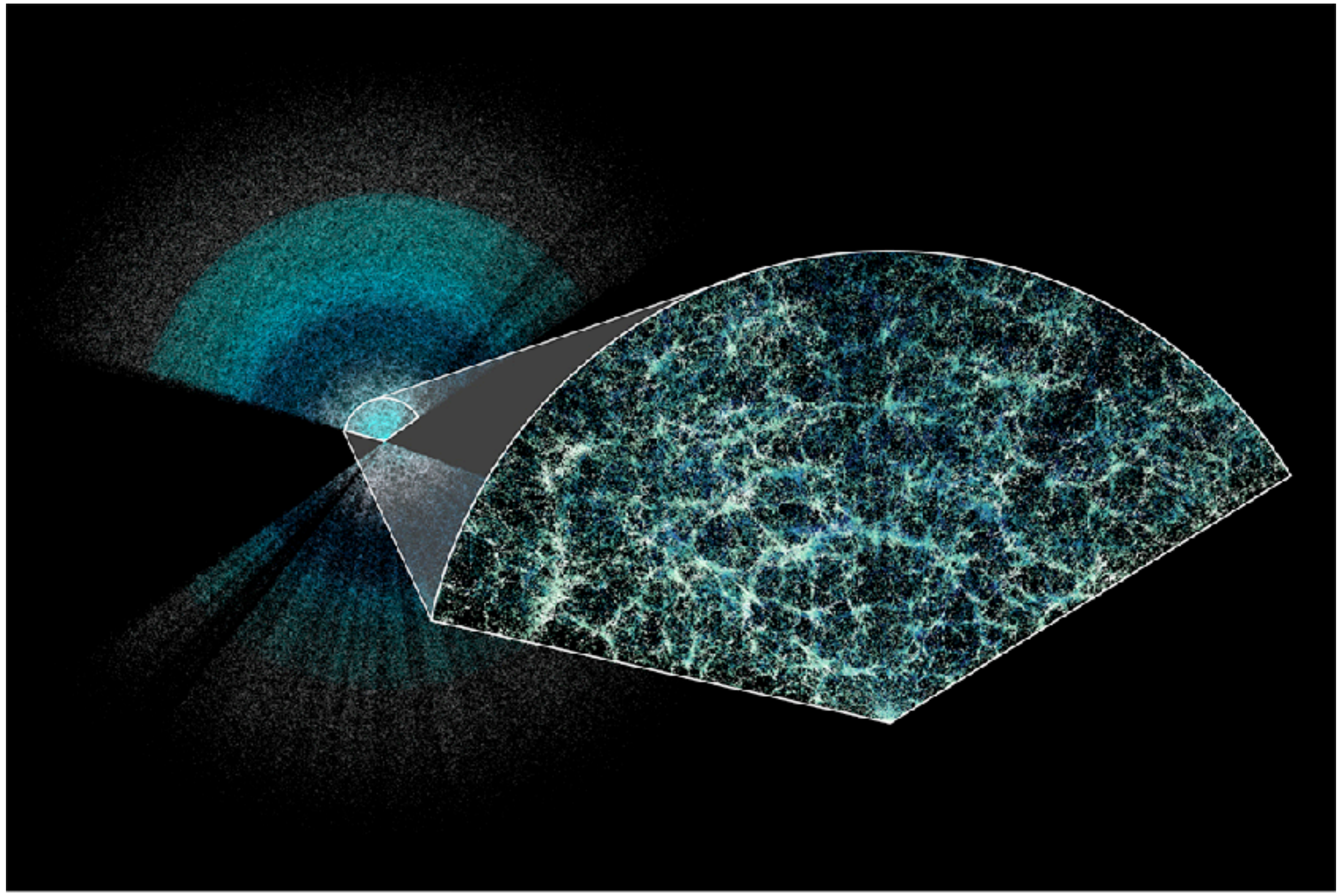
Important Note:
This subject has a long history
— my citations will be very incomplete!

Galaxy Catalog

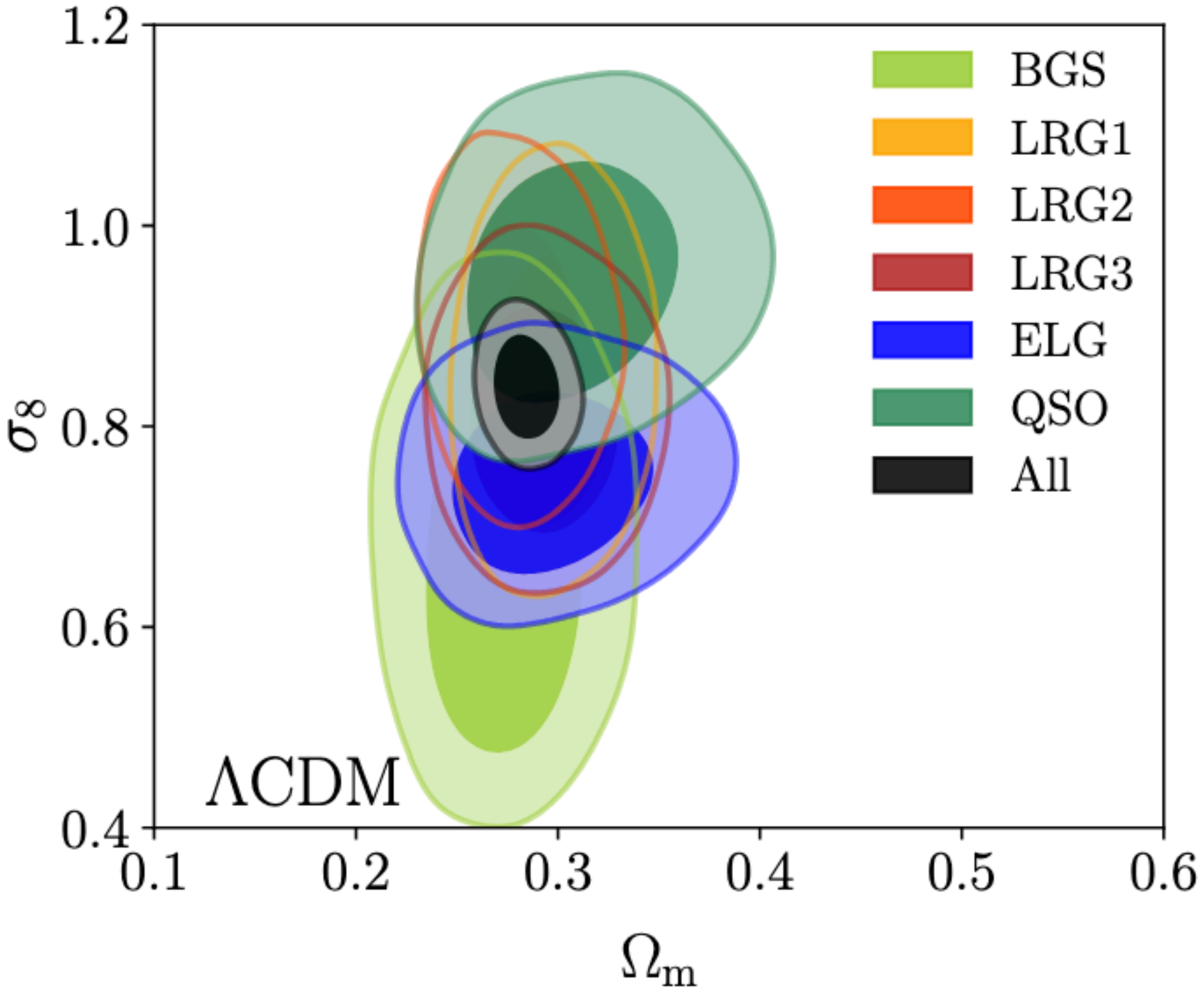


???

THESE

LECTURES

Parameter Constraints



DESI 2024

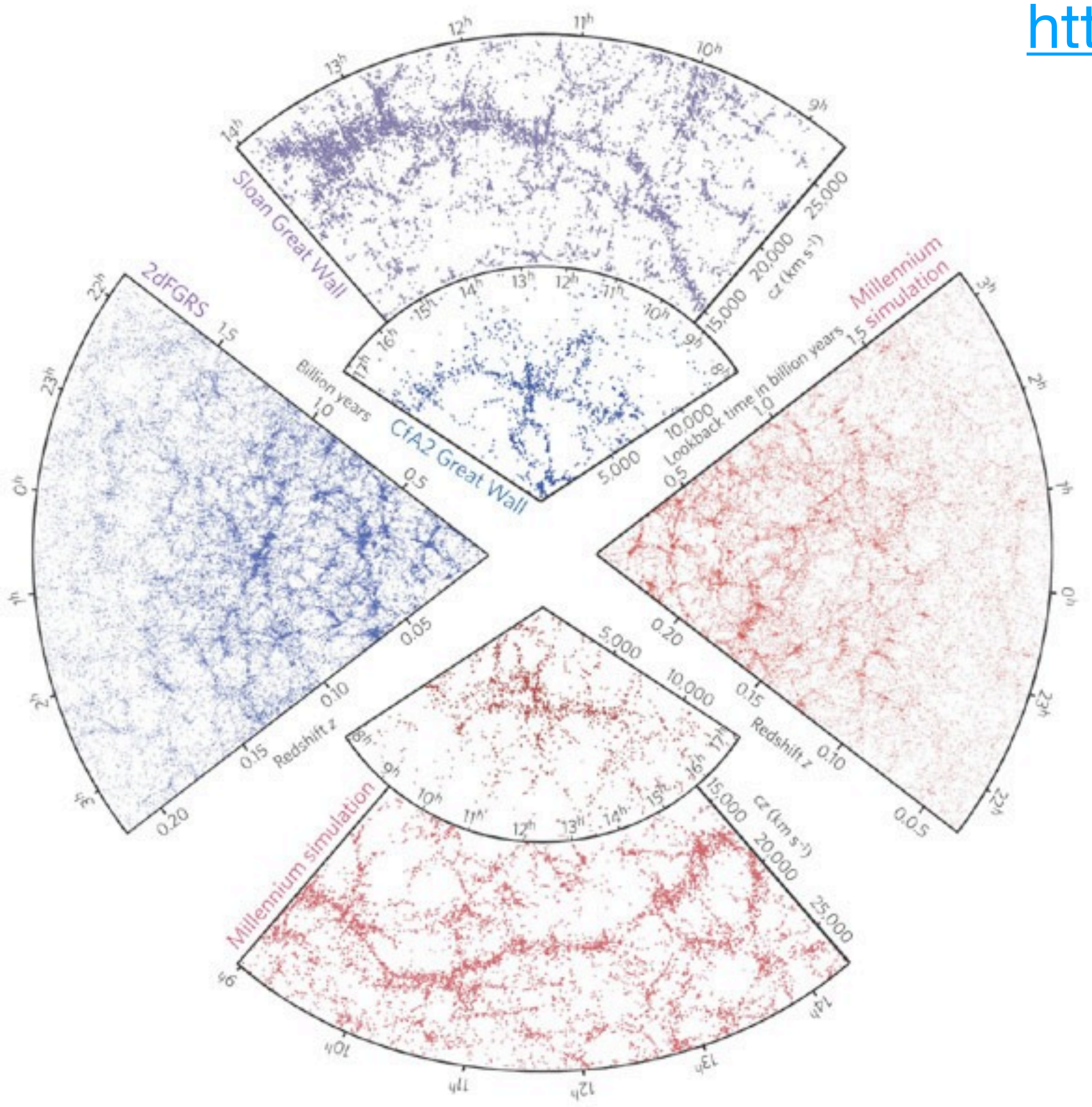
Practicalities

https://tinyurl.com/philcox-eft-notes

<ins>Further reading</ins>:

- **Oliver's** EFT Notes (see link)
- **Tobias Baldauf's** Notes
- **Daniel Baumann's** Notes

and many others!



SCAN ME

Lecture Notes

Plan:

- **Today:** what and why is **Effective Field Theory**
- **Tomorrow:** how can we use this to model for galaxy surveys?

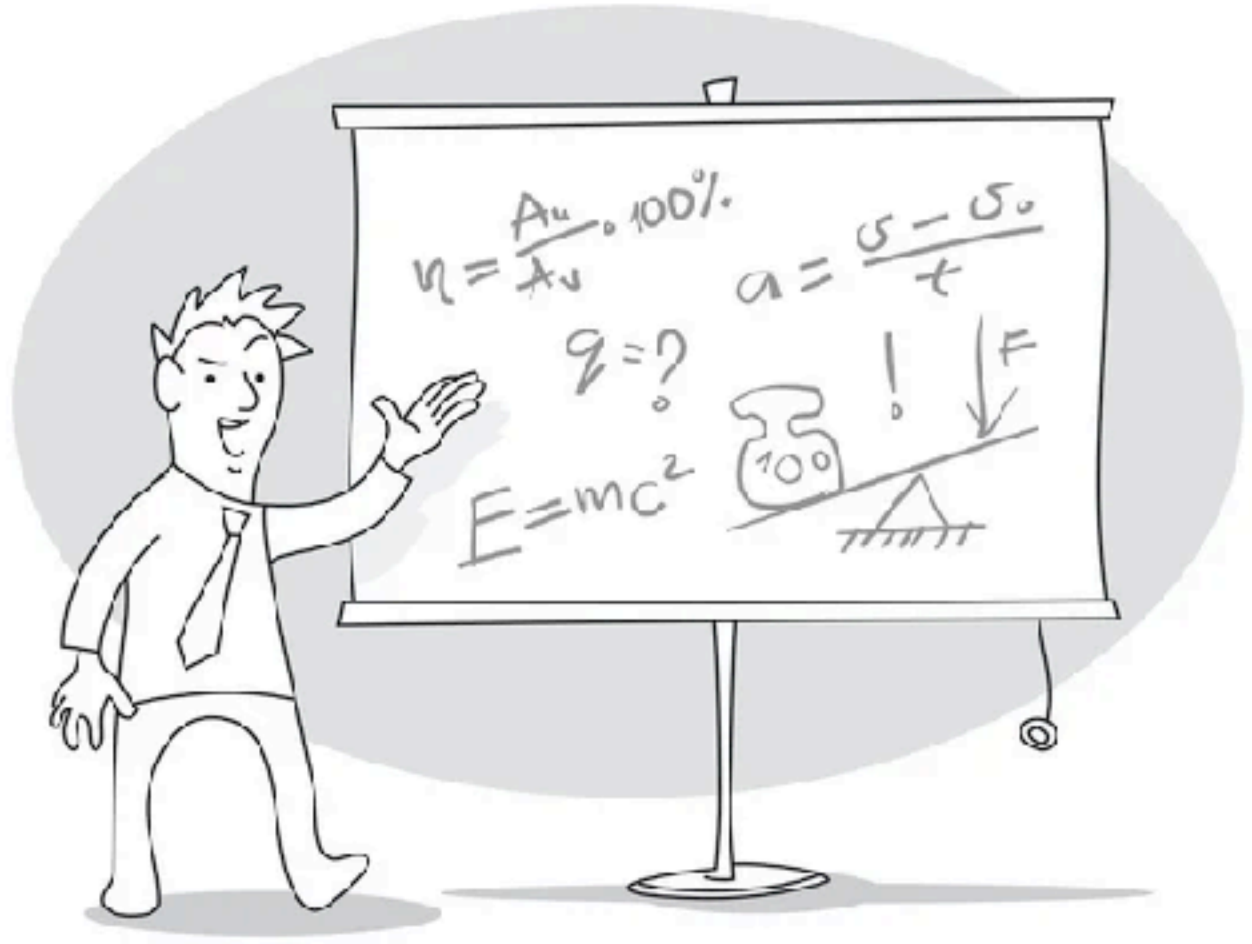
Please ask questions!

- This will be **theory-heavy** so we'll have a couple of breaks for some practical stuff!

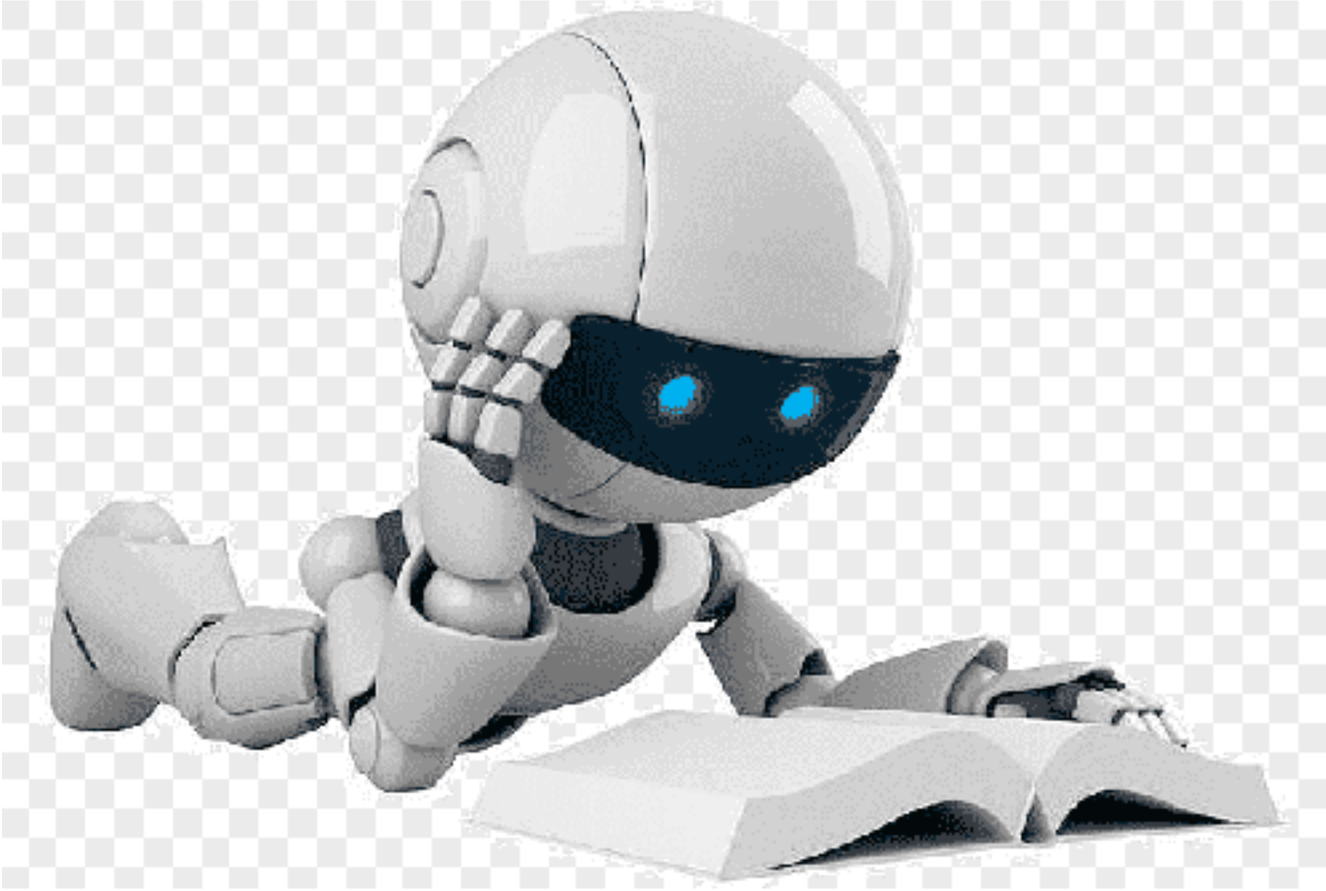
The Big Picture

The Big Picture — I

- Why is theory **useful**?
  - *Robust:* theory is accurate up to its assumptions
  - *Cheap:* no need for expensive N-body simulations
  - *Flexible:* easy to add new physical effects
- Why is theory **limited**?
  - *Failure:* Most theoretical models **break-down** on small scales
  - *Hard*: Modeling higher-point correlations is technically **challenging**
  - *Gaussian:* Analysis requires a known (Gaussian) likelihood
- SBI can ***improve*** on theory, but *only if the simulations are good enough!*



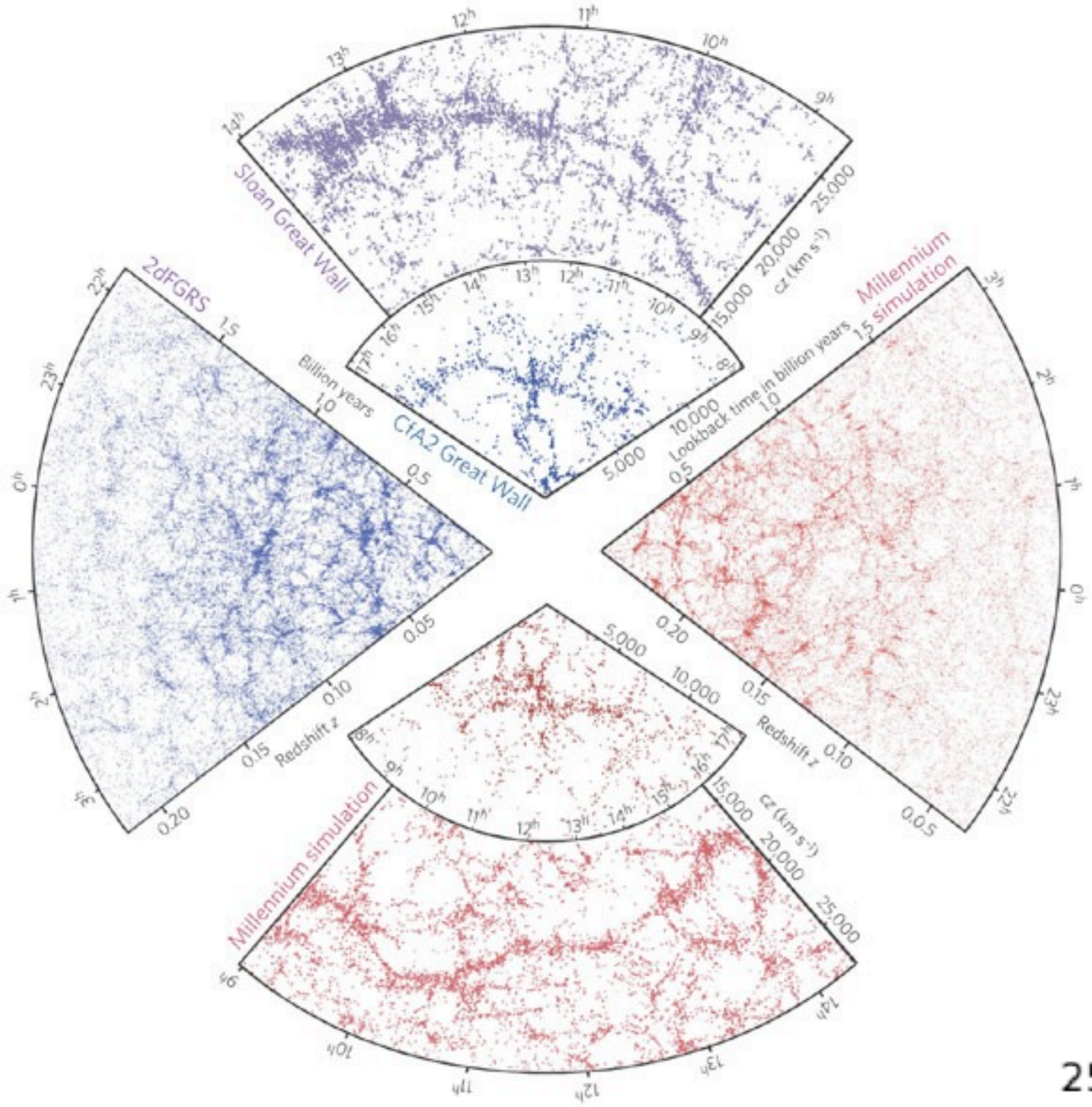
Me



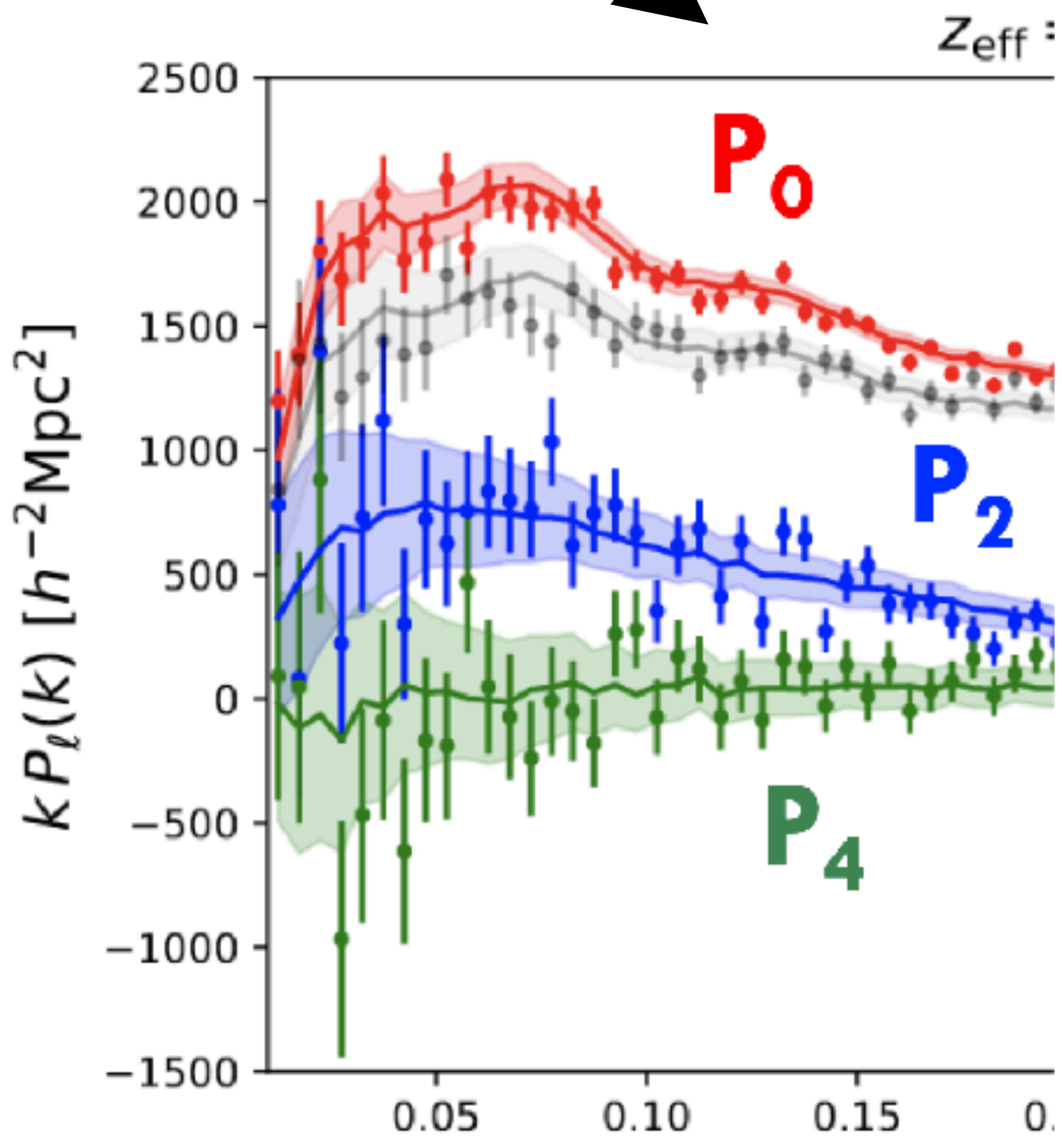
Francois

The Big Picture — II

- **What observables do we have?**
  - **Galaxy surveys** $\rightarrow \delta_g(\mathbf{x}, z), \mathbf{v}(\mathbf{x}, z)$
  - Galaxy shapes $\rightarrow I_{ij}(\mathbf{x}, z)$
  - Weak lensing $\rightarrow \rho_m(\mathbf{x}, z), I_{ij}(\mathbf{x}, z)$
- **How do we usually predict them?**
  - **Two-point functions**: $P(\mathbf{k}), \xi(\mathbf{r})$
  - Three-point functions: $B(\mathbf{k}_1, \mathbf{k}_2), \zeta(\mathbf{r}_1, \mathbf{r}_2)$
  - Cross-correlations, fields, marked spectra, voids, reconstructions, PDFs, *etc.*
- Actually measuring these statistics is a fun problem already!



Observables



Statistics

Ivanov, Philcox

The Big Picture — III

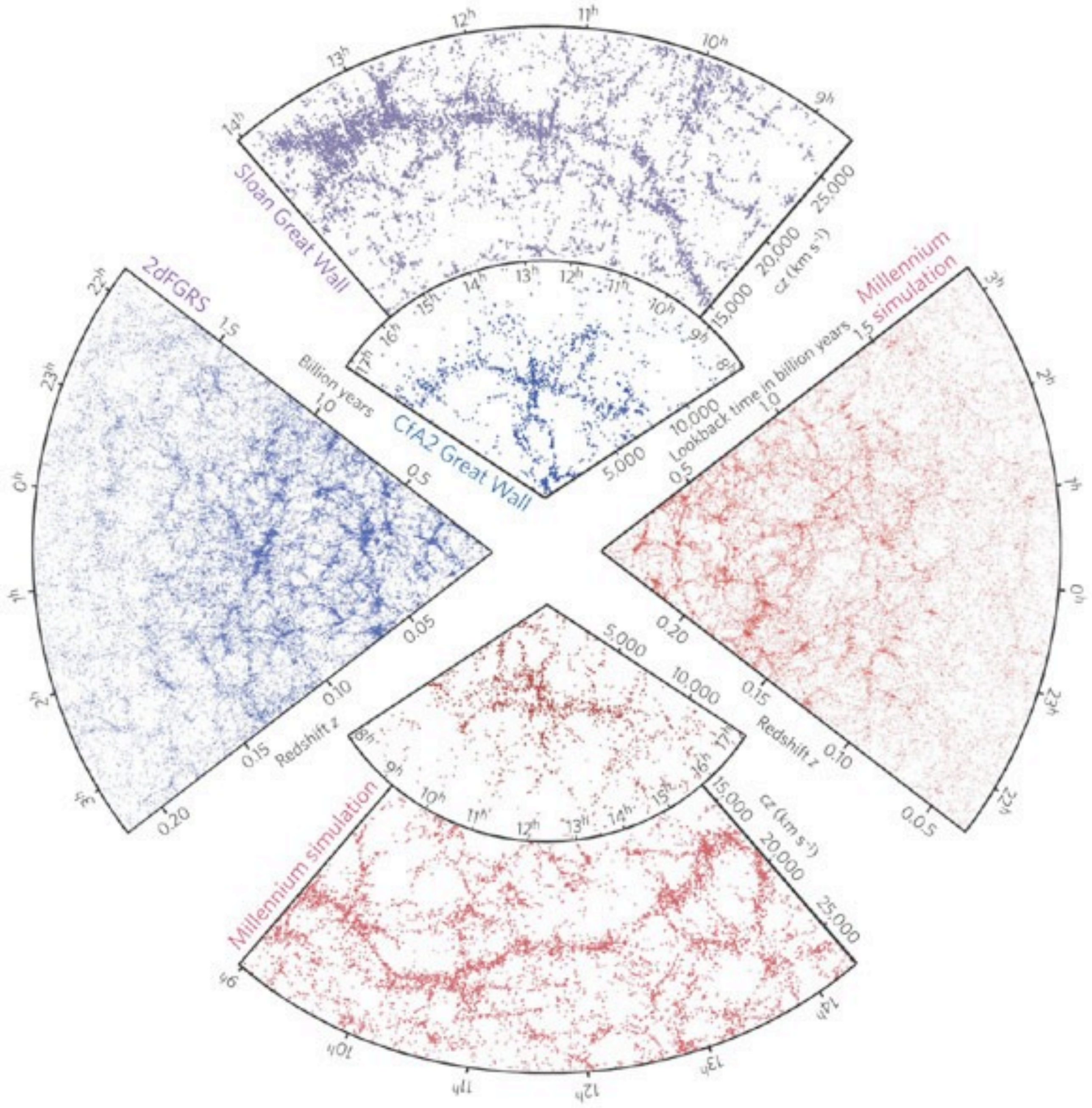
- Assuming Gaussianity, we can form a **likelihood** for a statistic X:

$$-2\log\mathscr{L}(\theta\,|\,\hat{X}) = [\hat{X}-X(\theta)]\cdot\mathrm{cov}_X^{-1}\cdot[\hat{X}-X(\theta)]$$

- Given a **theory model** $X(\theta)$**,** we can infer the underlying parameters θ *(Note: this is "full modeling" not ShapeFit)*

- **Note:** there's two options for treating the covariance:
  - Compute from simulations or analytic theory
  - Drop the Gaussianity assumption altogether [not needed here]

- ***The remainder of these lectures: derive*** $X(\theta)$***!***



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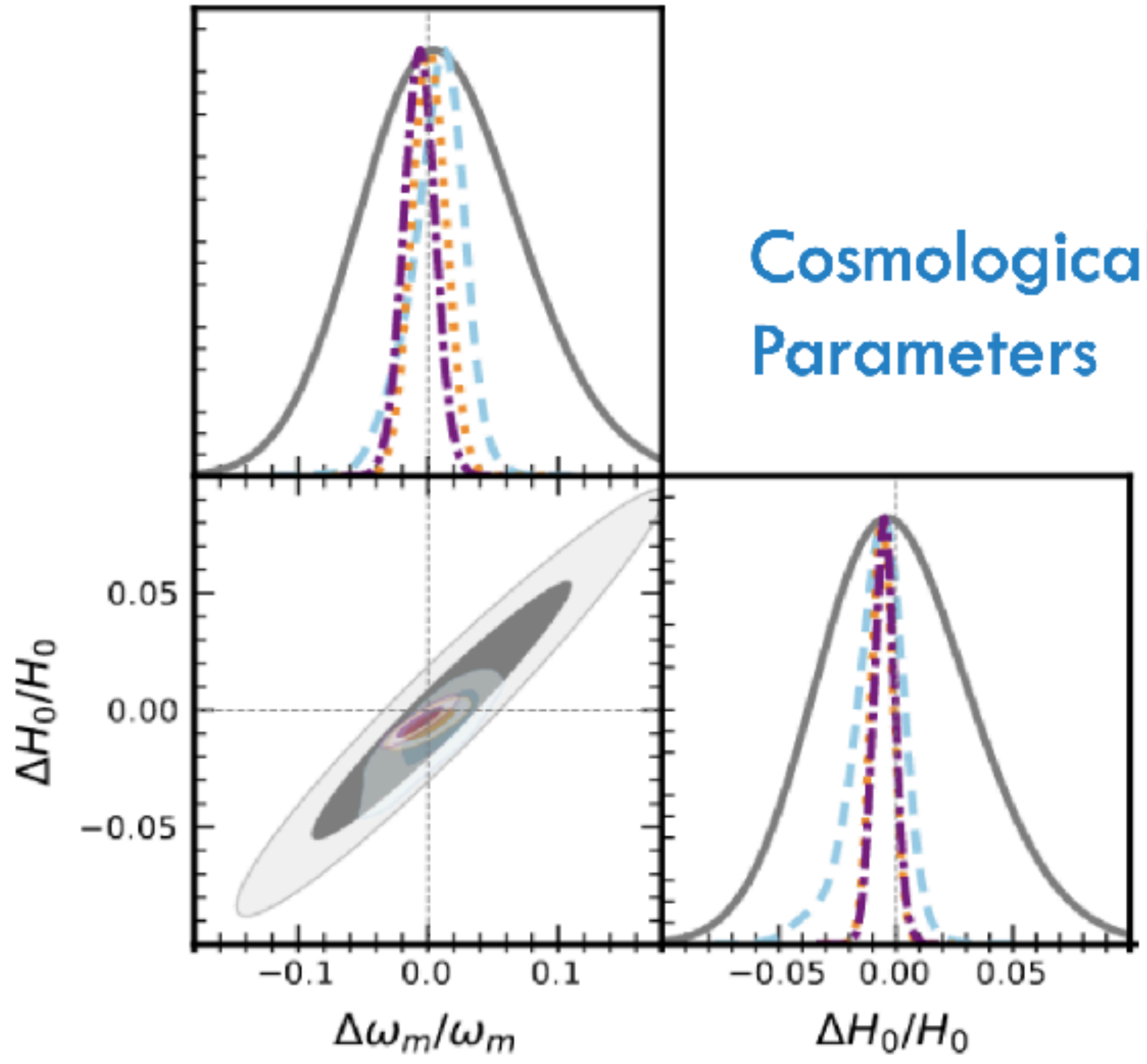


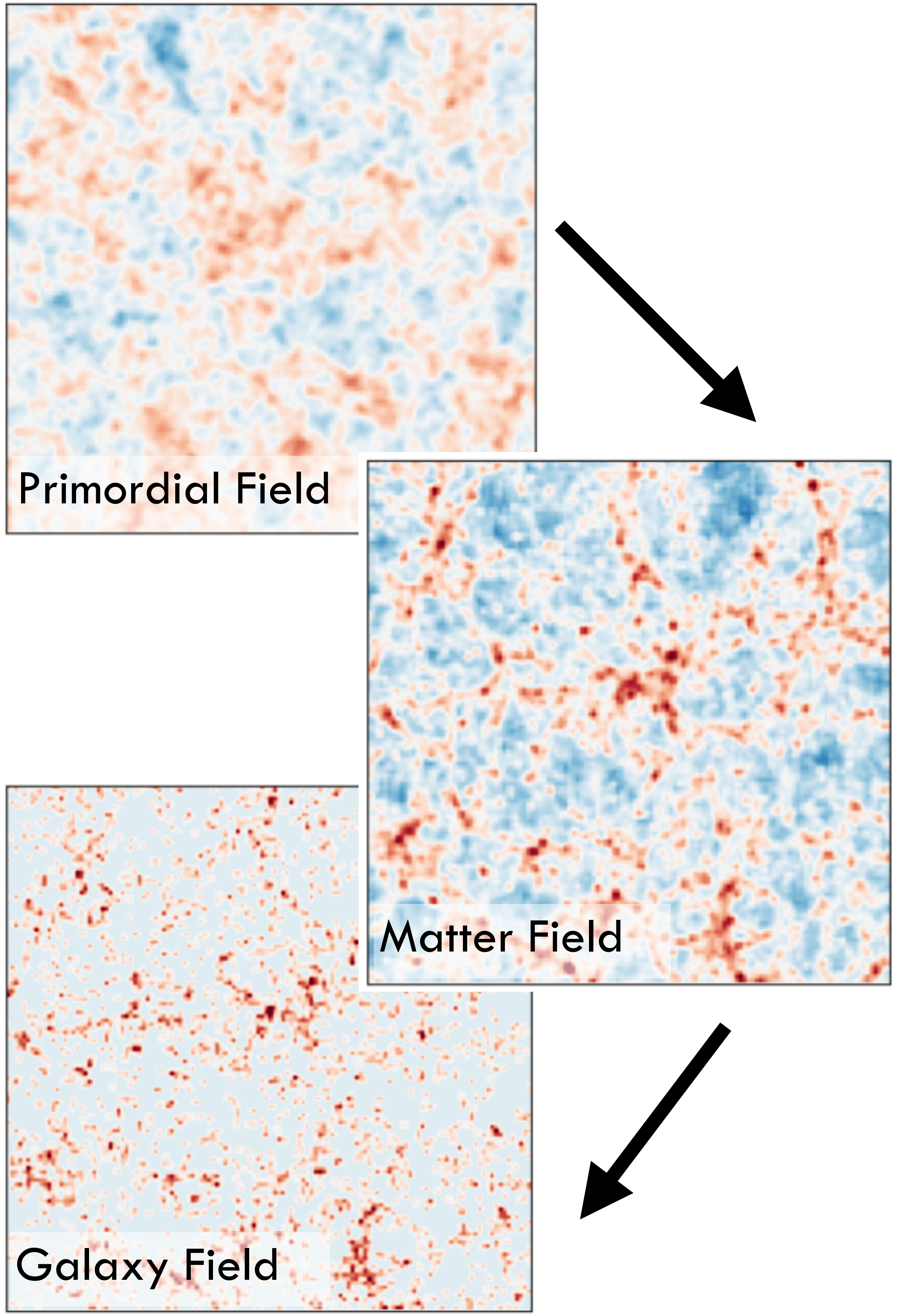
Cosmological Parameters

Modeling Basics

- To model the low-z galaxy distribution we need to model the Universe's:

    1. **Initial Conditions:** $A_s, n_s, f_{\rm NL}, \ldots$
    2. **Composition:** $\omega_b, \omega_c, M_\nu$
    3. **Evolution:** $\Omega_m, M_\nu, w_0, w_a, H_0$
    4. Velocities: $f(z)$
    5. Galaxy-Dark Matter Connection: $b(z), \ldots$

- **First-step**: predict the distribution of **matter in real-space:** $\rho(\mathbf{x}, z)$



Quijote: Villaescusa-Navarro

Standard Perturbation Theory

The Fluid Equations — I

- The late Universe is dominated by dark matter and baryons.

- For a *collisionless* system, neglecting *neutrinos* and baryonic effects, the dark matter-baryon "fluid" must obey:

  1. **Conservation of mass:** $\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$ [Continuity]

(where $\rho, \mathbf{v}, \phi$ are fluid density, velocity, potential, $\mathscr{H} = \dot{a}/a$, and $\dot{x} \equiv \partial x / \partial \tau$. $\boldsymbol{\sigma} = \sigma_{ij}$ is the stress tensor.)

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

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     Collisionless!

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   Collisionless!
3. **Conservation of energy:** $\nabla^2 \phi = 4\pi G \delta\rho$ [Poisson/Einstein]

(where $\rho, \mathbf{v}, \phi$ are fluid density, velocity, potential, $\mathscr{H} = \dot{a}/a$, and $\dot{x} \equiv \partial x / \partial \tau$. $\boldsymbol{\sigma} = \sigma_{ij}$ is the stress tensor.)

- These are the (Eulerian) **ideal fluid** = **Collisionless Boltzmann Moments** = **Vlasov** equations
- They are **ODEs** specifying the evolution (with **ICs** from inflation) — the same as those used in N-body codes!

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

The Fluid Equations — II

- There are *many* ways to extend these equations. These include
  - Dark-matter — dark-energy interactions
  - Dark-matter — baryon scattering
  - Fifth forces
  - Isocurvature modes
  - Radiation physics
  - Warm dark matter
- The basic implementation assumes an ***ideal*** fluid with $\sigma_{ij} = 0$ [we'll return to this later]

Continuity: $\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0$
Euler: $\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} = -\mathscr{H}\mathbf{v} - \nabla \phi$
Poisson: : $\nabla^2 \phi = 4\pi G \delta\rho$

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Linearized Solutions — I

- Define a **perturbation variable:** $\delta = (\rho - \bar{\rho})/\bar{\rho}$ which is (hopefully) small
- Compute fluid equations at linear order:
  - Continuity: $\dot{\delta}_1 + \nabla \cdot \mathbf{v}_1 = 0$
  - Euler: $\dot{\mathbf{v}}_1 = -\mathscr{H}\mathbf{v}_1 - \nabla\phi_1$
  - Poisson: : $\nabla^2\phi_1 = \frac{3}{2}\mathscr{H}^2\Omega_m\delta_1$

Continuity: $\dot{\delta} + \nabla \cdot [(1+\delta)\mathbf{v})] = 0$
Euler: $\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla\mathbf{v} = -\mathscr{H}\mathbf{v} - \nabla\phi$
Poisson: : $\nabla^2\phi = \frac{3}{2}\mathscr{H}^2\Omega_m\delta$

- To solve, we introduce the velocity **divergence** $\theta = \nabla \cdot \mathbf{v}$ and **vorticity** $\omega = \nabla \times \mathbf{v}$:

$$\ddot{\delta}_1 + \mathscr{H}\dot{\delta}_1 - \frac{3}{2}\mathscr{H}^2\Omega_m\delta_1 = 0, \qquad \theta_1 = -\dot{\delta}_1, \qquad \dot{\omega}_1 + \mathscr{H}\omega_1 = 0$$

- This is a second order ODE governing the time-dependence of δ_1
- At linear order, the vorticity **decays quickly** $(\omega_1 \sim a^{-1})$ so we can ignore it!

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Linearized Solutions — II

$$\theta_1 = -\dot{\delta}_1, \qquad \ddot{\delta}_1 + \mathscr{H}\dot{\delta}_1 - \frac{3}{2}\mathscr{H}^2\Omega_m\delta_1 = 0$$

- We can assume a **separable solution** with spatial parts set by the initial conditions:

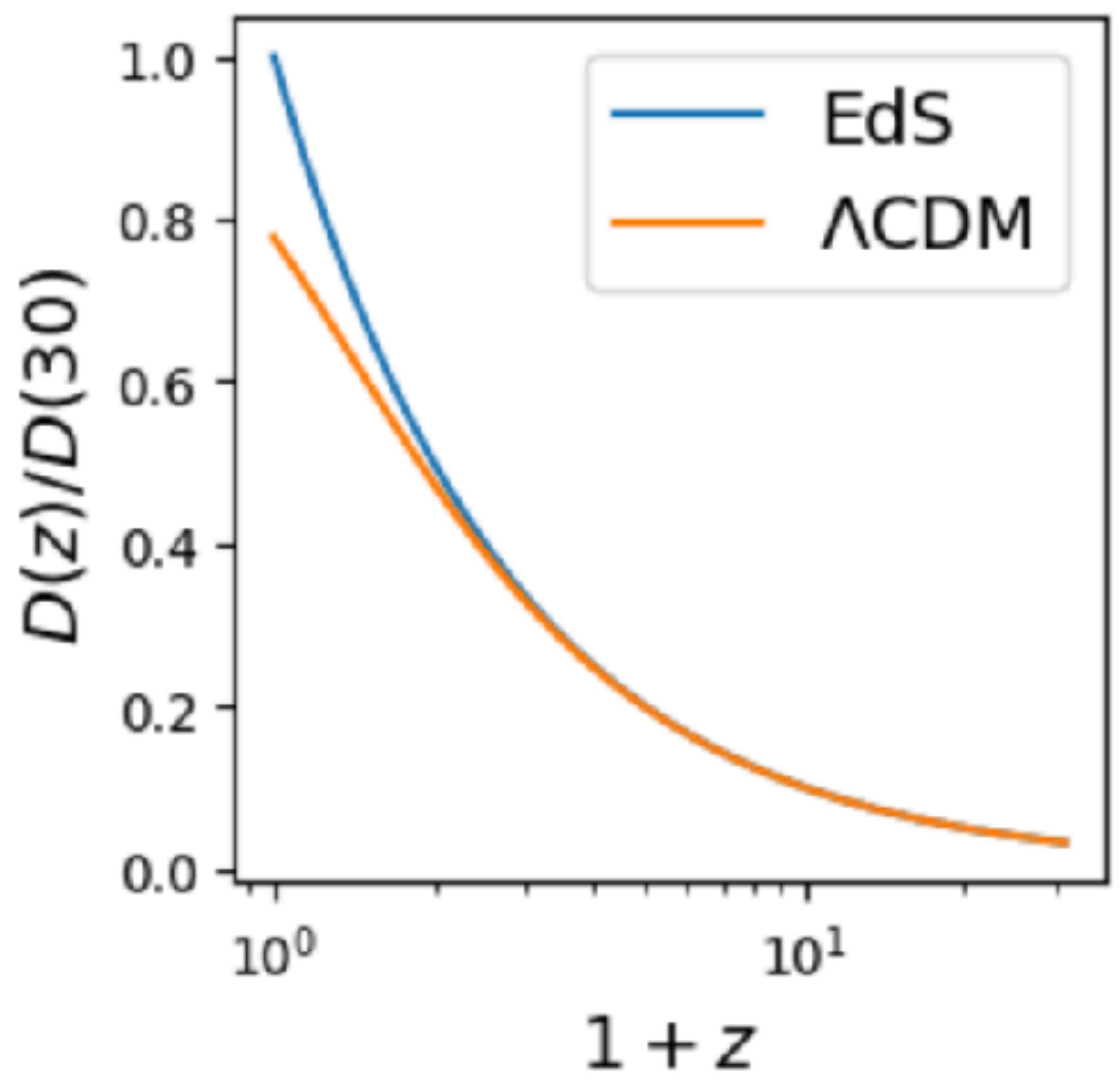
$$\delta_1(\mathbf{x}, z) = D(z)\delta_L(\mathbf{x}), \qquad \theta_1(\mathbf{x}, z) = -\mathscr{H}(z)f(z)D(z)\delta_L(\mathbf{x})$$

specializing to the ***growing mode*** solution (since the decaying mode quickly becomes negligible, $D_+ \sim a, \quad D_- \sim a^{-3/2}$)

- The growth rate and its derivative $f(z) = d\log D/d\log a$ are determined by the ODE

$$\ddot{D} + \mathscr{H}\dot{D} - \frac{3}{2}\mathscr{H}^2\Omega_m D = 0$$

- In an Einstein de Sitter Universe: $D(z) = a(z)$ [since $\Omega_m = 1, \quad a \sim \tau^2$]



Good agreement until Λ kicks in!

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Linearized Solutions — III

$$\delta_1(\mathbf{k}, z) = D(z)\delta_L(\mathbf{k}), \qquad \theta_1(\mathbf{k}, z) = -\mathcal{H}(z)f(z)D(z)\delta_L(\mathbf{k})$$

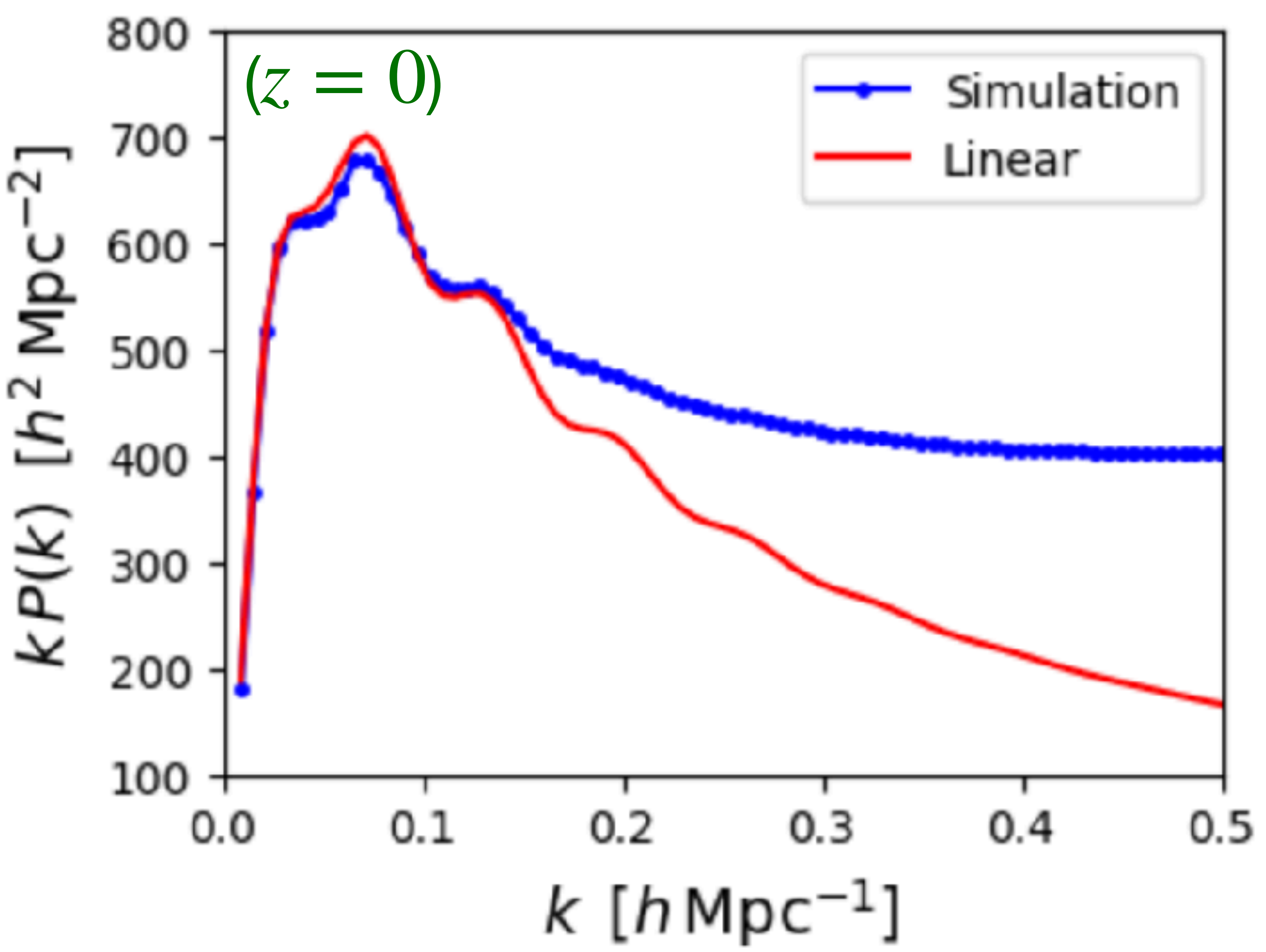
- From the **density fields** we can get the **statistics**:

$$P_{11}(k, z) = \langle \delta_1(\mathbf{k}, z)\delta_1(-\mathbf{k}, z)\rangle = D^2(z)P_L(k)$$

$$B_{111}(\mathbf{k}_1, \mathbf{k}_2, z) = \langle \delta_1(\mathbf{k}_1, z)\delta_1(\mathbf{k}_2, z)\delta_1(-\mathbf{k}_1 - \mathbf{k}_2, z)\rangle = D^3(z)B_L(\mathbf{k}_1, \mathbf{k}_2, z) = 0$$

- We relate the **late-time statistics** to the **growth rate** (⇒ evolution parameters) and the **primordial** correlators (⇒ inflation)

- P_{11} is also given by CAMB/CLASS, so we haven't done anything new yet…



Comparison to Quijote simulations

P_{11} does well on large scales only!

Bernardeau, Gaztanaga, Fry, Scoccimarro, Villaescusa-Navarro, …

Standard Perturbation Theory — I

- We can proceed by solving the equations **iteratively**
- Here, we will **ignore** the vorticity since:
  1. Primordial vorticity decays as a^{-1}
  2. It is sourced only from **small scales** and by σ_{ij}
- The resulting equations are a little messy:

$$\dot{\delta} + \theta = -\delta\theta - (\partial_i\delta)(\partial_i\nabla^{-2}\theta)$$

$$\dot{\theta} + \mathscr{H}\theta + \frac{3}{2}\mathscr{H}^2\Omega_m\delta = -(\partial_i\partial_j\nabla^{-2}\theta)^2 - (\partial_j\theta)(\partial_j\nabla^{-2}\theta)$$

- To solve them, we expand order-by-order in δ, θ, **assuming** δ_1 **is small!**

$$\delta(\mathbf{x}, z) = \sum_{n=1}^{\infty}\delta_n(\mathbf{x}, z), \qquad \theta(\mathbf{x}, z) = \sum_{n=1}^{\infty}\theta_n(\mathbf{x}, z) \qquad \delta_n, \theta_n = \mathcal{O}[(\delta_1)^n]$$

Continuity: $\dot{\delta} + \nabla\cdot[(1+\delta)\mathbf{v})] = 0$
Euler: $\dot{\mathbf{v}} + \mathbf{v}\cdot\nabla\mathbf{v} = -\mathscr{H}\mathbf{v} - \nabla\phi$
Poisson: : $\nabla^2\phi = \frac{3}{2}\mathscr{H}^2\Omega_m\delta$

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Standard Perturbation Theory — II

- At order n:

$$\dot{\delta}_n + \theta_n = -\sum_{m=1}^{n-1}\left[\delta_m\theta_{n-m} - (\partial_i\delta_m)(\partial_i\nabla^{-2}\theta_{n-m})\right]$$

$$\dot{\theta}_n + \mathscr{H}\theta_n + \tfrac{3}{2}\mathscr{H}^2\Omega_m\delta_n = -\sum_{m=1}^{n-1}\left[(\partial_i\partial_j\nabla^{-2}\theta_m)(\partial_i\partial_j\nabla^{-2}\theta_{n-m}) + (\partial_j\theta_m)(\partial_j\nabla^{-2}\theta_{n-m})\right]$$

- This is a recursive series: higher-order terms are generated by lower-order on the RHS — **it is a Taylor series in $k/k_{\rm NL}$!**

- Insert **separable ansatzes** for δ_n, θ_n:

$$\delta_n(\mathbf{k},z) = D_n(z)\int\frac{d\mathbf{p}_1}{(2\pi)^3}\cdots\frac{d\mathbf{p}_n}{(2\pi)^3}F_n(\mathbf{p}_1,\cdots,\mathbf{p}_n)\delta_L(\mathbf{p}_1)\cdots\delta_L(\mathbf{p}_n)\delta_{\rm D}(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{k})$$

$$\theta_n(\mathbf{k},z) = -\mathscr{H}(z)f(z)D_n'(z)\int\frac{d\mathbf{p}_1}{(2\pi)^3}\cdots\frac{d\mathbf{p}_n}{(2\pi)^3}G_n(\mathbf{p}_1,\cdots,\mathbf{p}_n)\delta_L(\mathbf{p}_1)\cdots\delta_L(\mathbf{p}_n)\delta_{\rm D}(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{k})$$

We usually assume $D_n(z) = D_n'(z) = D^n(z)$ which is true in Einstein-de-Sitter and an excellent approximation in ΛCDM.

- All of the physics dependence enters in the **kernels** F_n, G_n

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Standard Perturbation Theory — III

- Kernel recursion relations are found by inserting the ansatzes into the fluid equations:

$$F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)=\sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[(2n+1)\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)$$
$$+2\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big], \qquad (43)$$

$$\alpha(\mathbf{k}_1,\mathbf{k}_2)\equiv\frac{\mathbf{k}_{12}\cdot\mathbf{k}_1}{k_1^2}$$

$$G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)=\sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[3\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)$$
$$+2n\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big], \qquad (44)$$

$$\beta(\mathbf{k}_1,\mathbf{k}_2)\equiv\frac{k_{12}^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{2k_1^2k_2^2}$$

- These are just simple functions of the **momenta** / **wavenumbers** e.g.,

$$F_2(\mathbf{q}_1,\mathbf{q}_2)=\frac{5}{7}+\frac{1}{2}\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right)+\frac{2}{7}\frac{(\mathbf{q}_1\cdot\mathbf{q}_2)^2}{q_1^2q_2^2} \qquad G_2(\mathbf{q}_1,\mathbf{q}_2)=\frac{3}{7}+\frac{1}{2}\frac{\mathbf{q}_1\cdot\mathbf{q}_2}{q_1q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right)+\frac{4}{7}\frac{(\mathbf{q}_1\cdot\mathbf{q}_2)^2}{q_1^2q_2^2}$$

- **This is it**; we can now compute *standard* perturbation theory to any order!

(up to vorticity terms, stress terms, baryon effects, etc. etc.)

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Standard Perturbation Theory — IV

- Let's compute the **power spectrum** at next-to-leading order!

$$\delta(\mathbf{k},z) = D(z)\delta_L(\mathbf{k}) + D^2(z)\int_{\mathbf{p}} F_2(\mathbf{p},\mathbf{k}-\mathbf{p})\delta_L(\mathbf{p})\delta_L(\mathbf{k}-\mathbf{p}) + D^3(z)\int_{\mathbf{p},\mathbf{p}'} F_3(\mathbf{p},\mathbf{p}',\mathbf{k}-\mathbf{p}-\mathbf{p}')\delta_L(\mathbf{p})\delta_L(\mathbf{p}')\delta_L(\mathbf{k}-\mathbf{p}-\mathbf{p}')\cdots$$

- Linear theory generates $\mathcal{O}(2)$ terms, so next-to-leading needs $\mathcal{O}(4)$ [$\mathcal{O}(3)$ vanishes, since $\langle\delta_L^n\rangle = 0$ for odd n]

$$P_{\mathrm{SPT}}(k,z) = P_{11}(k,z) + P_{22}(k,z) + 2P_{13}(k,z) + \cdots$$

- The higher-order involve one **loop integral**:

$$P_{22}(k,z) = D^4(z)\int_{\mathbf{p},\mathbf{p}'} F_2(\mathbf{p},\mathbf{k}-\mathbf{p})F_2(\mathbf{p}',-\mathbf{k}-\mathbf{p}')\langle\delta_L(\mathbf{p})\delta_L(\mathbf{p}')\delta_L(\mathbf{k}-\mathbf{p})\delta_L(\mathbf{k}-\mathbf{p}')\rangle = 2D^4(z)\int_{\mathbf{p}} F_2(\mathbf{p},\mathbf{k}-\mathbf{p})^2 P_L(p)P_L(|\mathbf{k}-\mathbf{p})$$

$$P_{13}(k,z) = D^4(z)\int_{\mathbf{p},\mathbf{p}'} F_3(\mathbf{p},\mathbf{p}',\mathbf{k}-\mathbf{p}-\mathbf{p}')\langle\delta_L(\mathbf{p})\delta_L(\mathbf{p}')\delta_L(\mathbf{k}-\mathbf{p}-\mathbf{p}')\delta_L(-\mathbf{k})\rangle = 3D^4(z)\int_{\mathbf{p}} F_3(\mathbf{p},-\mathbf{p},\mathbf{k})P_L(p)P_L(k)$$

- Note that the integrals involve **all** modes, not just those at large-scales (small $\mathbf{p}$)

Bernardeau, Gaztanaga, Fry, Scoccimarro, …

Standard Perturbation Theory — V

- We can compute the **bispectrum** similarly!

$$B_{\rm SPT}(\mathbf{k}_1, \mathbf{k}_2, z) = B_{211}(\mathbf{k}_1, \mathbf{k}_2, z) + \cdots$$

- This is produced <u>only</u> by non-linear physics, but starts at **tree-level** (no loop integrals!)

$$B_{211}(\mathbf{k}_1, \mathbf{k}_2, z) = D^4(z)\int_{\mathbf{p}} F_2(\mathbf{p}, \mathbf{k}_1 - \mathbf{p})\delta_L(\mathbf{p})\delta_L(\mathbf{k}_1 - \mathbf{p})\delta_L(\mathbf{k}_2)\delta_L(-\mathbf{k}_1 - \mathbf{k}_2) + 2\text{ perm.} = 2D^4(z)F_2(\mathbf{k}_1, \mathbf{k}_2)P_L(k_1)P_L(k_2) + 2\text{ perm.}$$

- It is straightforward to extend to **higher-loops** (capturing more small-scale physics) and **higher-order statistics**

- **However**, the computation gets expensive!
  - $n-$loop requires $(3n-1)-$dimensional integration for each bin of interest
  - There are **efficient** numerical schemes for **1**-loop [FFTLog, COBRA, Propagators] but higher-order is hard!

Bernardeau, Gaztanaga, Fry, Scoccimarro, Simonovic, Anastasiou, Braganca, Senatore, Zheng, Zaldarriaga, McEwen, Fang, …

Standard Perturbation Theory — VI

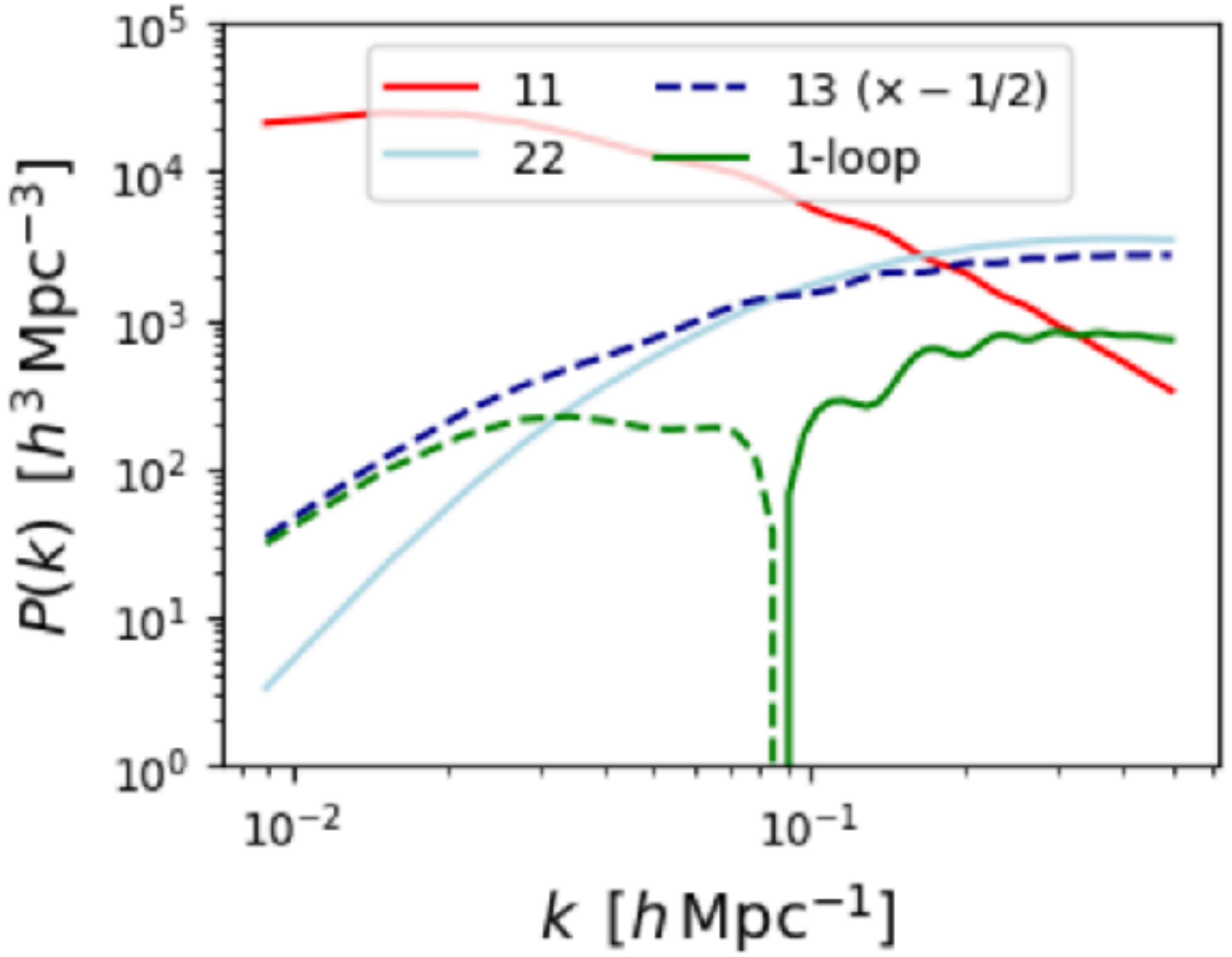
- Let's compare the results to data.

- For this, we can rewrite the 1-loop results as low-dimensional integrals:

$$P_{22}(k,z) = \frac{k^3}{2\pi^2} D^4(z) \int_0^\infty r^2 dr \int_{-1}^{1} d\mu \left(\frac{7\mu + r(3-10\mu^2)}{14r(1+r^2-2r\mu)}\right)^2 P_L\left(k\sqrt{1+r^2-2r\mu}\right) P_L(kr)$$

$$2P_{13}(k,z) = \frac{k^3}{252(2\pi)^2} P_L(k) D^4(z) \int_0^\infty r^2 dr \left[\frac{12}{r^4} - \frac{158}{r^2} + 100 - 42r^2 + \frac{3}{r^5}(7r^2+2)(r^2-1)^3 \log\frac{r+1}{r-1}\right] P_L(kr)$$

P_{22} and P_{13} *almost* cancel!



Bernardeau, Gaztanaga, Fry, Scoccimarro, Baldauf, Villaescusa-Navarro…

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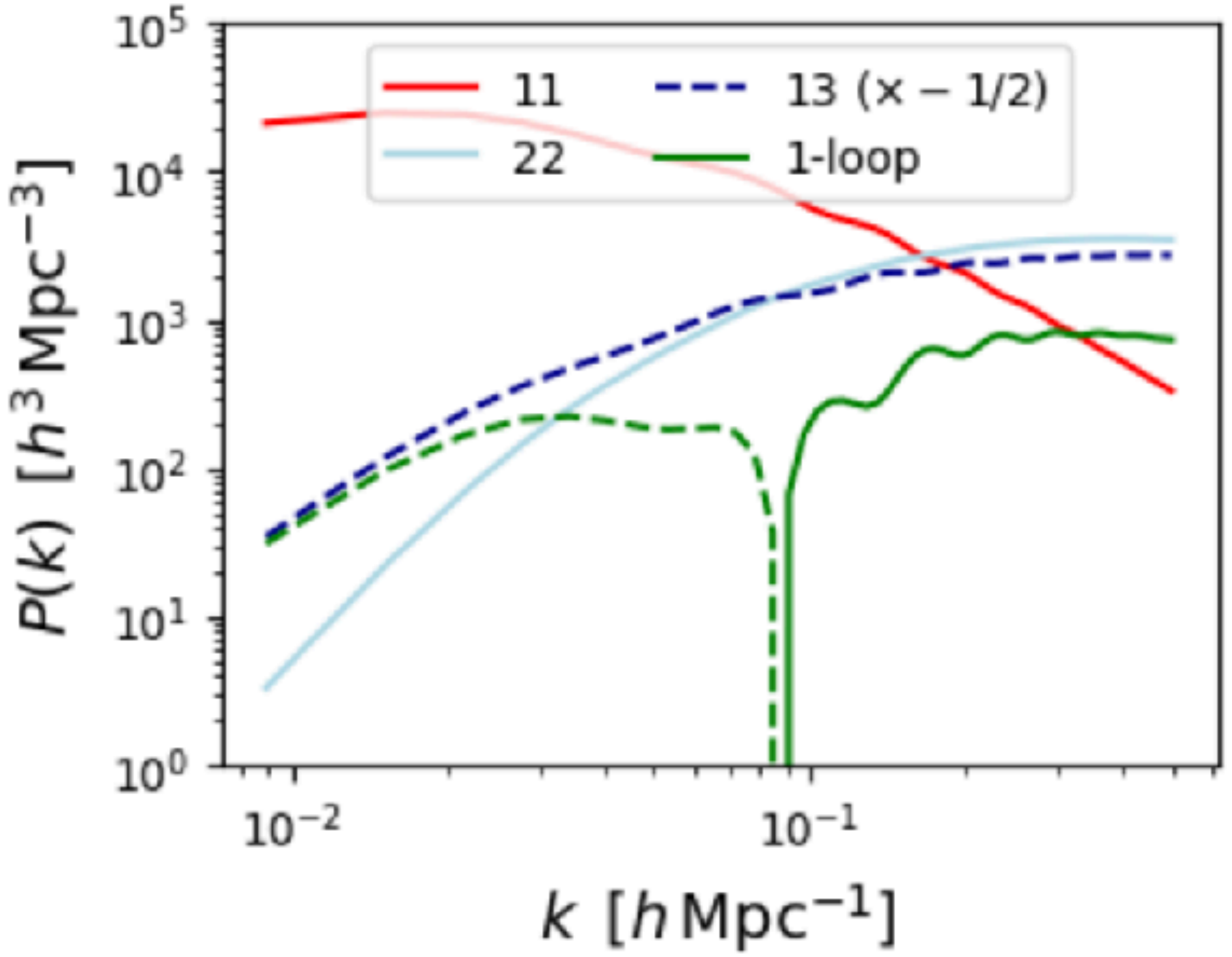
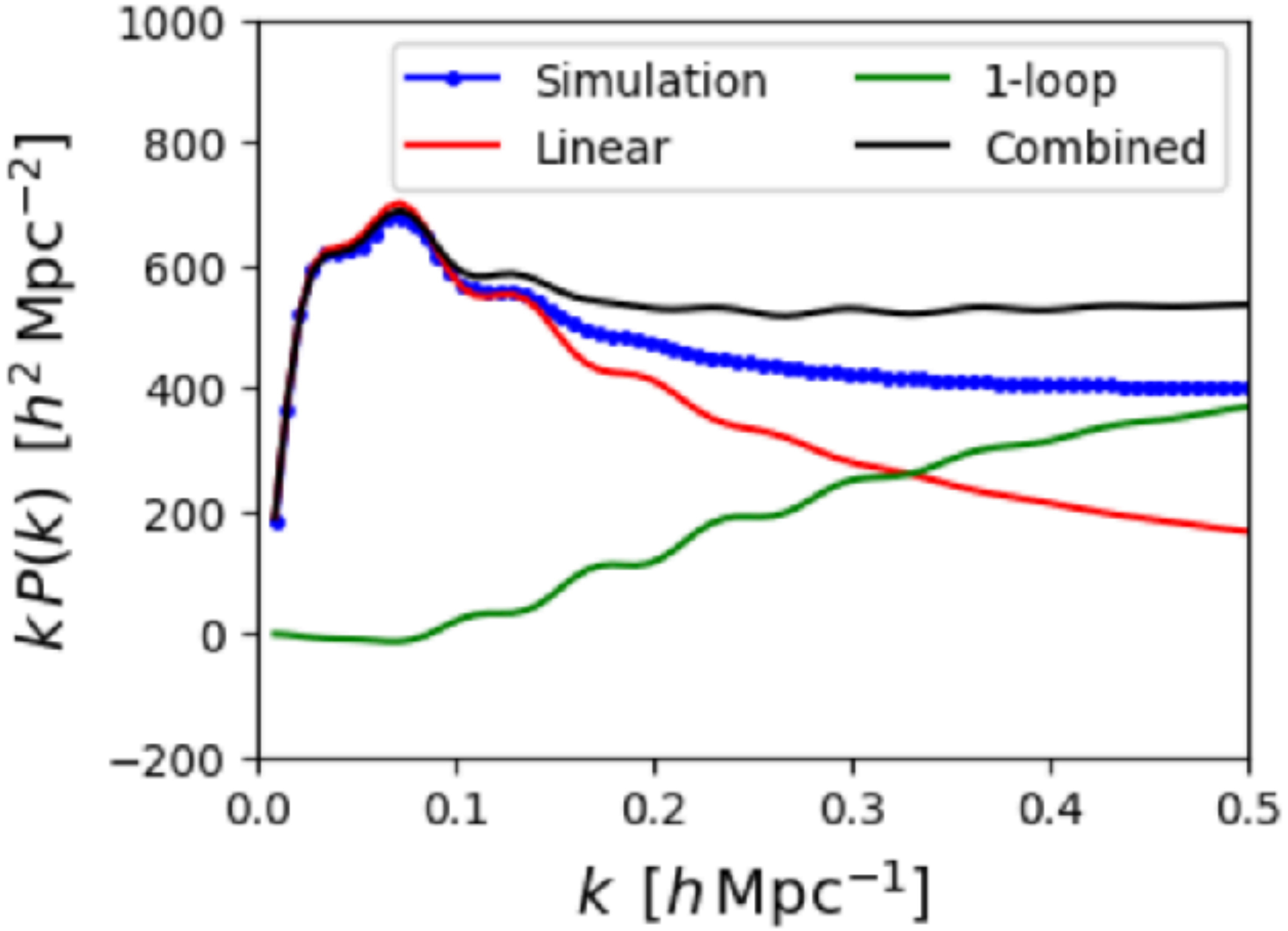




1-loop is no better than linear theory!!

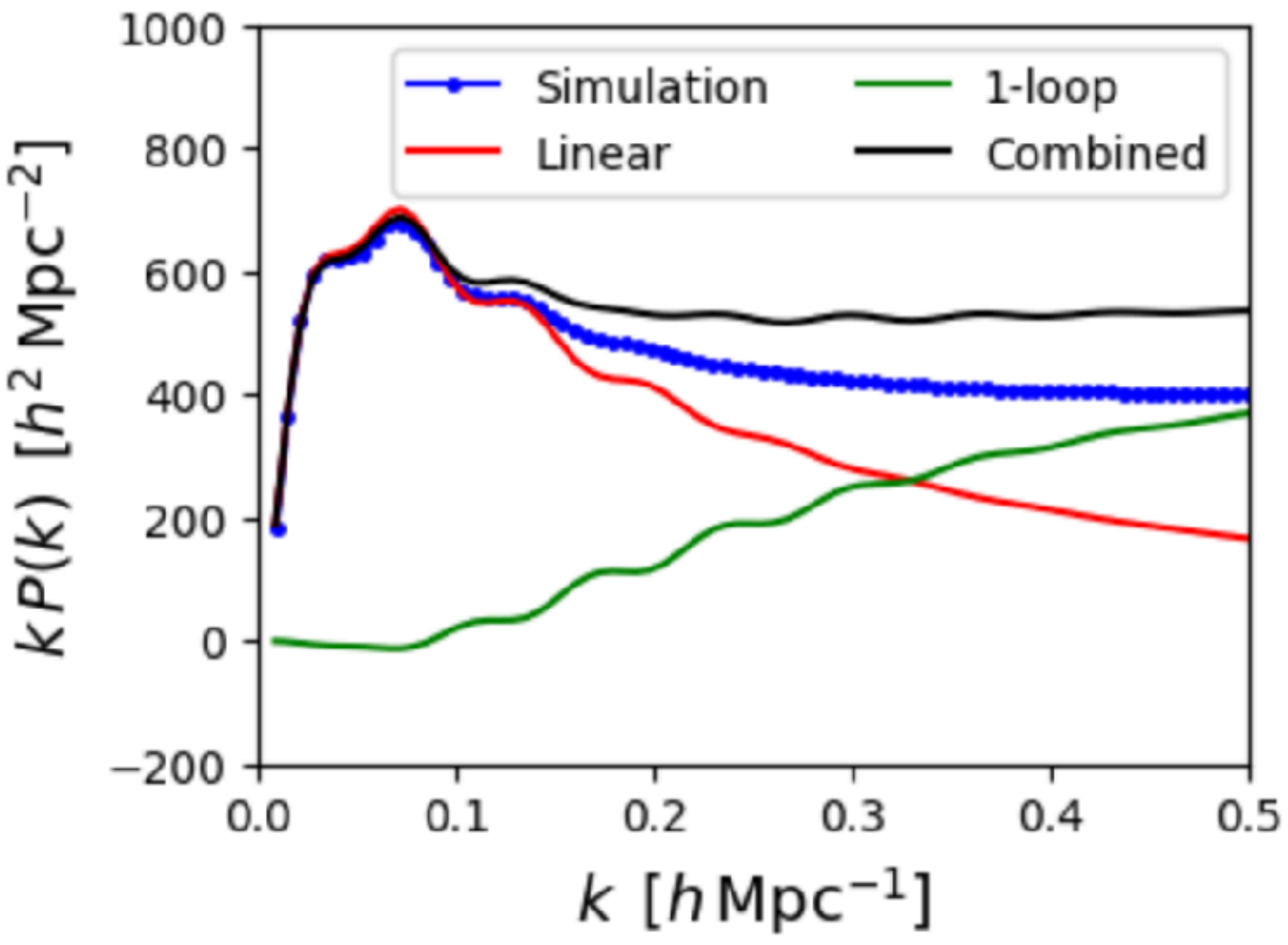
Bernardeau, Gaztanaga, Fry, Scoccimarro, Baldauf, Villaescusa-Navarro…

The Failure of SPT — I

- What went wrong?

    1. **SPT does not give the right answer**
        - Adding the 1-loop corrections does **not** improve the fit to simulations!
        - The fit is **not** improved with the 2-loop corrections either!
        - (Many bells and whistles have been added to try and fix this, e.g. RPT, GRPT, etc.)

    2. **SPT is not general**
        - The loop integrals can be **divergent**. Let's set $P(k) = k^n$.
        - The loop integrals converge in the UV ($k \ll p$) only for $n < -1$
        - The loop integrals converge in the IR ($k \gg p$) only for $n > -3$



The theory *blows up* for many choices of $P_L(k)$!

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

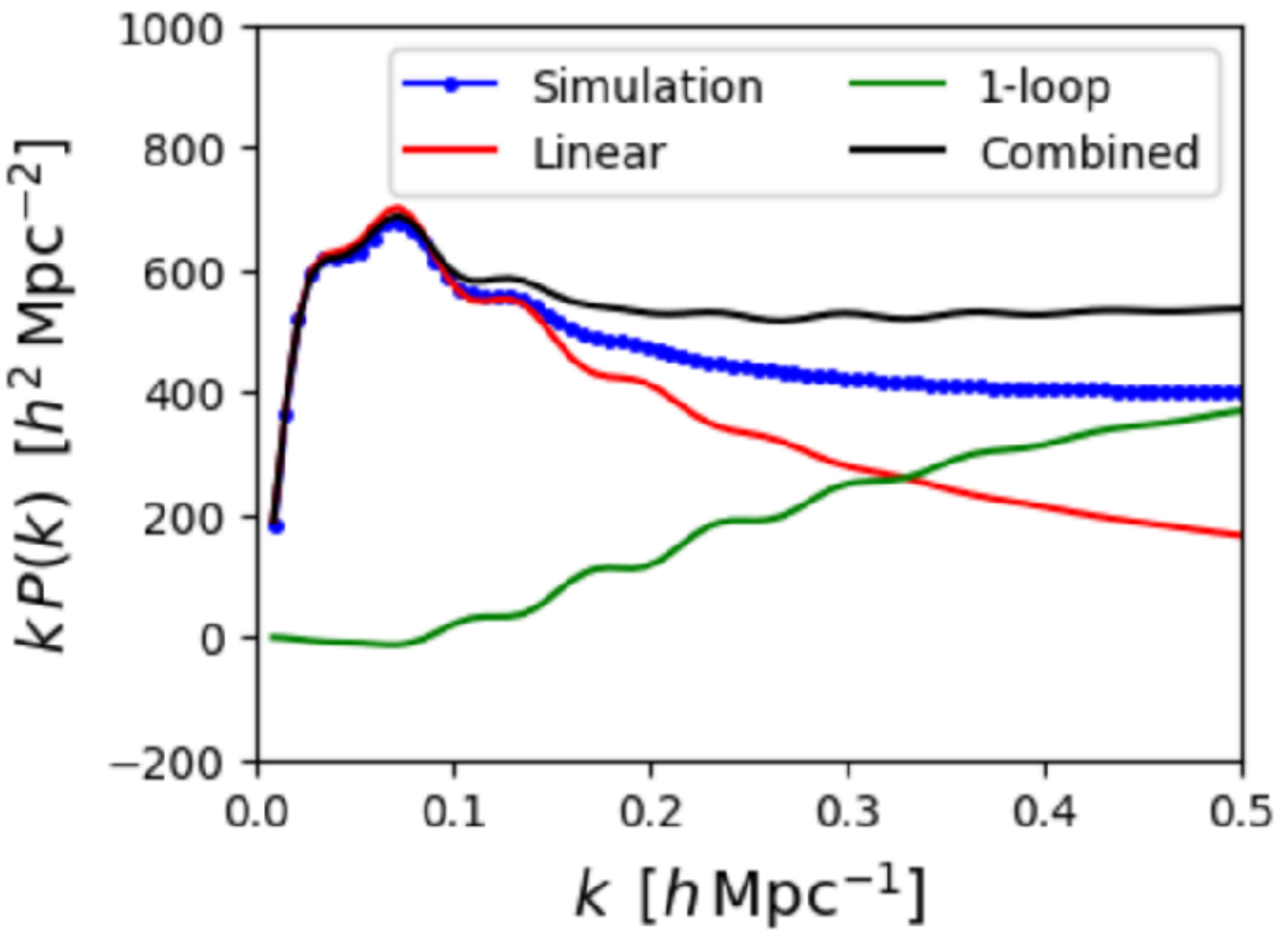
The Failure of SPT — II

- What went wrong?

3. **The expansion is *not* well-defined!**
   - The fluid equations expand in the *small-parameter* δ
   - $\mathrm{rms}(\delta) \equiv \sigma$ can be arbitrarily large!
4. **We are integrating over UV modes in the non-linear regime!**

$$P_{13}(k,z) = 3D^4(z)P_L(k)\int^{\infty}\frac{d\mathbf{p}}{(2\pi)^3}F_3(\mathbf{p},-\mathbf{p},\mathbf{k})P_L(p)$$

   - The theory is not well controlled!
5. **We have a assumed an ideal fluid**
   - Is this valid on small scales??



Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Effective Field Theory

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg, Simonovic, Ivanov, Chen, White, Philcox, d'Amico, Zhang, Donath, Colas, Vlah, de Belsunce, Mirbabayi, Baldauf, Foreman, Angulo, Perko, Green, Lewandowski, Aviles, …

Introducing the Effective Field Theory

- SPT fails since it solves the **wrong equations** expanding in the **wrong variable**
- How do we fix this?
  1. Work with the **non-ideal fluid equations** (keeping σ_{ij})
  2. Expand in terms of the **smoothed** density, δ_Λ. Define $\delta_\Lambda(\mathbf{k}, z) = W_\Lambda(k)\delta(\mathbf{k}, z)$ to **coarse-grain** the theory.
- This **cuts off** any contributions from $k > \Lambda \sim k_{\rm NL}$ making the theory **well-controlled**!
- Let's return to the fluid equations, including the stress, $\tau_{ij} = \rho\sigma_{ij}$:

$$\dot{\delta} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$

$$\dot{\mathbf{v}} + \mathscr{H}\mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla\phi - \frac{1}{\rho}\nabla\boldsymbol{\tau}$$

$$\nabla^2\phi = \frac{3}{2}\mathscr{H}^2\Omega_m\delta$$

- The next step is the **smooth** the equations, so we have equations for $\delta_\Lambda, \mathbf{v}_\Lambda, \phi_\Lambda$

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Matter Effective Field Theory — II

- Smoothing is easy to apply to linear terms: $\delta \to \delta_\Lambda, \phi \to \phi_\Lambda, \ldots$. We find:

$$\dot{\delta}_\Lambda + \nabla \cdot [(1+\delta_\Lambda)\mathbf{v}_\Lambda] = 0$$
$$\dot{\mathbf{v}}_\Lambda + \mathscr{H}\mathbf{v}_\Lambda + \frac{1}{\rho_\Lambda}[\rho\mathbf{v}\cdot\nabla\mathbf{v}]_\Lambda = -\frac{1}{\rho_\Lambda}[\rho\nabla\phi]_\Lambda - \frac{1}{\rho_\Lambda}\nabla\boldsymbol{\tau}_\Lambda$$
$$\nabla^2\phi_\Lambda = \frac{3}{2}\mathscr{H}^2\Omega_m\delta_\Lambda$$

- This is **identical** to the usual fluid equations, except for the terms in **red**
- Smoothing the **product** of two fields isn't equivalent to smoothing each: $[XY]_\Lambda \neq X_\Lambda Y_\Lambda$
  - *In other words: the product of **small-scale** features can give rise to **large-scale** behavior!*
- We can **collect** these small-scale additions into a new **stress-tensor** $\tau_\Lambda^{\mathrm{UV}}$

$$\dot{\mathbf{v}}_\Lambda + \mathscr{H}\mathbf{v}_\Lambda + \mathbf{v}_\Lambda\cdot\nabla\mathbf{v}_\Lambda = -\nabla\phi_\Lambda - \frac{1}{\rho_\Lambda}\nabla(\boldsymbol{\tau}_\Lambda + \boldsymbol{\tau}_\Lambda^{\mathrm{UV}})$$

Technical note: we smooth $\rho\mathbf{v}$ instead of $\mathbf{v}$, defining $\mathbf{v}_\Lambda = [\rho\mathbf{v}]_\Lambda/\rho_\Lambda$

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Matter Effective Field Theory — III

- The new stress tensor collects up the **back-reaction** of small onto large scales:

$$\nabla \tau_\Lambda^{\mathrm{UV}} = \left([\rho \nabla \phi]_\Lambda - \rho_\Lambda \nabla \phi_\Lambda\right) + \left([\rho \mathbf{v} \cdot \nabla \mathbf{v}]_\Lambda - \rho_\Lambda \mathbf{v}_\Lambda \cdot \nabla \mathbf{v}_\Lambda\right)$$

- We now have a set of equations for the **smoothed** DM+baryon fluid!
  - These are **identical** to the non-ideal fluid equations!
  - Smoothing **generates** a stress-tensor $\tau_{\mathrm{NL}}^{\mathrm{UV}}$
  - The **new** and **true** stress-tensors are **indistinguishable!**
- An analogy:
  - **Viscosity** in fluid flow → small scales (atomic motions) impact large scales (flow) via a stress tensor (viscosity)

Continuity:

$$\dot{\delta}_\Lambda + \nabla \cdot [(1+\delta_\Lambda)\mathbf{v}_\Lambda] = 0$$

Euler:

$$\dot{\mathbf{v}}_\Lambda + \mathscr{H}\mathbf{v}_\Lambda + \mathbf{v}_\Lambda \cdot \nabla \mathbf{v}_\Lambda = -\nabla \phi_\Lambda - \frac{1}{\rho_\Lambda}\nabla(\boldsymbol{\tau}_\Lambda + \boldsymbol{\tau}_\Lambda^{\mathrm{UV}})$$

Poisson:

$$\nabla^2 \phi_\Lambda = \tfrac{3}{2}\mathscr{H}^2 \Omega_m \delta_\Lambda$$

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Matter Effective Field Theory — IV

$$\nabla \tau_\Lambda^{\rm UV} = \left([\rho \nabla \phi]_\Lambda - \rho_\Lambda \nabla \phi_\Lambda\right) + \left([\rho \mathbf{v} \cdot \nabla \mathbf{v}]_\Lambda - \rho_\Lambda \mathbf{v}_\Lambda \cdot \nabla \mathbf{v}_\Lambda\right)$$

- To use the theory, we need to **model** the stress tensor $\tau_\Lambda^{\rm UV}$
  - This is sourced by **small-scale** physics we can't predict from our theory!
  - **Symmetry to the rescue!** Expand in all relevant quantities, respecting symmetry, *i.e.* $\tau_{\Lambda,ij}^{\rm UV} = F_{ij}(\delta_\Lambda, \mathbf{v}_\Lambda, \partial)$

$$\tau_{\Lambda,ij}^{\rm UV} = \left[p(\Lambda) + \bar{\rho} c_s^2(\Lambda)\delta_\Lambda\right] \delta_{ij}^{\rm K} - \bar{\rho} c_v^2(\Lambda)\left[\partial_i v_{\Lambda,j} - \partial_j v_{\Lambda,i}\right] + \cdots$$

  - This is an expansion in **long-wavelength** **fields** and **counterterms**.
  - The **counterterms** are $\Lambda-$dependent constants encapsulating the small-scale physics we left behind
  - The contributions to $\tau_\Lambda^{\rm UV}$ look just like the usual non-ideal fluid terms τ_Λ

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Matter Effective Field Theory — V

- Let's put our long-wavelength stress tensor into the fluid equations.

$$\dot{\delta}_\Lambda + \nabla \cdot [(1+\delta_\Lambda)\mathbf{v}_\Lambda] = 0$$
$$\dot{\mathbf{v}}_\Lambda + \mathscr{H}\mathbf{v}_\Lambda + \mathbf{v}_\Lambda \cdot \nabla \mathbf{v}_\Lambda = -\nabla\phi_\Lambda - \frac{1}{\rho_\Lambda}\nabla\boldsymbol{\tau}_\Lambda - c_s^2(\Lambda)\nabla\delta_\Lambda + \cdots$$
$$\nabla^2\phi_\Lambda = \frac{3}{2}\mathscr{H}^2\Omega_m\delta_\Lambda$$

- The **only** change (at leading order) is a new derivative term $c_s^2(\Lambda)\nabla\delta_\Lambda$ in the Euler equation!

- The equations can be solved perturbatively in δ_Λ (which is now **small** if $\Lambda < k_{\rm NL}$). The solution has two parts:

$$\delta(\mathbf{k},z) = \sum_n \left[\delta_n(\mathbf{k},z;\Lambda) + \delta_n^{\rm ct}(\mathbf{k},z;\Lambda)\right]$$

where the **counterterm** contributions $\delta_n^{\rm ct}$ come from the **stress-tensor = back reaction** of short-scale physics.

- At leading order: $\delta_3^{\rm ct}(\mathbf{k},z) = -c_s^2(\Lambda,z)k^2\delta_L(\mathbf{k})$ — all other terms match SPT!

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

Matter Effective Field Theory — VI

- The solution for the power spectrum is as follows:

New EFTofLSS term!

$$P(\mathbf{k},z) = P_{11}(\mathbf{k},z) + P_{22}(\mathbf{k},z,\Lambda) + 2P_{13}(\mathbf{k},z,\Lambda) - 2c_s^2(\Lambda,z)k^2P_L(k) + \cdots \quad (k < \Lambda)$$

- The first three terms match SPT, *except* we know integrate only up to Λ (due to the smoothing).

$$P_{22}(k,z,\Lambda) = 2D^4(z)\int_{|\mathbf{p}|\leq\Lambda} F_2(\mathbf{p},\mathbf{k}-\mathbf{p})^2 P_L(p)P_L(|\mathbf{k}-\mathbf{p})$$

$$P_{13}(k,z,\Lambda) = 3D^4(z)\int_{|\mathbf{p}|\leq\Lambda} F_3(\mathbf{p},-\mathbf{p},\mathbf{k})P_L(p)P_L(k)$$

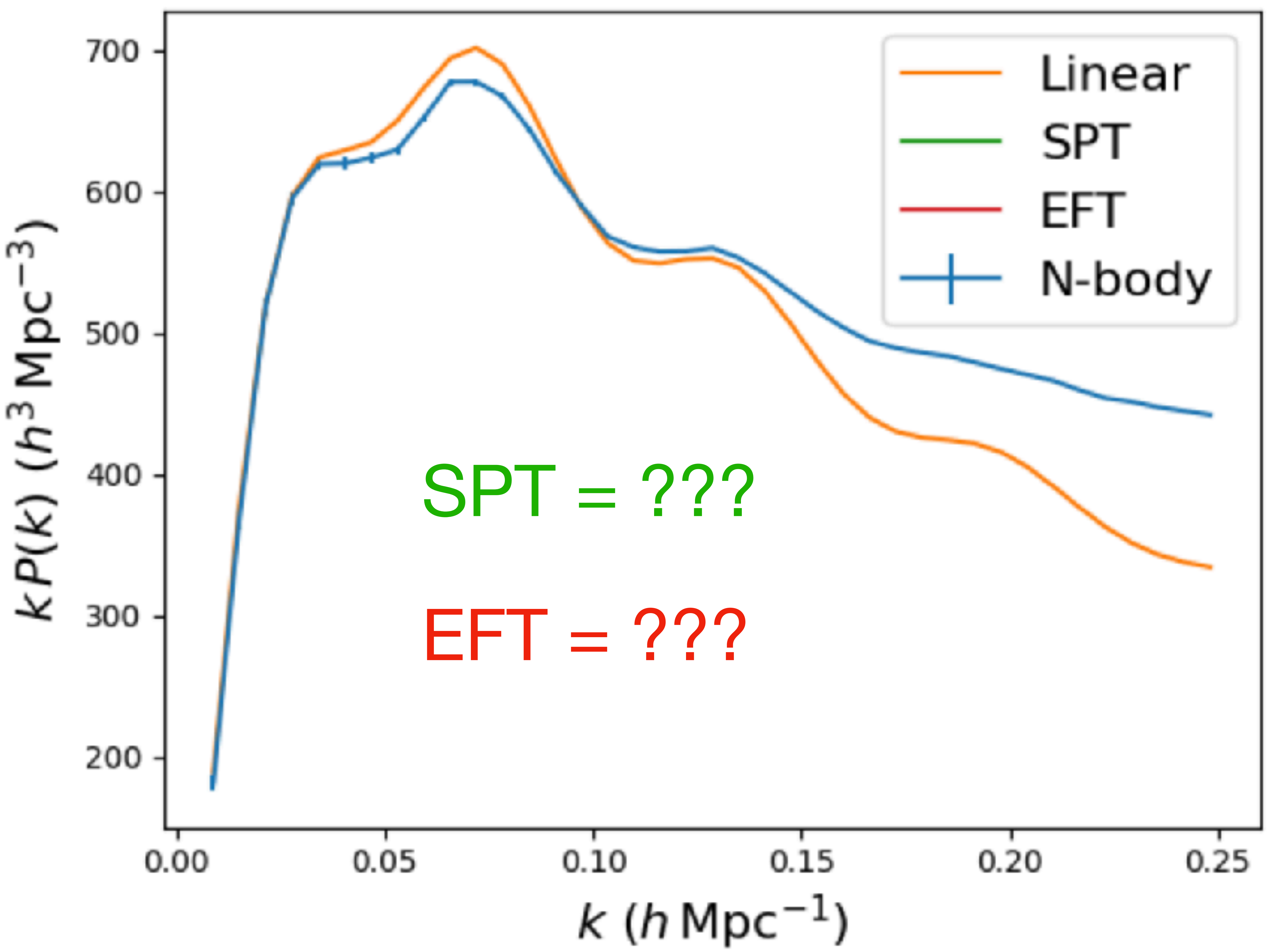
- What is the **value of the counterterm** $c_s^2(\Lambda)$?
  - We don't know! [Cannot be predicted perturbatively, cf. viscosity]
  - Must either **match to simulations** or **fit from data**

EFT gives a model for the matter power spectrum at 1-loop depending on one physical (yet unknown) parameter!

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg…

TIME FOR A BREAK!

- How does this work in practice?



https://tinyurl.com/myfirstpt

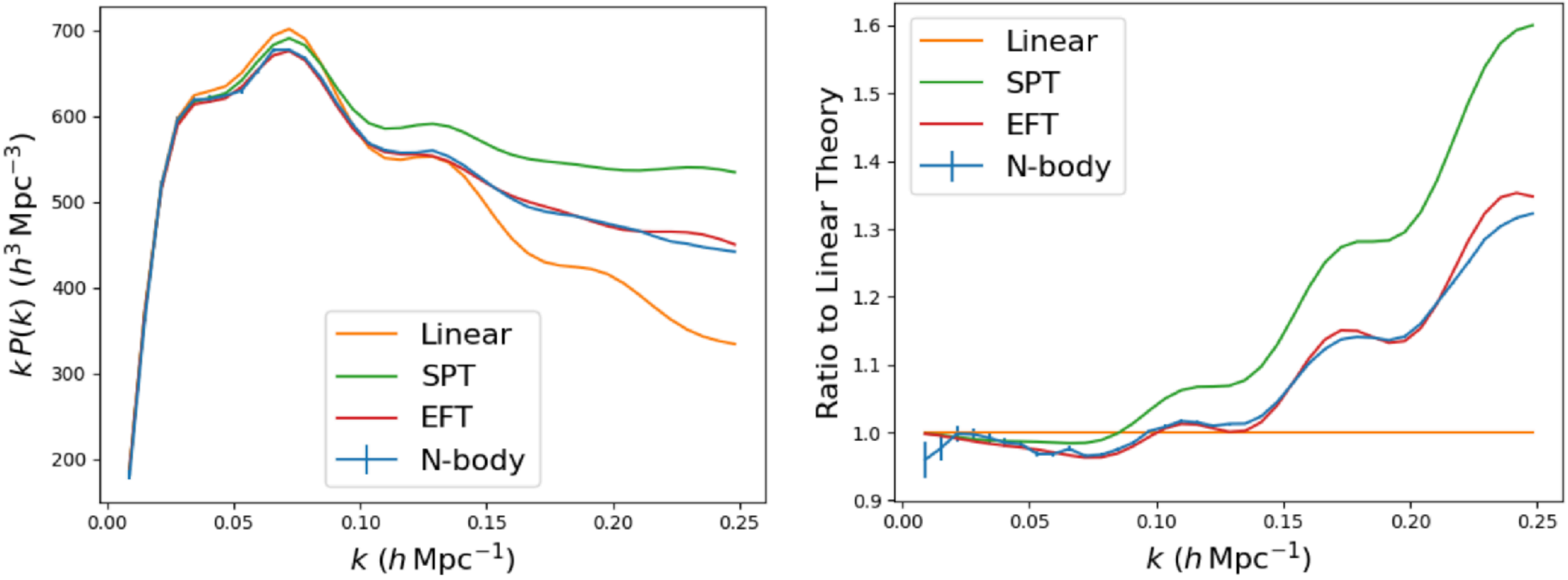
SCAN ME

Build your own 1-loop theory here!

TIME FOR A BREAK!

- The solution:

https://tinyurl.com/myfirstpt-solution



1-loop **EFT** fits much better than 1-loop **SPT**!

Renormalization — I

- Both the 1-loop integrals and the counterterms explicitly depend on the cut-off scale Λ.
- **However**, this is not a physical parameter — we introduced it only to make perturbation theory easier!

- Let's see how P_{13} changes when we shift Λ a little bit (restricting to $k \ll \Lambda$):

$$P_{13}(k,z,\Lambda') - P_{13}(k,z,\Lambda) = -\frac{61}{210}D^4(z)k^2P_L(k)\int_\Lambda^{\Lambda'}\frac{p^2dp}{6\pi^2}\frac{P_L(p)}{p^2} = k^2P_L(k)[f(\Lambda',z)-f(\Lambda,z)]$$

- This is **exactly** the same form as the change in the counterterm:

$$P_{\rm ct}(k,z,\Lambda') - P_{\rm ct}(k,z,\Lambda) = -2k^2P_L(k)[c_s^2(\Lambda',z) - c_s^2(\Lambda,z)]$$

- A small change in Λ leads to **opposite** changes in the **loop integrals** and the **counterterms**

$$2P_{13}(k,z,\Lambda) + P_{\rm ct}(k,z,\Lambda) = 2P_{13}(k,z,\Lambda') + P_{\rm ct}(k,z,\Lambda')$$

- We can play a similar game for P_{22}: this changes as k^4, matching a (neglected) stochastic counterterm.

***Technical note**: this is on-shell Wilson renormalization*

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg, Schmidt…

Renormalization — II

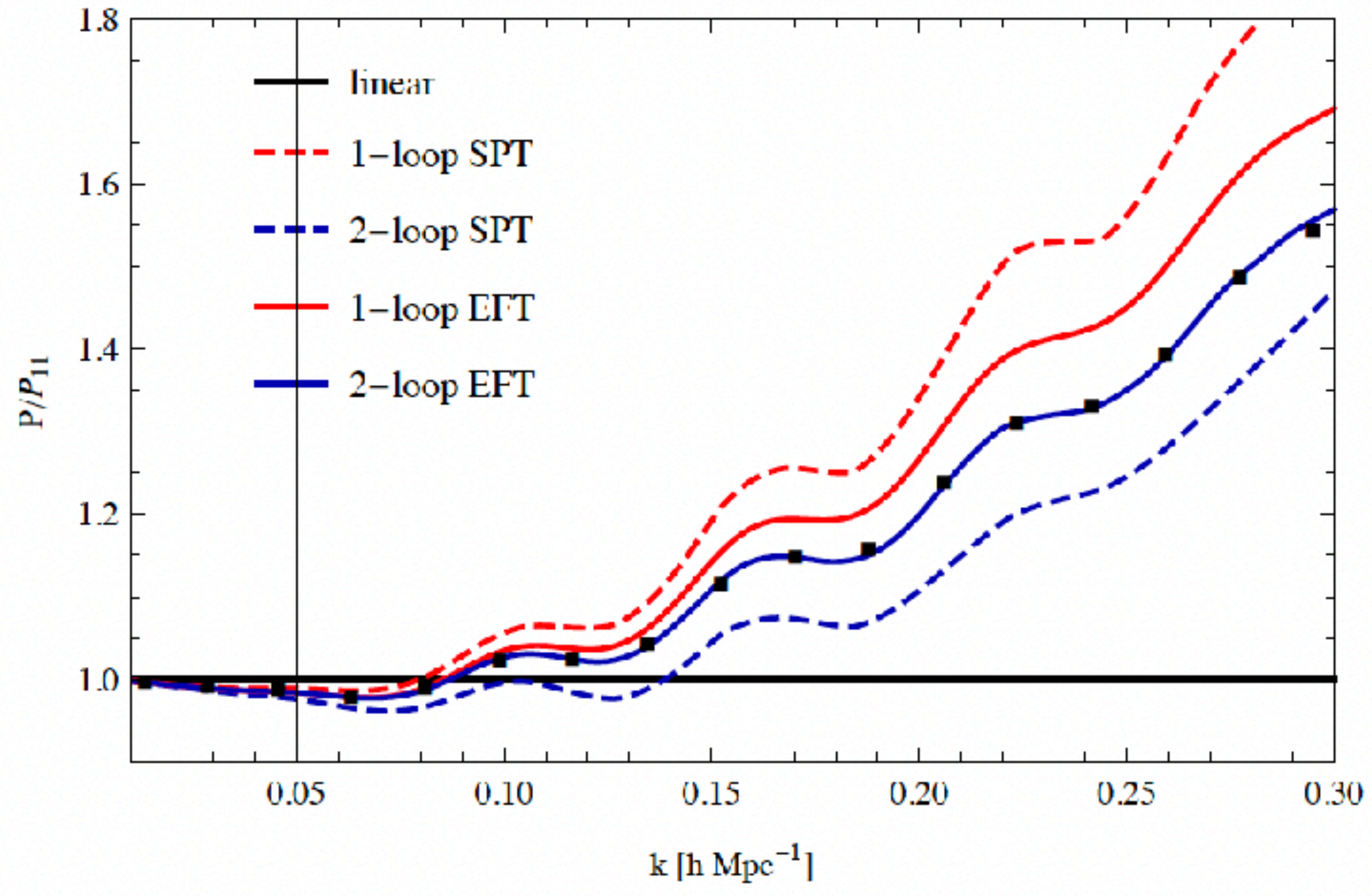
- This is the essence of **renormalization**: due to the counterterms, the total power spectrum $P_{\rm EFT}(k,z)$ does not depend on the cut-off Λ
  - In other words, we capture the **UV divergences** of the theory (modes with $p > k_{\rm NL}$) via a set of free **counterterm coefficients**
- This is an equivalent (and often easier) way to introduce the counterterms
  - Just check that the cut-off dependence of the loops is captured by a counterterm contribution
- In practice, the counterterms have two contributions: $c_s^2(\Lambda) = c_{s,\rm UV}^2(\Lambda) + c_{s,\rm phys}^2$
  - The **physical** part (a true sound-speed, for example) is indistinguishable from the **renormalization** piece.

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg, Schmidt…

Matter EFT Round-Up

- Effective Field Theory can be extended to **higher-loops** and **higher-orders**
- All we need to do is:
  a. Calculate the SPT diagrams, integrating up to Λ
  b. Add all the relevant counterterms (from τ_{ij}^{UV} or cut-off dependence)
- For matter, it has been computed for:
  - $P(k)$: 1-loop, 2-loop, 3-loop
  - $B(k_1, k_2, k_3)$: 1-loop, 2-loop
  - $T(k_1, k_2, k_3, k_4, K)$: 1-loop

2-loop EFT is great for $k < 0.3$!



- Eventually the **loops** are expensive, the **dimensionality** explodes, and the number of **counterterms** is large!

Senatore, Baumann, Nicolis, Zaldarriaga, McDonald, Carrasco, Hertzberg, Schmidt…

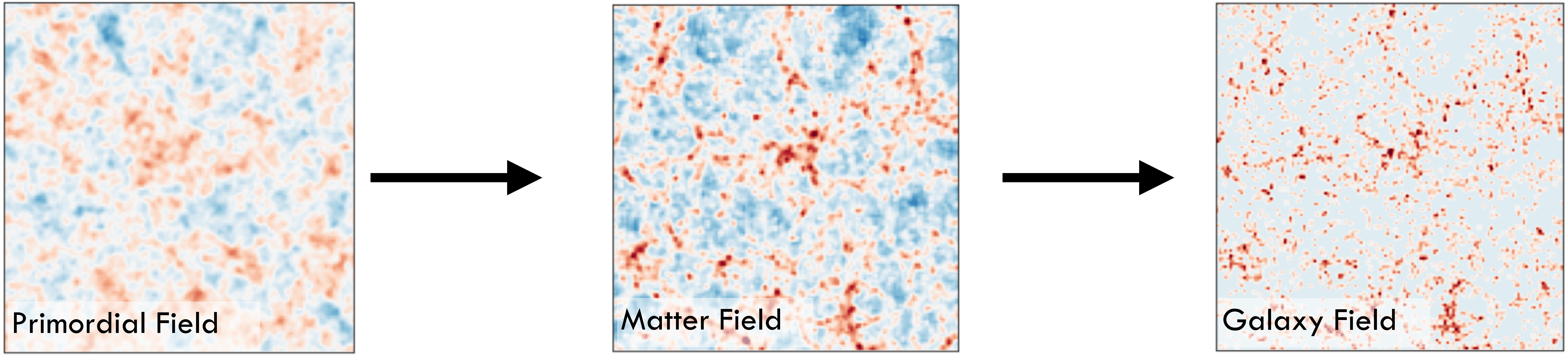
How to Model a Galaxy Survey

The whys, hows and woes of Effective Field Theory

Oliver Philcox
Postdoc @ Columbia
Junior Fellow @ Simons Foundation
(Acting) Assistant Prof @ Stanford

Important Note:
This subject has a long history
— my citations will be very incomplete!

Where Did We Get To Last Time?



Last Time: Modeling **Matter**

This Time: Modeling **Galaxies**

Lecture Notes

https://tinyurl.com/philcox-eft-notes

SCAN ME

Summary of the EFT of LSS

- EFT is a **perturbative** solution of the **non-ideal** fluid equations
    - A ***controlled*** Taylor series in $k/k_{\rm NL}, kR_{\rm halo}, \dots$
    - ***Agnostic*** to UV physics

L. Senatore

- Includes all effects relevant to symmetry (including **baryonic** effects!)

- It is **maximally conservative** theory $\Rightarrow$ we'd do better if we knew the counterterms!

Lagrangian Perturbation Theory

Senatore, Vlah, White, Kokron, Zel'dovich, Chen, Reid, Carlson, Fry, Bertschinger, Zaldarriaga, Matsubara, …

Lagrangian Perturbation Theory — I

- We can describe a fluid in two frames:
  - **Observer Frame**: Track external properties $\rho(\mathbf{x},t), \mathbf{v}(\mathbf{x},t)$
  - **Fluid Frame**: Track fluid displacement: $\mathbf{x}(\mathbf{q},z) = \mathbf{q} + \Psi(\mathbf{q},z)$ from initial position $\mathbf{q}$
- The Newtonian **geodesic equation** gives:

$$\ddot{\boldsymbol{\Psi}} + \mathscr{H}\dot{\boldsymbol{\Psi}} = -\nabla_{\mathbf{x}}\phi(\mathbf{q}+\boldsymbol{\Psi}), \qquad \nabla^2_{\mathbf{x}}\phi = \frac{3}{2}\mathscr{H}^2\Omega_m\delta$$

for an ideal fluid, as before

- To relate to density, we can use conservation of matter:

$$[1+\delta(\mathbf{x},z)]d\mathbf{x} = d\mathbf{q} \qquad \Rightarrow \qquad \delta(\mathbf{k},z) = \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\left(e^{i\mathbf{k}\cdot\boldsymbol{\Psi}(\mathbf{q},z)} - 1\right)$$

Senatore, Vlah, White, Kokron, Zel’dovich, Chen, Reid, Carlson, Fry, Bertschinger, Zaldarriaga, Matsubara, …

Lagrangian Perturbation Theory — II

$$\ddot{\mathbf{\Psi}} + \mathcal{H}\dot{\mathbf{\Psi}} = -\nabla_{\mathbf{x}}\phi(\mathbf{q}+\mathbf{\Psi}), \qquad \nabla^2_{\mathbf{x}}\phi = \frac{3}{2}\mathcal{H}^2\Omega_m\delta, \qquad \delta(\mathbf{k},z) = \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\left(e^{i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q},z)} - 1\right)$$

- At linear order:

$$\boxed{\ddot{\mathbf{\Psi}}_1 + \mathcal{H}\dot{\mathbf{\Psi}}_1 = -\frac{3}{2}\mathcal{H}^2\Omega_m \nabla\nabla^{-2}\delta_1}, \qquad \boxed{\delta_1(\mathbf{k},z) = i\mathbf{k}\cdot\mathbf{\Psi}_1(\mathbf{k})}$$

- Inserting the **blue** into the **red** gives:

$$\ddot{\delta}_1 + \mathcal{H}\dot{\delta}_1 - \frac{3}{2}\mathcal{H}^2\Omega_m\delta_1 = 0$$

  - This **matches** the Eulerian expression (as expected)!

- To obtain the **Zel'dovich solution** we solve the **red** equation for $\mathbf{\Psi}_1$ but do **not** expand the exponential:

$$\mathbf{\Psi}_1(\mathbf{k},z) = D(z)\frac{i\mathbf{k}}{k^2}\delta_L(\mathbf{k}), \qquad \delta_{\rm Zel}(\mathbf{k},z) = \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\left(e^{-D(z)\int_{\mathbf{p}}\frac{\mathbf{k}\cdot\mathbf{p}}{p^2}\delta_L(\mathbf{p})} - 1\right)$$

Senatore, Vlah, White, Kokron, Zel'dovich, Chen, Reid, Carlson, Fry, Bertschinger, Zaldarriaga, Matsubara, …

Lagrangian Perturbation Theory — III

- We can expand **perturbatively** as before:

$$\boldsymbol{\Psi}(\mathbf{q},z) = \sum_{n=1}^{\infty} \boldsymbol{\Psi}_n(\mathbf{q},z), \qquad \boldsymbol{\Psi}_n(\mathbf{q},z) = D^n(z)\frac{i}{n!}\int_{\mathbf{p}_1\cdots\mathbf{p}_n} \mathbf{L}_n(\mathbf{p}_1,\cdots,\mathbf{p}_n)\delta_L(\mathbf{p}_1)\cdots\delta_L(\mathbf{p}_n)\delta_{\rm D}(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{k})$$

- This is analogous to SPT but with new **kernels**, e.g., $\mathbf{L}_1(\mathbf{k}) = \mathbf{k}/k^2, \quad \mathbf{L}_2(\mathbf{p}_1,\mathbf{p}_2) = \frac{3}{7}\frac{\mathbf{p}_{12}}{p_{12}^2}\left[1-\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{p_1^2p_2^2}\right]$

- To compute **correlators** we use the expression for $\delta(\mathbf{k},z)$, e.g., for Zel'dovich:

$$P_{\rm Zel}(k,z) = \int d\mathbf{q}_1 d\mathbf{q}_2 e^{i\mathbf{k}\cdot(\mathbf{q}_1-\mathbf{q}_2)}\left\langle\left(e^{i\mathbf{k}\cdot\boldsymbol{\Psi}_1(\mathbf{q}_1,z)}-1\right)\left(e^{-i\mathbf{k}\cdot\boldsymbol{\Psi}_1(\mathbf{q}_2,z)}-1\right)\right\rangle$$

- This is more painful to simplify, but we can use the **cumulant theorem:** $\left\langle e^{iX}\right\rangle = e^{-\sigma_X^2/2}$. We eventually find:

$$P_{\rm Zel}(k,z) = \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\exp\left(-D^2(z)k^2\int\frac{p^2dp}{2\pi^2}\frac{P_L(p)}{p^2}\left[\frac{1}{3}(1-j_0(pq)-j_2(pq))+(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2 j_2(pq)\right]\right)$$

- Higher orders are computed similarly, *but* we expand them from the exponential

Senatore, Vlah, White, Kokron, Zel'dovich, Chen, Reid, Carlson, Fry, Bertschinger, Zaldarriaga, Matsubara, …

Lagrangian Perturbation Theory — IV

- As for Eulerian PT, the basic formulation of LPT is **pathological** and **inaccurate!**

- We need to **coarse-grain** the theory via: $\boldsymbol{\Psi} \to \boldsymbol{\Psi}_\Lambda, \phi \to \phi_\Lambda$

- This leads to **modifications** of the equation of motion:

$$\ddot{\boldsymbol{\Psi}}_\Lambda(\mathbf{q}, z) + \mathscr{H}(z)\dot{\boldsymbol{\Psi}}_\Lambda(\mathbf{q}, z) = -\nabla_{\mathbf{x}}\phi_\Lambda(\mathbf{q} + \boldsymbol{\Psi}_\Lambda(\mathbf{q}, z), z) + \mathbf{a}_\Lambda(\mathbf{q}, \boldsymbol{\Psi}_\Lambda, z)$$

- The **acceleration** term comes from small-scale fluctuations and depends on Λ

- We can expand it using **symmetry** as before: $\mathbf{a}_\Lambda = \mathbf{a}_0(z) + \mathbf{a}_1(z)\,\nabla_{\mathbf{x}}\delta_\Lambda(\mathbf{q} + \boldsymbol{\Psi}_\Lambda(\mathbf{q}, z), z) + \cdots$
  (i.e. isotropy, homogeneity, equivalence)

- This has the same effect as for Eulerian PT: $P_{\rm EFT}(k, z) = P_{\rm LPT}(k, z, \Lambda)\left[1 - k^2\alpha(\Lambda, z)\right] + \cdots$

- As before, it ensures that the theory is **cut-off independent!**

Senatore, Vlah, White, Chen, Zaldarriaga, …

Infrared Resummation — I

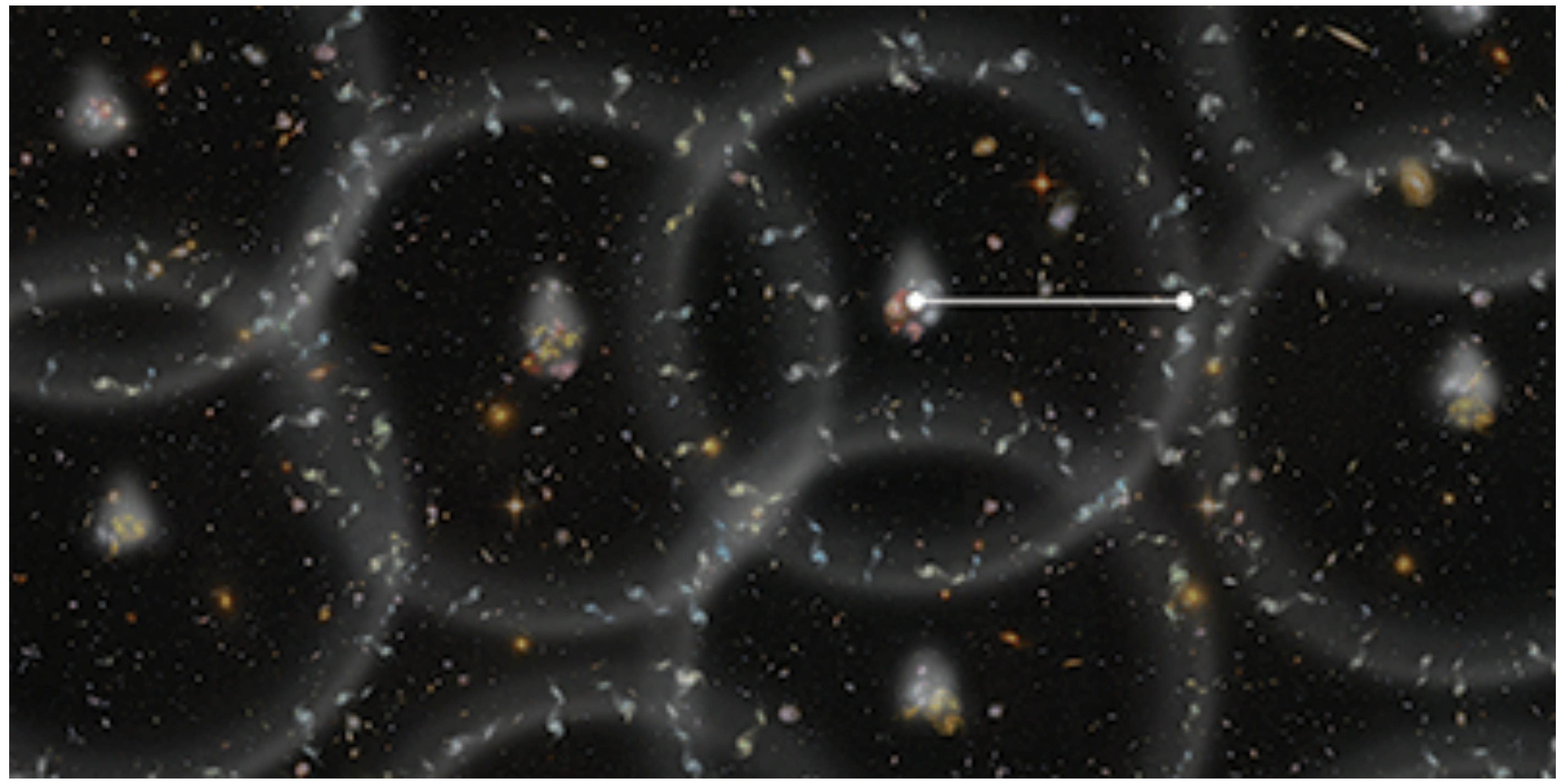
- There's a slight **error** in our perturbative treatments!
- In **Eulerian** PT, we essentially **expand the displacement exponential**

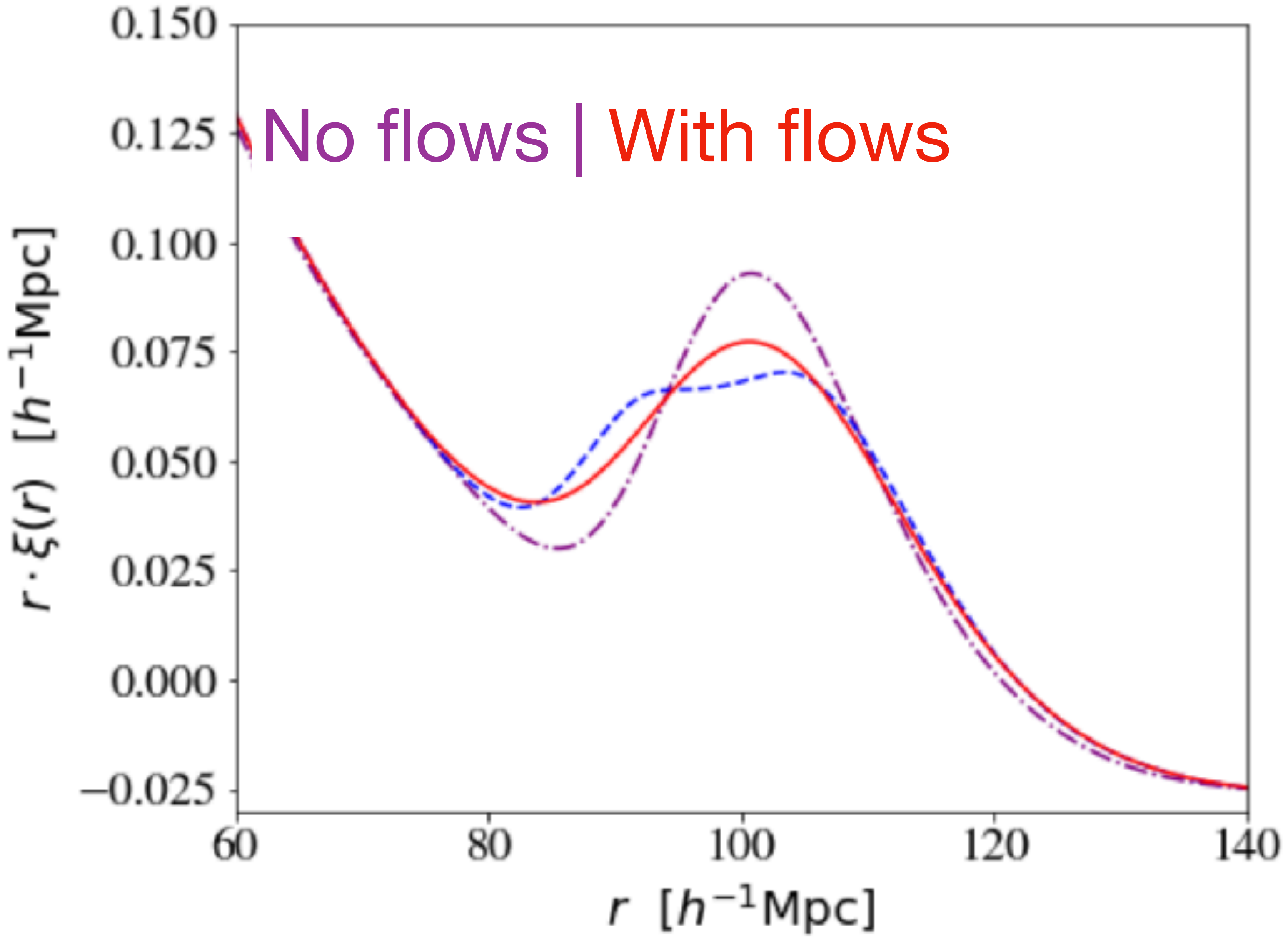
$$\delta_\Lambda(\mathbf{k},z) = \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\left(e^{i\mathbf{k}\cdot\mathbf{\Psi}_\Lambda}-1\right) \approx \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{q}}\left(ik_i\Psi^i_\Lambda - \tfrac{1}{2}k_ik_j\Psi^i_\Lambda\Psi^j_\Lambda + \cdots\right)$$

- **However,** the exponent isn't necessarily small!

$$\frac{1}{3}\langle\mathbf{\Psi}\cdot\mathbf{\Psi}^*\rangle_{\rm Zel} = \frac{D^2(z)}{6\pi^2}\int\frac{p^2dp}{2\pi^2}\frac{P_L(p)}{p^2} \approx (20\,\mathrm{Mpc})^2$$

- On large-scales, this is the **distance** a particle moves since **inflation** — it is not small!
- These **bulk flows** smooth out any **sharp features** in the spectrum!





*Technically, there is a **second** dimensionless parameter in the problem that we ignored!*

Zaldarriaga, Senatore, Blas, Ivanov, Sibiryakov, Baldauf…

Infrared Resummation — II

- Note that this **doesn't** affect smooth features due to mass and momentum conservation!

- **Solution**: do **not** Taylor expand the long-modes!

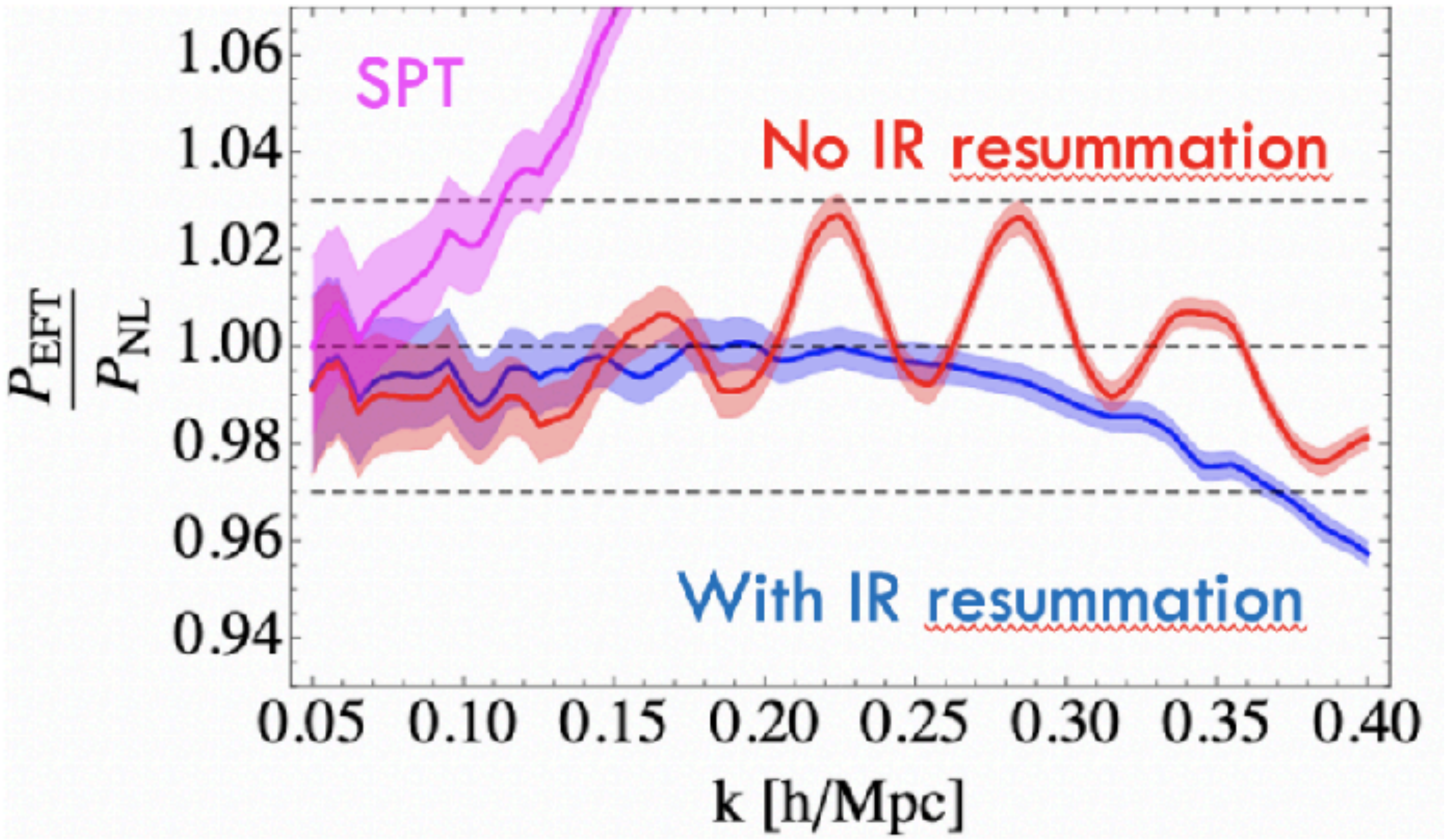
- This is **technical**. At tree-level, we find:

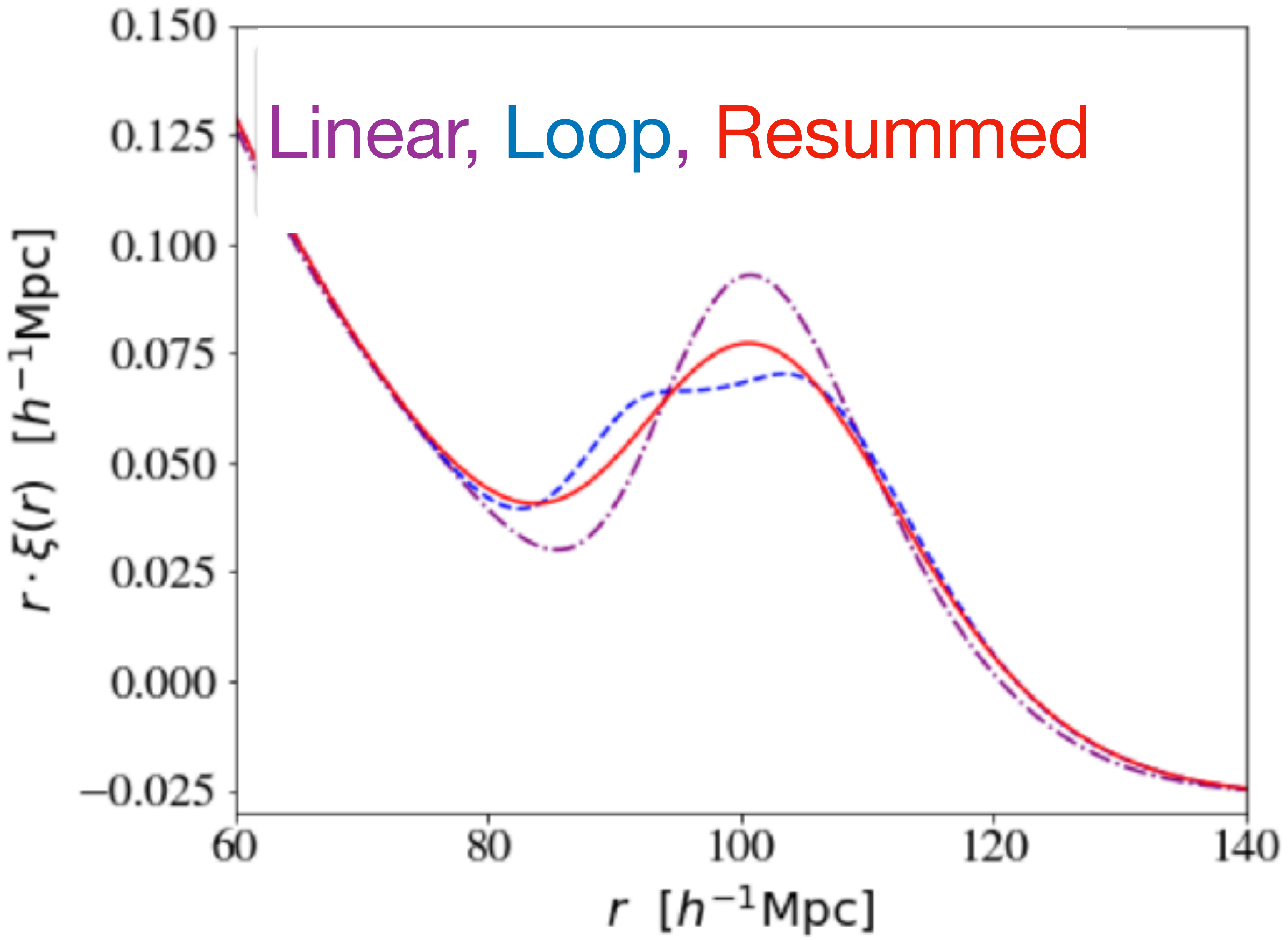
$$P_{\mathrm{IR}}(k,z) = P_L^{\mathrm{nw}}(k,z) + e^{-k^2\Sigma^2(z)} P_L^{\mathrm{w}}(k,z)$$

where only the **wiggly** pieces P^w are damped!

*(This is formalized in **Time-Sliced Perturbation Theory**)*

- The result: **EFT matches the data!**





Notes: we can also do this in LPT and for higher-orders

Zaldarriaga, Senatore, Blas, Ivanov, Sibiryakov, Baldauf…

Galaxy Bias

Schmidt, Desjaques, Senatore, McDonald, Zaldarriaga, Ivanov, Philcox, Chen, d'Amico, Zhang, Kokron, Assassi, Simonovic…

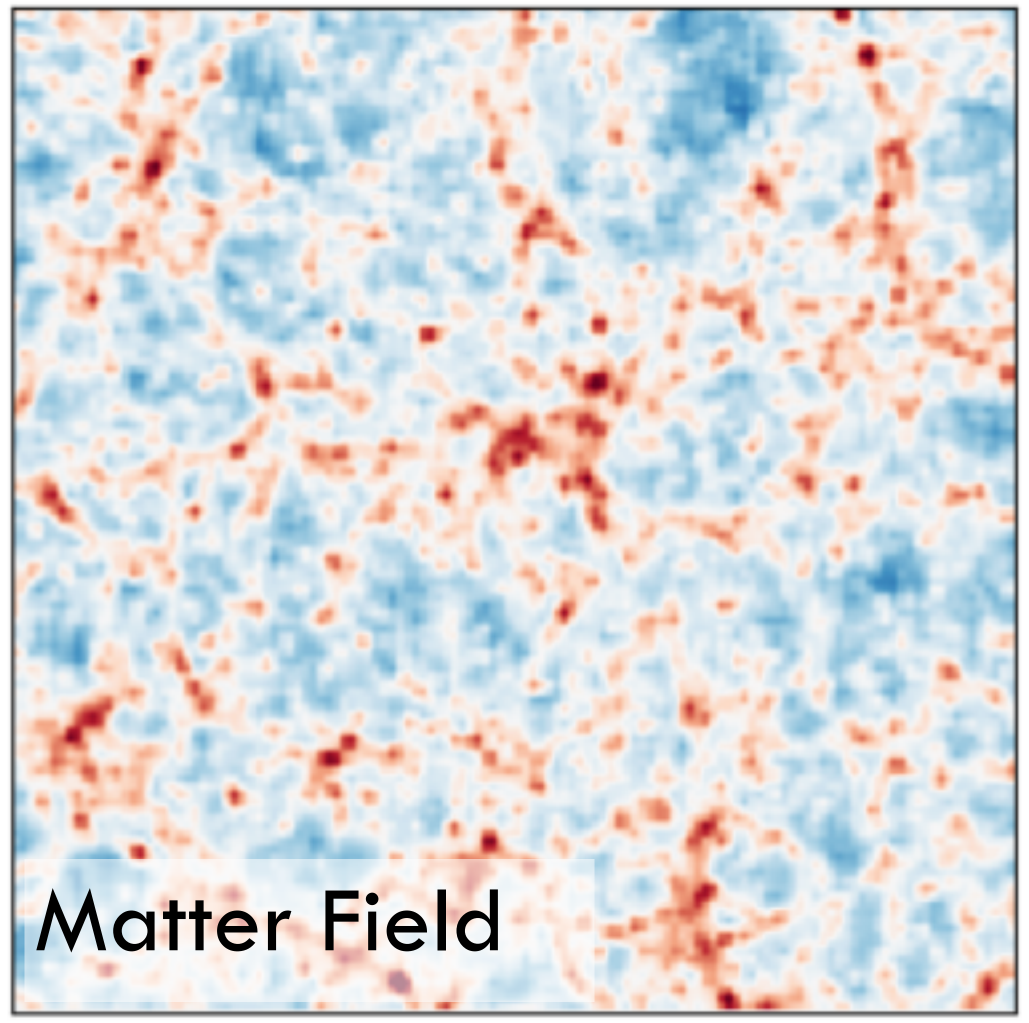
Galaxy Bias — I

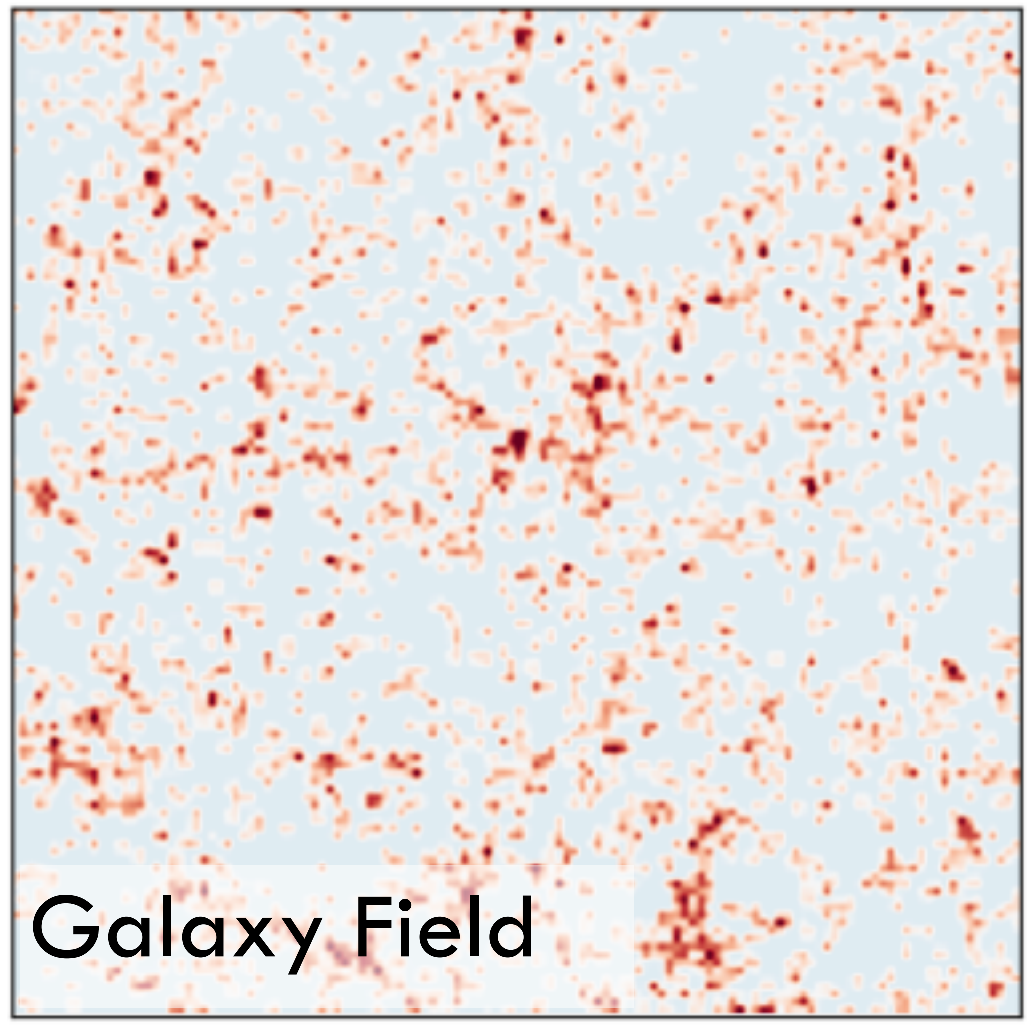
- We know how to predict the **dark matter** (+ baryon) field $\delta(\mathbf{x}, z)$
- Our observable is the **galaxy** overdensity!

$$n_g(\mathbf{x}, z) = \bar{n}_g(z)[1 + \delta_g(\mathbf{x}, z)]$$

- **The EFT approach:**
  - $\delta_g(\mathbf{x}, z)$ must be a function of **local** variables, e.g., $\delta(\mathbf{x}, z), \mathbf{v}(\mathbf{x}, z), \nabla\delta(\mathbf{x}, z), s_{ij}(\mathbf{x}, z), \cdots$
    (integrated over a light cone)
  - Expand δ_g **perturbatively** in all possible **operators**
  - **Symmetries**: translation invariance, rotation invariance, Galilean invariance
    (can't have less than two ϕ derivatives!)

- At lowest order: $\delta_g = b_1\delta + \cdots$ for **linear bias** b_1





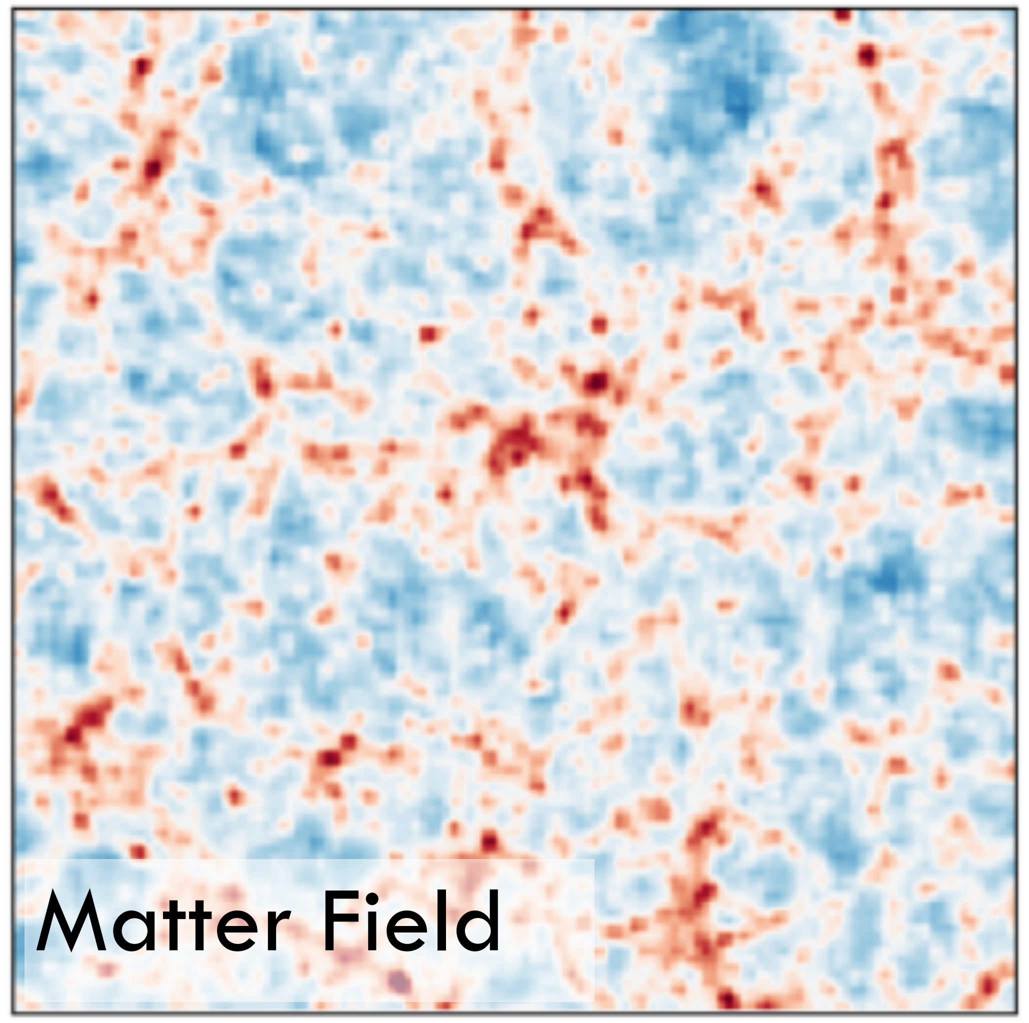
Schmidt, Desjaques…

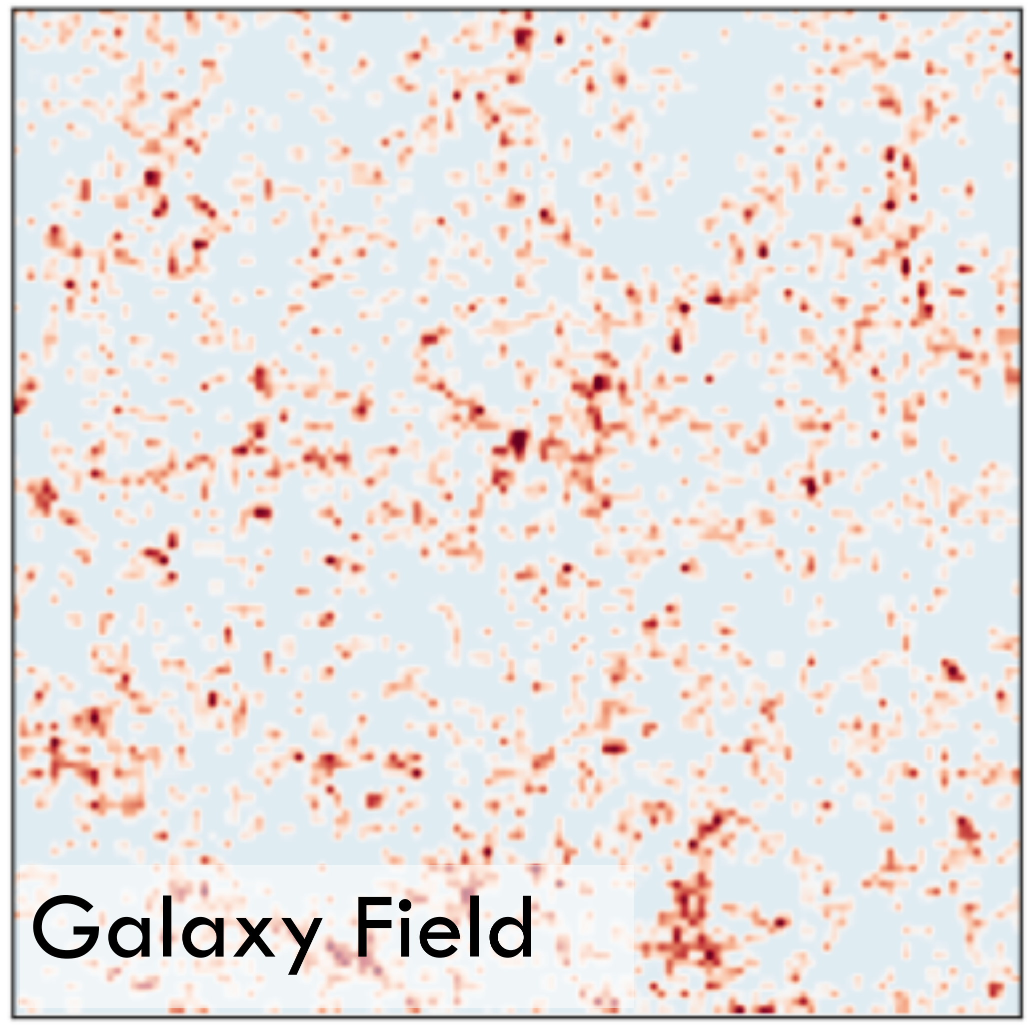
Galaxy Bias — II

- At third-order, **many** more terms are possible, including the **tidal field** $s_{ij} \sim \partial_i \partial_j \nabla^{-2} \delta$

$$\delta_g = b_1 \delta + \frac{b_2}{2} \delta^2 + b_{s^2} s_{ij} s^{ij} + b_{\nabla^2 \delta} \nabla^2 \delta + \cdots$$

- This is a **Taylor expansion** in $kR_{\rm halo}$!
- There are two pieces:
  - **Physical operators** — which fields do δ_g depend on?
  - **Bias coefficients** — how does δ_g depend on each field?
- All of the fun galaxy-formation physics is encoded in the biases: $\{b_1, b_2, b_{s^2}, b_{\nabla^2\delta}, \cdots\}$





Schmidt, Desjaques…

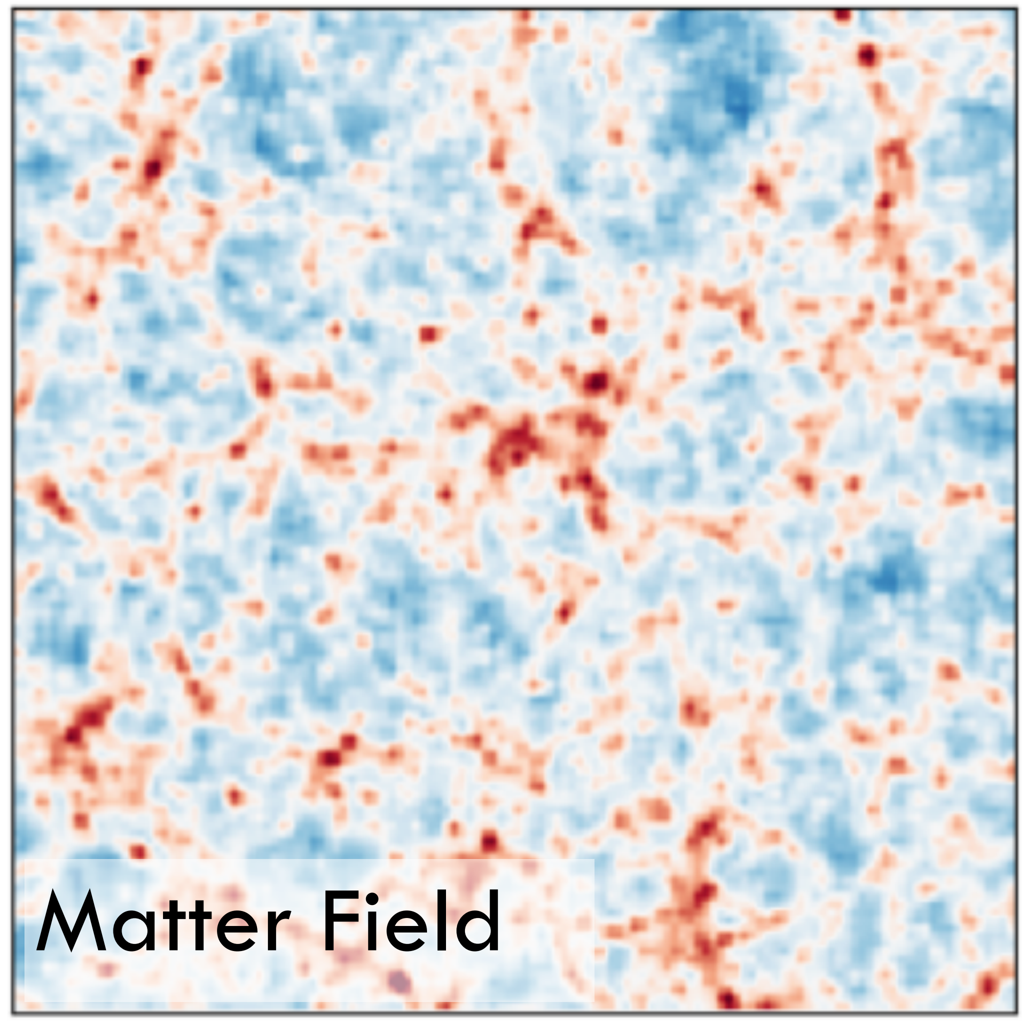
Galaxy Bias — III

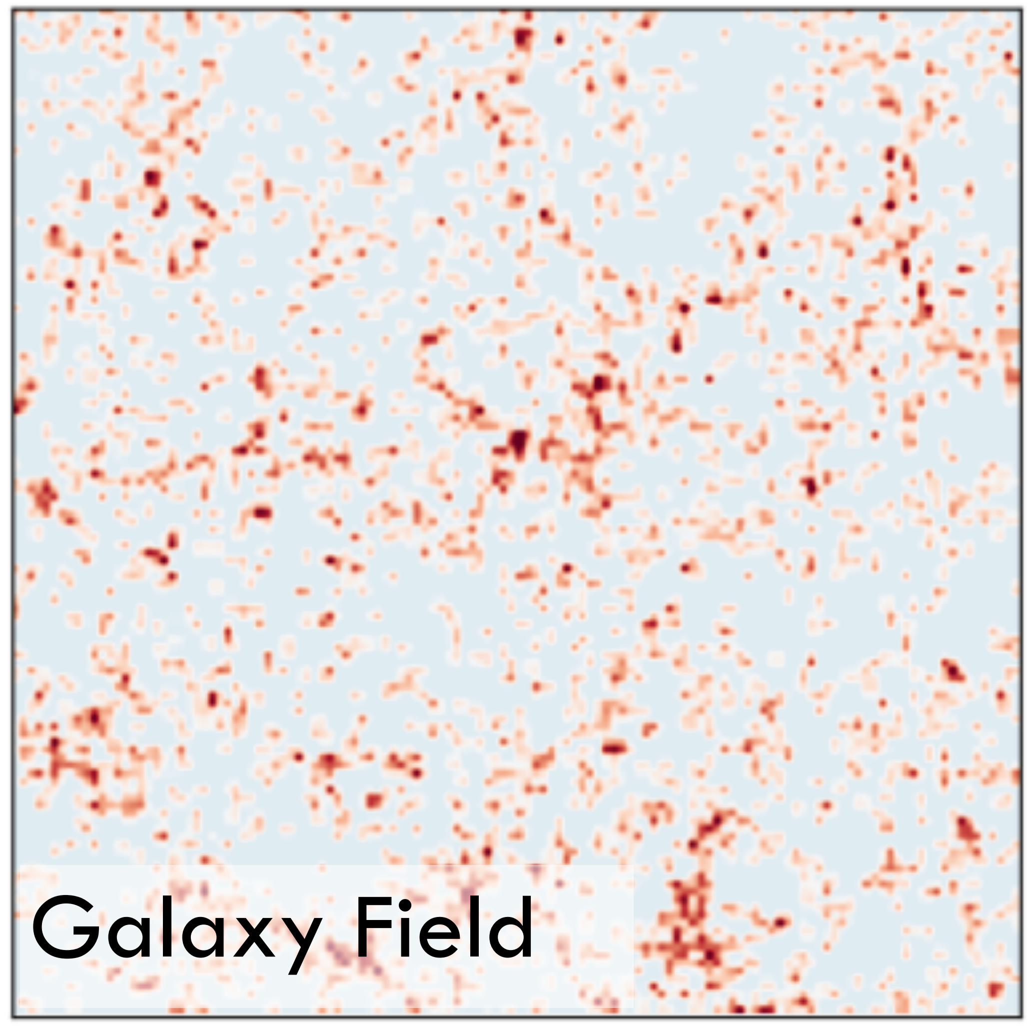
- In the EFT language, we are expanding in **smoothed** fields, e.g., δ_Λ.

- **Small-scale physics** generates **new terms** $\delta_g(\mathbf{x}, z) \supset \epsilon(\mathbf{x}, z) + \cdots$
  - These come from **renormalization** of non-linear pieces, e.g., $[\delta^2]_\Lambda \neq \delta^2_\Lambda$
  - Unlike for matter, they **don't** have to conserve mass and momentum!

- At leading-order we find the **stochastic** contribution:
  - $\langle \epsilon(\mathbf{k}, z)\epsilon(-\mathbf{k}, z)\rangle = P_\epsilon(k, z) = a_0 + a_2 k^2 + a_4 k^4 + \cdots$
  - This is (scale-dependent) **shot-noise!**

(For matter, this term looks like $P_\epsilon \sim k^4$)





Schmidt, Desjaques, Assassi, Senatore, Zaldarriaga…

Galaxy Bias — IV

- Let's combine everything together. To model the **power spectrum** at one-loop, we'll need:

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + b_{s^2}s_{ij}s^{ij} + b_{\nabla^2\delta}\nabla^2\delta + \epsilon + \cdots$$

Deterministic *Stochastic*

- The **tree-level** power spectrum is given by $P_{gg}(k,z) \equiv \langle |\delta_g(\mathbf{k},z)|^2 \rangle$:

$$P_{gg}^{\rm tree}(k,z) = D^2(z)b_1^2(z)\left[P_L^{\rm nw}(k) + e^{-k^2\Sigma^2(z)}P_L^{\rm w}(k)\right] + P_{\rm shot}$$

Bias *IR-resummed spectra* *Stochasticity*

- At higher-order, we will need **loop integrals** and **perturbative expansions…**

Schmidt, Desjaques…

Galaxy Bias — V

- We can expand δ_g directly in terms of δ_L as in matter PT!

$$\delta_{g,n}(\mathbf{k}, z) = D_n(z) \int \frac{d\mathbf{p}_1}{(2\pi)^3} \cdots \frac{d\mathbf{p}_n}{(2\pi)^3} K_n(\mathbf{p}_1, \cdots, \mathbf{p}_n; z) \delta_L(\mathbf{p}_1) \cdots \delta_L(\mathbf{p}_n) \delta_\mathrm{D}(\mathbf{p}_1 + \cdots + \mathbf{p}_n - \mathbf{k})$$

New kernels: now depend on biases!

- The **power spectra** look **exactly** the same as before, just with K_n instead of F_n!

$$P_{22}(k, z) = 2D^4(z) \int_\mathbf{p} |K_2(\mathbf{p}, \mathbf{k} - \mathbf{p}; z)|^2 P_L(p) P_L(|\mathbf{k} - \mathbf{p}) \qquad P_{13}(k, z) = 3D^4(z) \int_\mathbf{p} K_1(z) K_3(\mathbf{p}, -\mathbf{p}, \mathbf{k}; z) P_L(p) P_L(k)$$

- We can similarly compute **bispectra**. At tree-level:

$$B_{211}(\mathbf{k}_1, \mathbf{k}_2, z) = \left[2D^4(z) K_1(z) K_1(z) K_2(\mathbf{k}_1, \mathbf{k}_2; z) P_L(k_1) P_L(k_2) + D^2(z) P_\epsilon(z) P_L(k) + B_\epsilon(z)\right] + 2 \text{ perm.}$$

(Note: stochasticity more complex here!)

(Note: we need to add IR-resummation and UV cut-offs in practice!)

Schmidt, Desjaques…

Redshift-Space Distortions

Redshift-Space Distortions — I

- **Finally**, we observe the galaxy field in **redshift space**
- **Observed position** is related to **true position** by **radial velocity**

$$\mathbf{s} = \mathbf{x} + \frac{1}{\mathcal{H}(z)}\mathbf{v}_{\parallel}$$

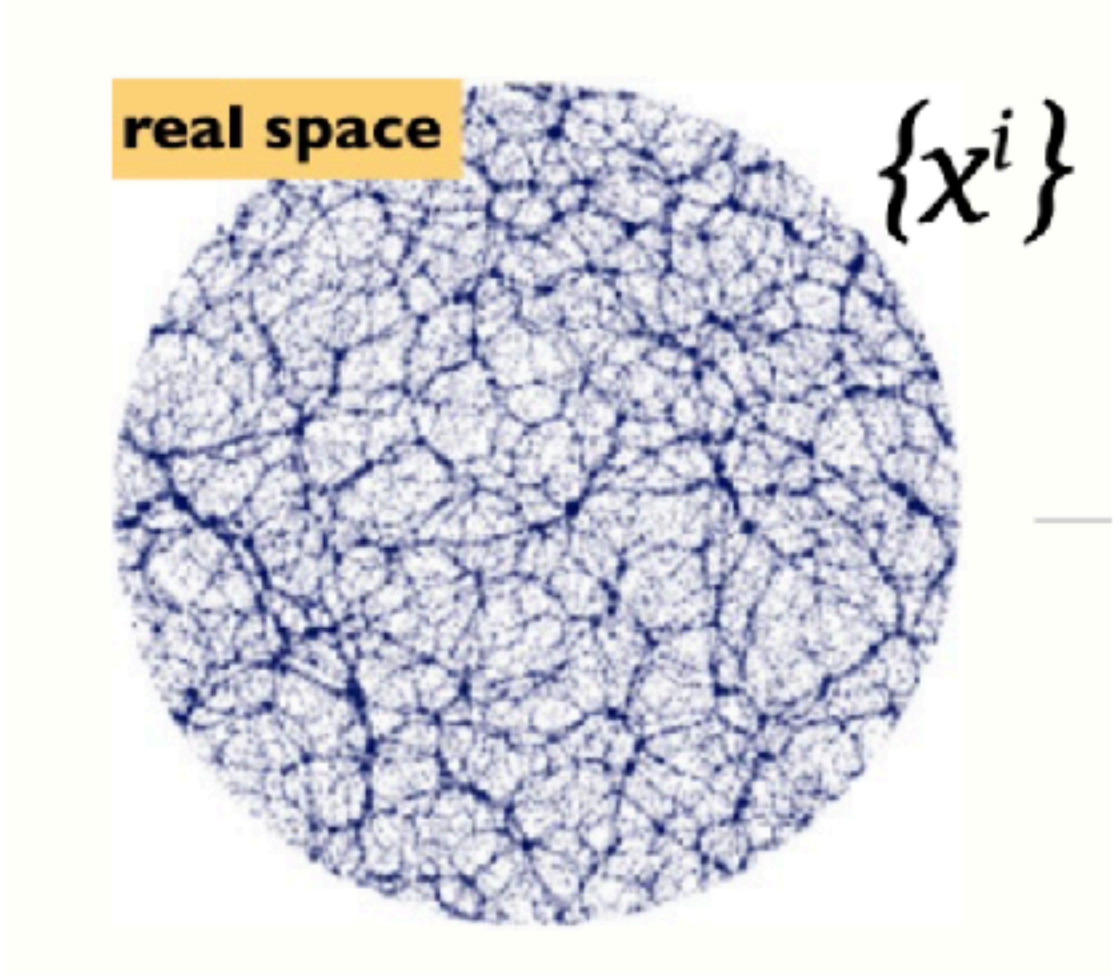
$(\mathbf{v}_{\parallel} = (\mathbf{v}\cdot\hat{z})\hat{z})$

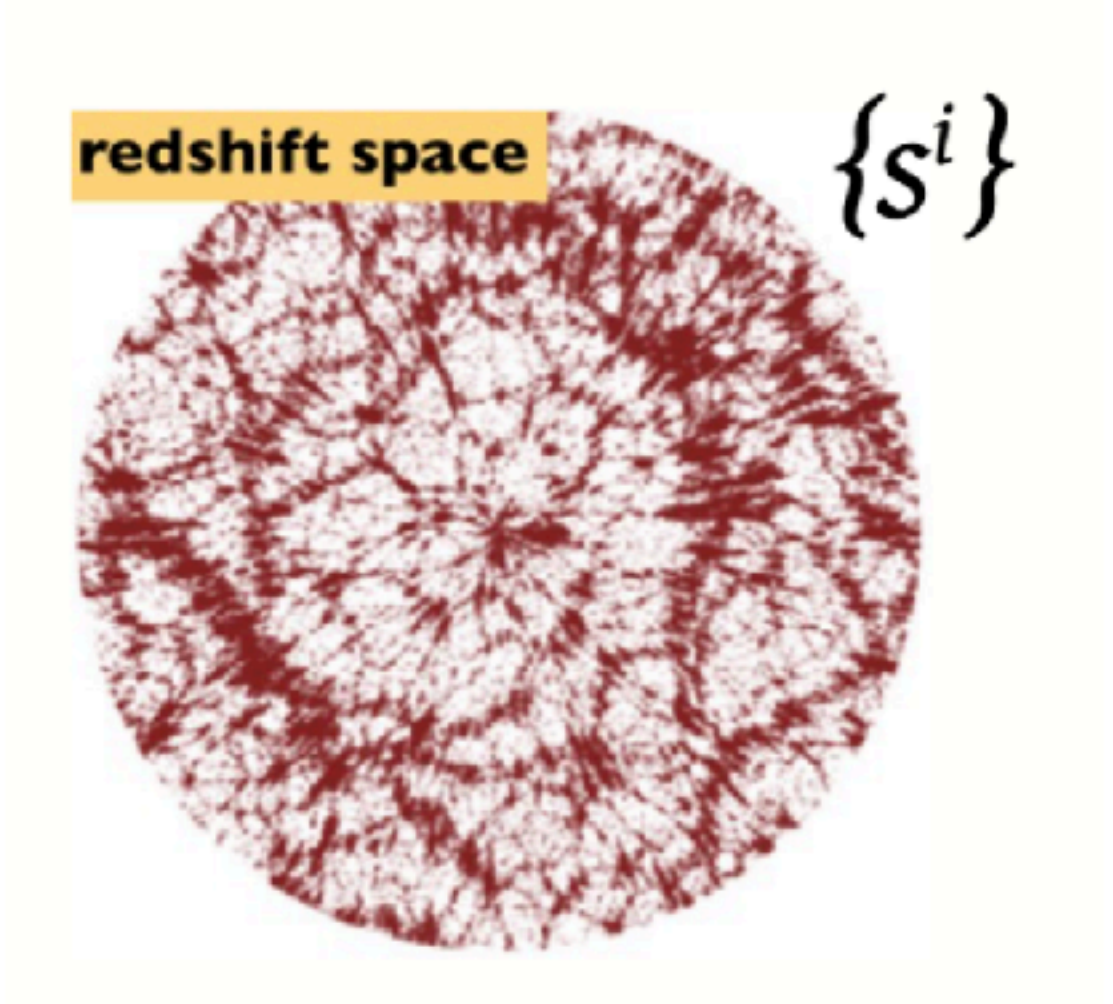
- This **remaps** the galaxy density:

$$\delta_g(\mathbf{x}) \to \delta_{g,s}(\mathbf{s}) = \delta_g(\mathbf{x}+\mathbf{v}_{\parallel}/\mathcal{H}(z))$$

- In Fourier-space:

$$\delta_{g,s}(\mathbf{k}) = \int d\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\left\{[(1+\delta_g(\mathbf{x})]e^{i\mathbf{k}\cdot\mathbf{v}_{\parallel}/\mathcal{H}(z)} - 1\right\}$$





Fonseca de la Bella, Kaiser, Peebles, …

Redshift-Space Distortions — II

$$\delta_{g,s}(\mathbf{k}) = \int d\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \left\{ [(1+\delta_g(\mathbf{x})] e^{i\mathbf{k}\cdot\mathbf{v}_{||}/\mathscr{H}(z)} - 1 \right\}$$

- To solve, we now **Taylor expand** in $\mathbf{v}_{||}$ as well as δ!
- At leading order:

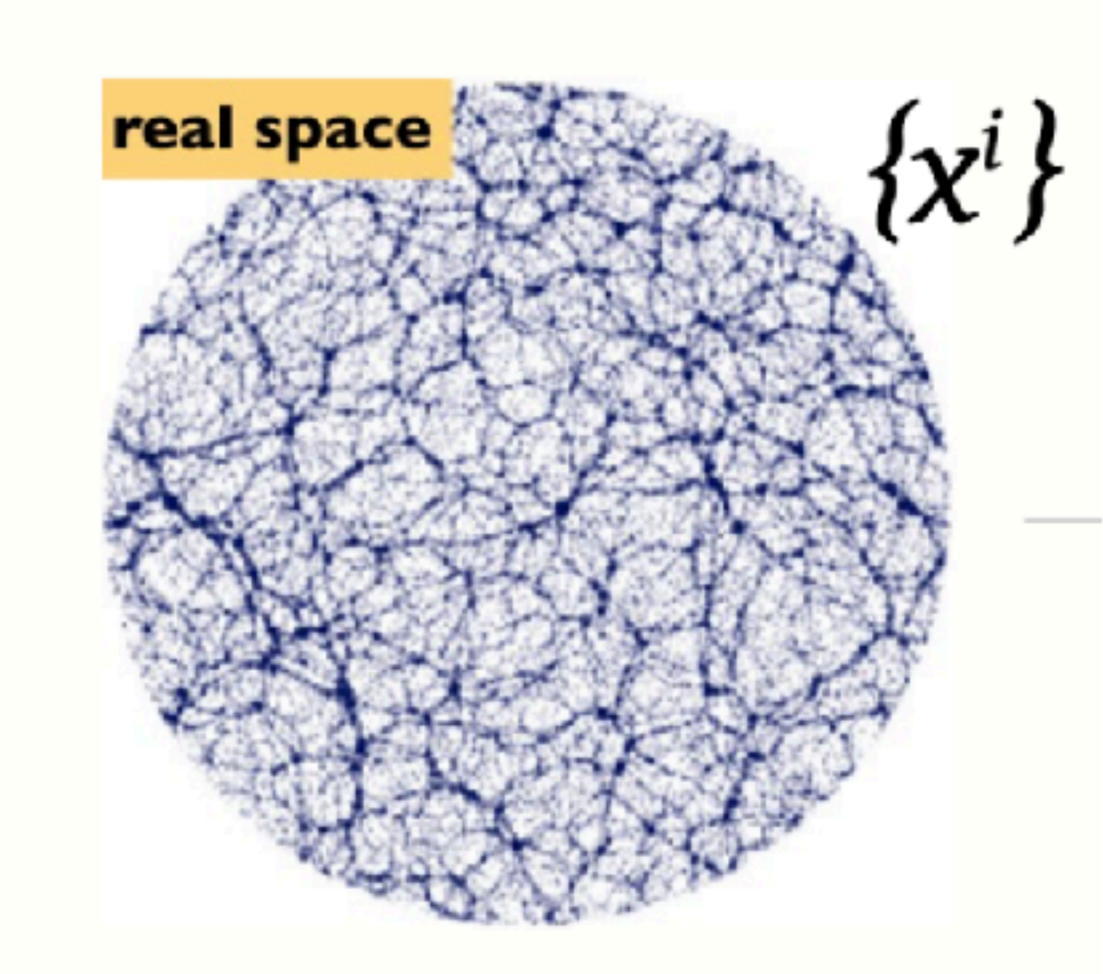
$$\delta_{g,s}(\mathbf{k},z) = \delta_g(\mathbf{k},z) + \frac{i}{\mathscr{H}(z)}\mathbf{k}\cdot\mathbf{v}_{||}(\mathbf{k},z)$$

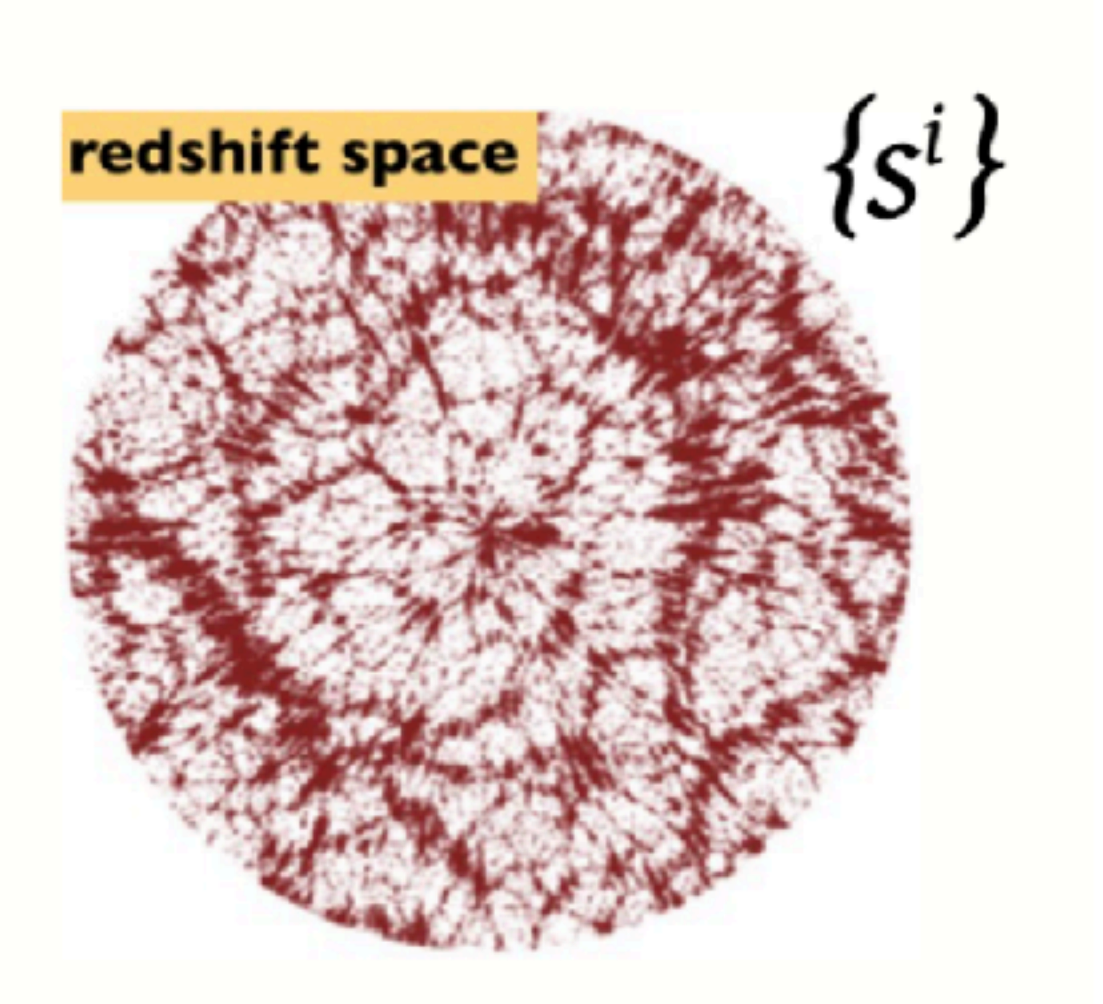
- Remembering $\delta_g = b_1\delta$, $\mathbf{v}(\mathbf{k},z) = i\mathbf{k}/k^2\theta(\mathbf{k},z)$, we get the **Kaiser** formula:

$$\delta_{g,s}(\mathbf{k},z) = \left[b_1(z) + f(z)\mu^2\right] D(z)\delta_L(\mathbf{k})$$

$\mu = \hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$ (line of sight angle)

- Of course, we can go to **arbitrary higher orders!**





Fonseca de la Bella, Kaiser, Peebles, …

Redshift-Space Distortions — III

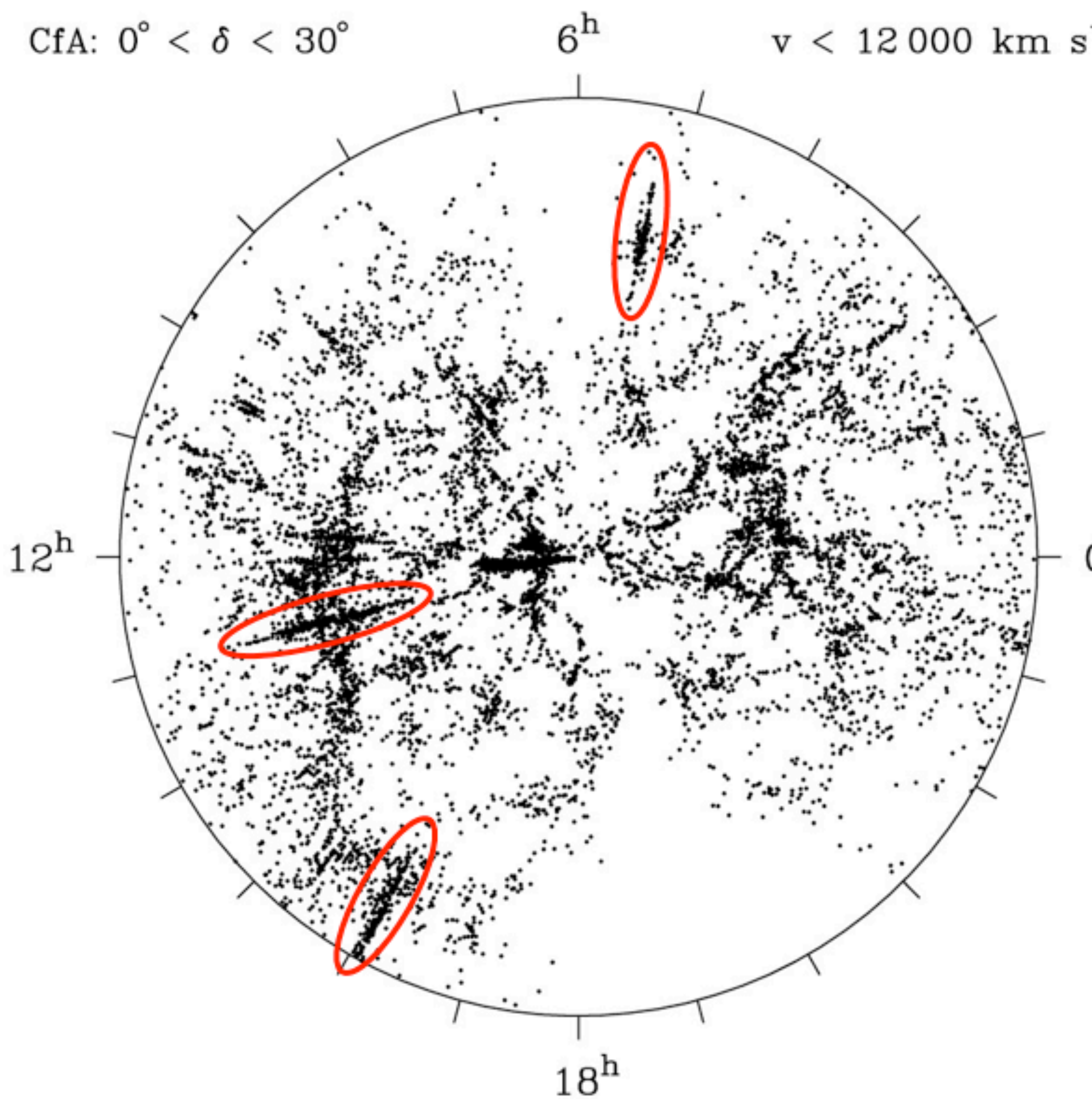
$$\delta_{g,s}(\mathbf{k}) = \int d\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}} \left\{ [(1+\delta_g(\mathbf{x})] e^{i\mathbf{k}\cdot\mathbf{v}_\parallel/\mathscr{H}(z)} - 1 \right\}$$

- The **result**: we get **new kernels** relating $\delta_{g,s}$ to the linear power spectrum

$$\delta_{g,s,n}(\mathbf{k},z) = D_n(z) \int \frac{d\mathbf{p}_1}{(2\pi)^3} \cdots \frac{d\mathbf{p}_n}{(2\pi)^3} Z_n(\mathbf{p}_1,\cdots,\mathbf{p}_n;z)\delta_L(\mathbf{p}_1)\cdots\delta_L(\mathbf{p}_n)\delta_{\mathrm{D}}(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{k})$$

New kernels: now depend on biases and $f(z)$!

- Of course, we need to be careful about **small-scales**
  - **Small-scale** velocities change these expressions: $[\mathbf{v}_\parallel^2]_\Lambda \to \mathbf{v}^2_{\parallel,\Lambda}$ + short scales
  - This gives **new counterterms** which depend on **direction**
  - $\delta_{g,s}(\mathbf{k},z) \supset c(z)D(z)k^2\mu^2\delta_L(\mathbf{k},z) + \cdots$
  - This is the **fingers-of-God** effect



Fingers of God!

Fonseca de la Bella, Kaiser, Peebles, Senatore, Zaldarriaga, …

Redshift-Space Distortions — IV

- We can form **power spectra** as before. At tree-level (***Kaiser***):

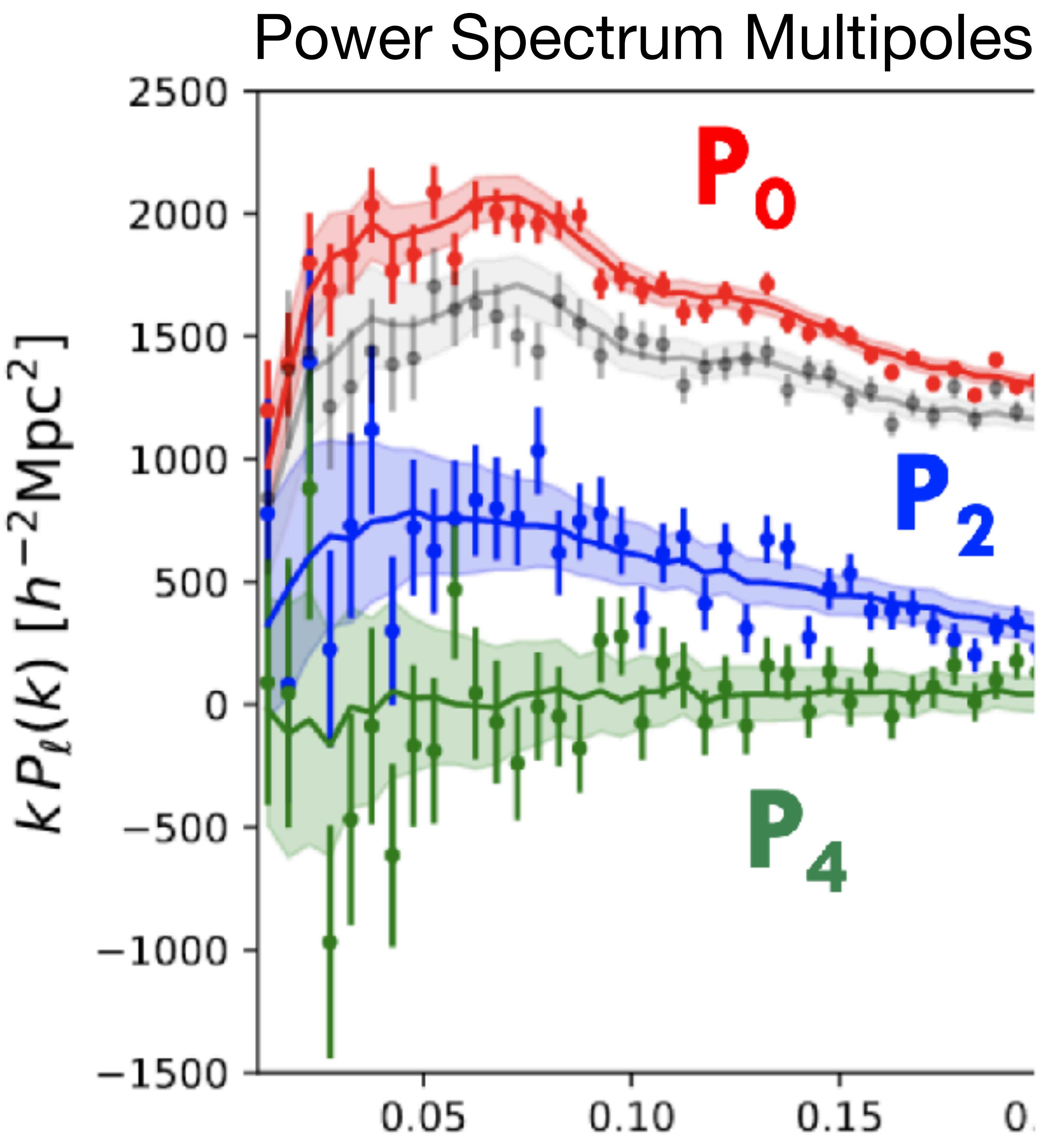
$$P^{\rm tree}_{gg,s}(k,\mu,z) = D^2(z)\left[b_1(z)+f(z)\mu^2\right]^2\left[P_L^{\rm nw}(k)+e^{-k^2\Sigma^2(z)}P_L^{\rm w}(k)\right]+P_{\rm shot}$$

- This is usually as **multipoles:** $P(k,\mu) = \sum_\ell P_\ell(k)\mathscr{L}_\ell(\mu)$

- At **one-loop** we just do some more loop integrals…

- The **main point**: our galaxy density now depends on δ ***and*** $\mathbf{v}_\parallel$

- This is **useful** — $\mathbf{v}_\parallel$ doesn't need any bias parameter — it is directly $\propto \sigma_8^{1/2}$
  - ***This is why DESI can measure σ_8!***



*Note: velocity bias **can** exist…*

Philcox, Ivanov, Zaldarriaga, Simonovic, d'Amico, Senatore, Zhang, Lewandowski…

Codes & Spectra

Power Spectra — I

- We can compute the **full** IR-resummed, UV-renormalized power spectra for galaxies at **1-loop**

- **Ingredients for $P_\ell(k)$ $(\ell \leq 4)$:**
  - **Cosmology**: $D(z), f(z), P_L(k)$
  - **Third-order** bias: $b_1, b_2, b_{\mathcal{G}_2}, b_{\Gamma_3}$
  - **Leading-order** counterterms: c_0, c_2, c_4

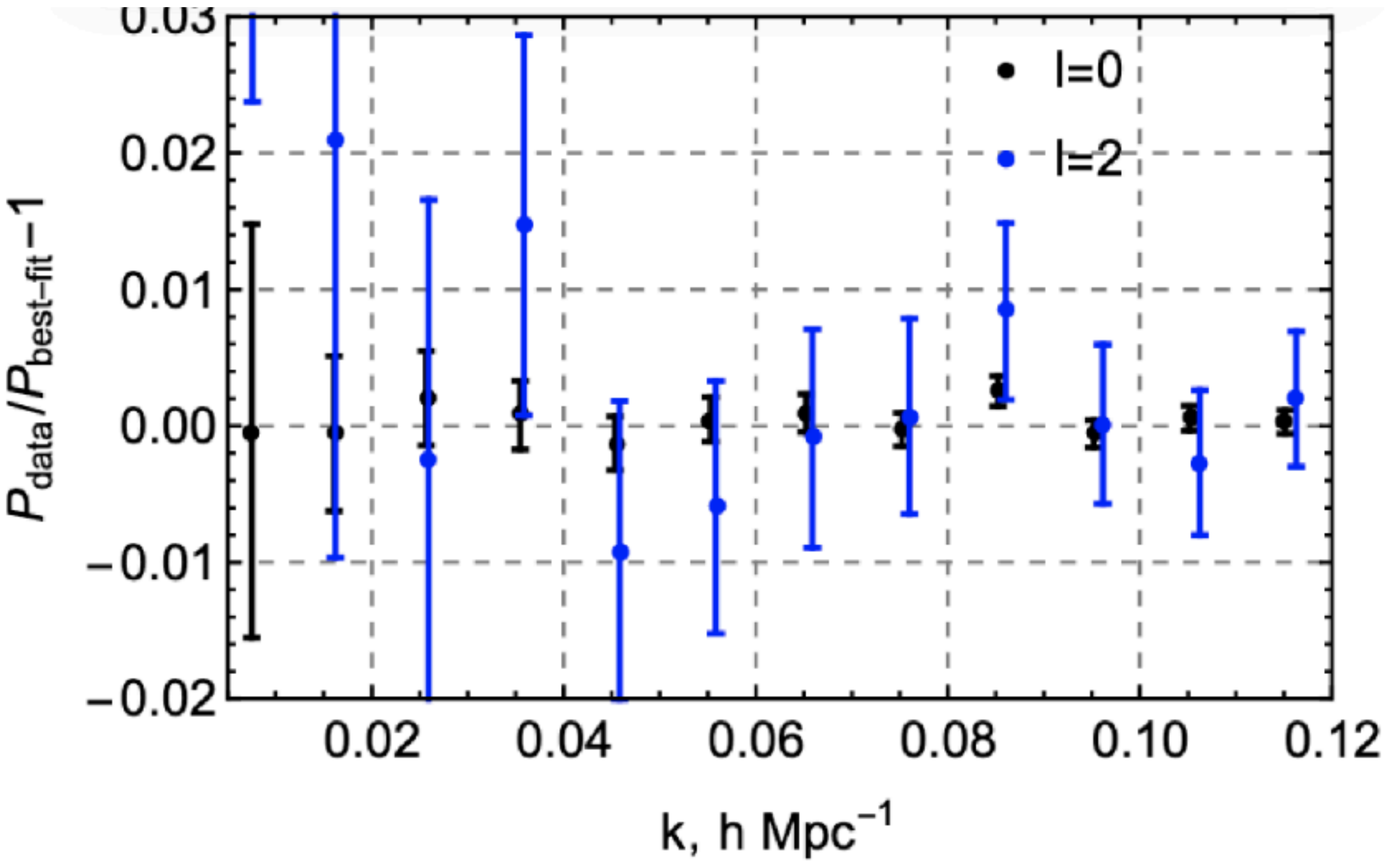
    $[c_0 + c_2\mu^2 + c_4\mu^4]k^2P_L(k)$
  - **Next-to-leading-order** stochasticity: $P_{\text{shot}}, a_0, a_2$

    $P_{\text{shot}}[1 + a_0k^2 + a_2k^2\mu^2]$
  - **Alcock-Paczynski** parameters: $\alpha_\parallel, \alpha_\perp$

    $P(k_\parallel, k_\perp) \to P(\alpha_\parallel k_\parallel, k_\perp \alpha_\perp)$

10 “nuisance” parameters

P_ℓ ratio: $k \lesssim 0.15\, h\text{Mpc}^{-1}$



PT better than 1 % accurate!

Ivanov, Nishimichi, Senatore, Zaldarriaga…

Power Spectra — II

- There are various **public codes** computing power spectrum multipoles using **numerical tricks.**

1. **CLASS-PT** [Ivanov, Chudaykin, Simonovic, Cabass, Philcox, Zaldarriaga]
    - A **modified** version of **CLASS**
    - Computes all 1-loop PT integrals for matter/galaxies in < 1 s
    - Includes **non-Gaussianity** ($f_{\rm NL}$) and **public Montepython likelihoods** (including **bispectra**)
2. **PyBird** [Zhang, d'Amico, Senatore]
    - A **standalone** code taking input from **CLASS/CAMB**
    - Computes 1-loop PT integrals for matter/galaxies
    - Includes **public Montepython likelihoods**
3. **Velocileptors** [Chen, Vlah, White]
    - A **standalone** code taking input from **CLASS/CAMB**
    - Includes both **Eulerian** and **Lagrangian** perturbation theory for **galaxies**

Michalychforever / **CLASS-PT** Public

Nonlinear perturbation theory extension of the Boltzmann code CLASS

17 stars 10 forks

pierrexyz / **pybird** Public

Python code for Biased tracers in redshift space

pybird.readthedocs.io/en/latest/

MIT license

17 stars 12 forks

sfschen / **velocileptors** Public

A code for velocity-based Lagrangian and Eulerian PT expansions of redshift-space distortions.

MIT license

12 stars 3 forks

https://github.com/michalychforever/class-pt

Ivanov, d'Amico, Philcox, Chen, Chudaykin, Simonovic, d'Amico, Zhang…

Power Spectra — III

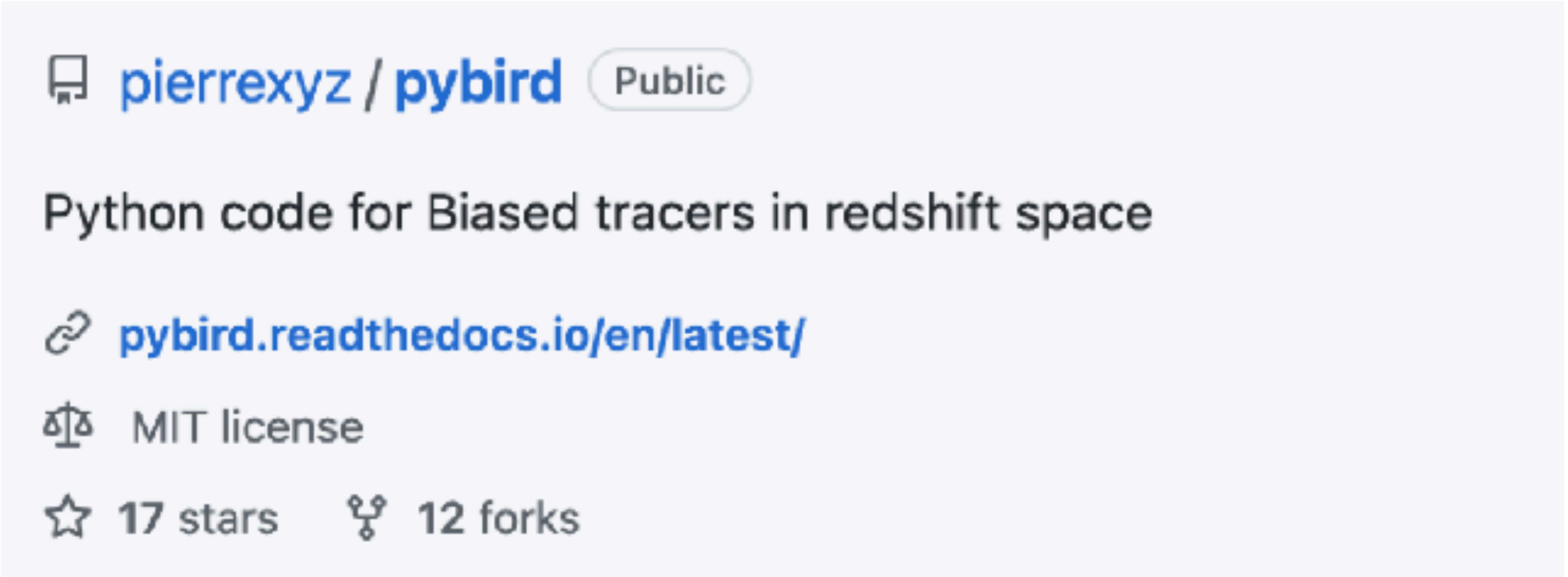
- There are various **public codes** computing power spectrum multipoles using **numerical tricks.**

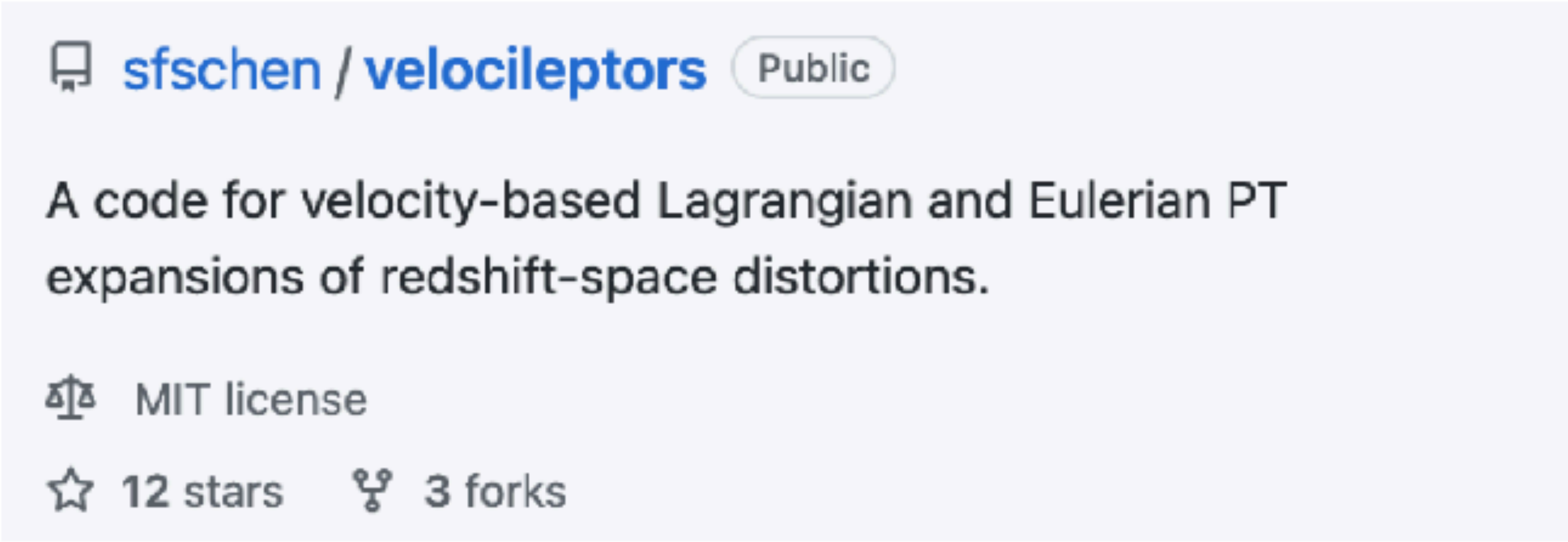
1. **CLASS-PT**
2. **PyBird**
3. **Velocileptors**

- **Many new variants:** PBJ, FOLPSν, CLASS One-Loop…

- These have been **heavily validated** against each other
    - They solve the ***same equations*** in **different ways**

      (biases, priors, IR resummation, Euler/Lagrangian)





https://github.com/michalychforever/class-pt

Ivanov, d'Amico, Philcox, Chen, Chudaykin, Simonovic, d'Amico, Zhang…

Bispectra — I

- We can compute the **full** IR-resummed bispectra for galaxies at **tree-level**

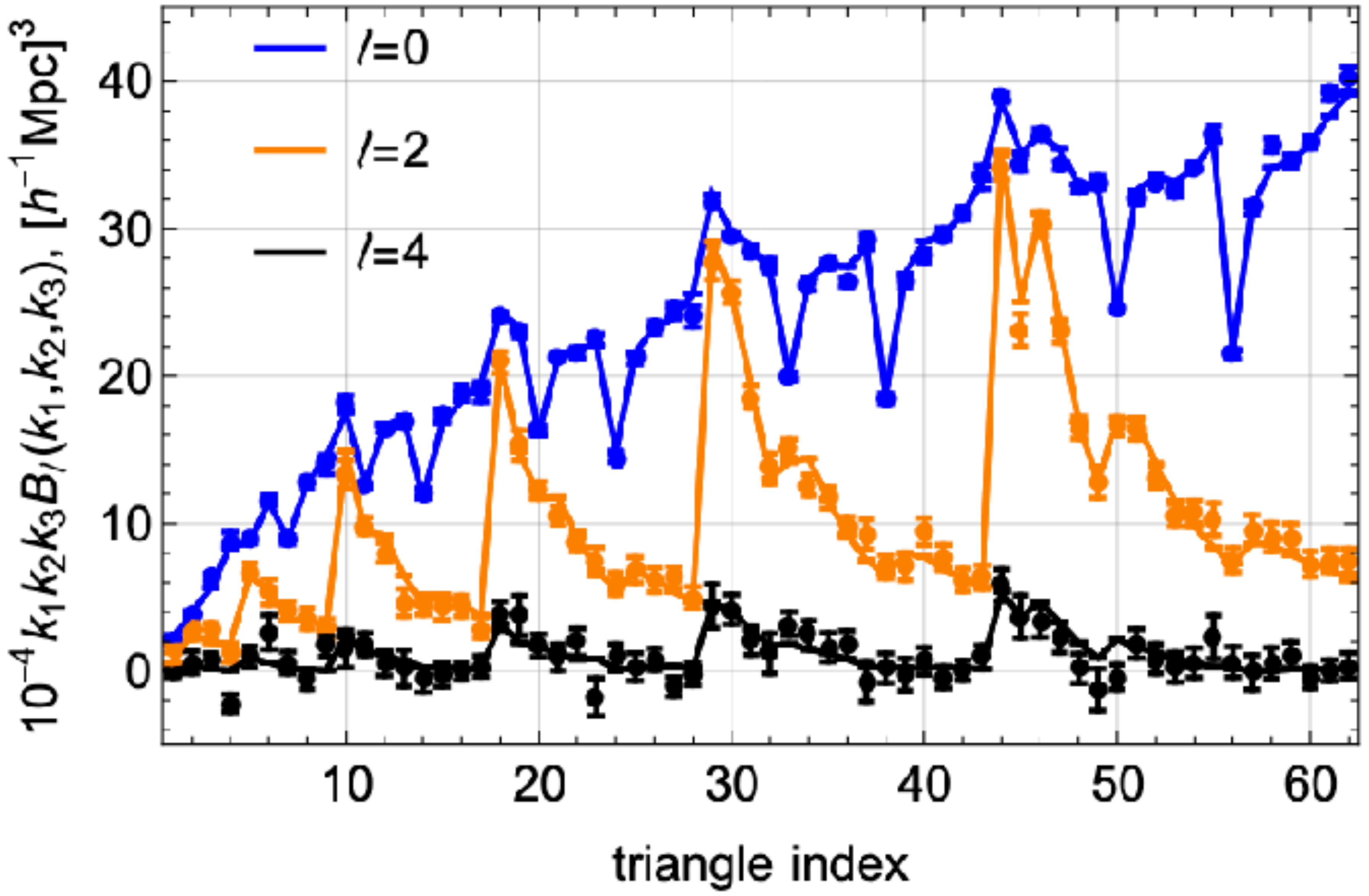
- **Ingredients for $B_\ell^{\rm tree}(k)$ $(\ell \leq 4)$:**
    - **Cosmology**: $D(z), f(z), P_L(k)$
    - **Second-order** bias: $b_1, b_2, b_{\mathcal{G}_2}$
    - **Leading-order** counterterm: c_1 $\quad B_{211} \to c_1 k^2 \mu^2 B_{211}$
    - **Leading-order** stochasticity: $P_{\rm shot}, B_{\rm shot}, A_{\rm shot}$

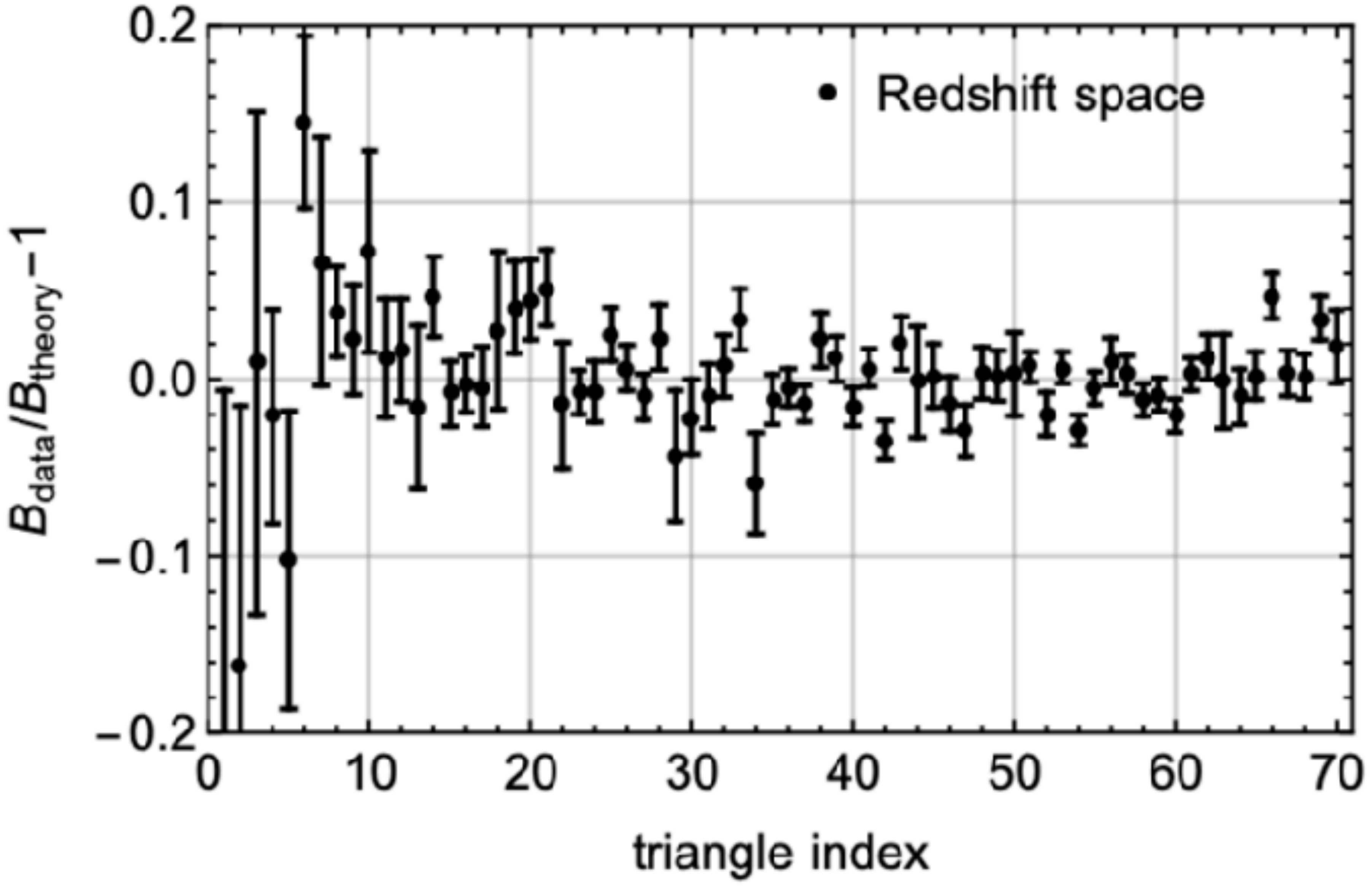
      $[B_{\rm shot} + (1 + P_{\rm shot}) f\mu^2] Z_1 P_L(k) + A_{\rm shot}$
    - **Alcock-Paczynski** parameters: $\alpha_\parallel, \alpha_\perp$

13 “nuisance” parameters

(All included in our public likelihoods!)

https://github.com/oliverphilcox/full_shape_likelihoods





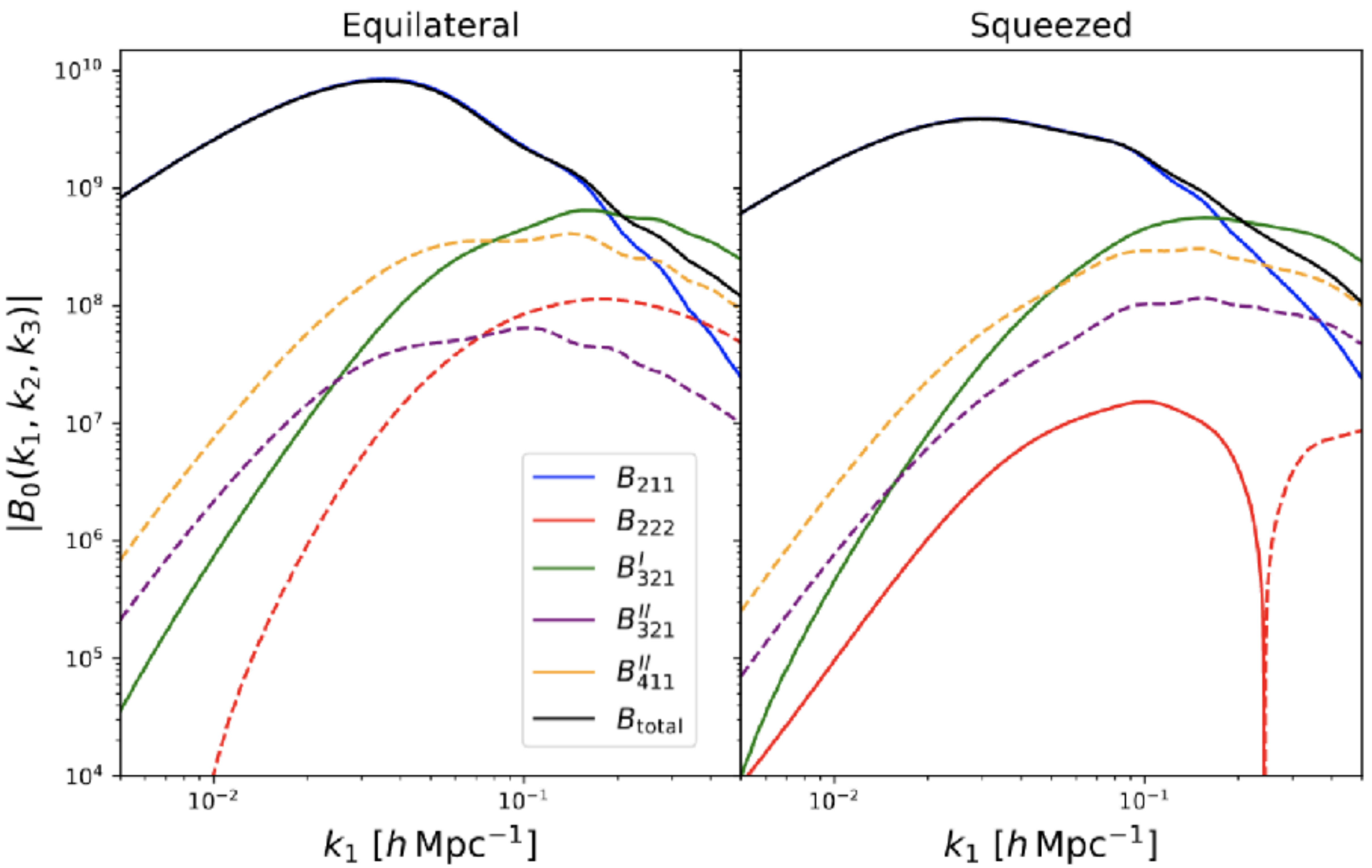
$k \lesssim 0.08\, h\text{Mpc}^{-1}$

Ivanov, Philcox, Nishimichi, Simonovic, Zaldarriaga…

Bispectra — II

$$B_0 : k \lesssim 0.15\, h\text{Mpc}^{-1}$$

- We can **also** compute the **full** bispectra at **one-loop**
- This gets **expensive!** (lots of loops, lots of integrals)
- **Ingredients for $B_\ell^{1-\text{loop}}(k)$ $(\ell \leq 4)$:**
    - **Cosmology**: $D(z), f(z), P_L(k)$
    - **Fourth-order** bias: $b_1, b_2, b_{\mathcal{G}_2}, b_{\mathcal{G}_3}, b_{\Gamma_3}, b_{\delta^3}, \cdots$
    - **Next-to-leading-order** counterterm ($\sim k^2, k^2\mu^2$)
    - **Next-to-leading-order** stochasticity ($\sim 1, k^2, k^2\mu^2$)
    - **Alcock-Paczynski** parameters: $\alpha_{\|}, \alpha_\perp$



https://github.com/oliverphilcox/OneLoopBispectrum

Ivanov, Philcox, d'Amico, Simonovic, Senatore, Zaldarriaga…

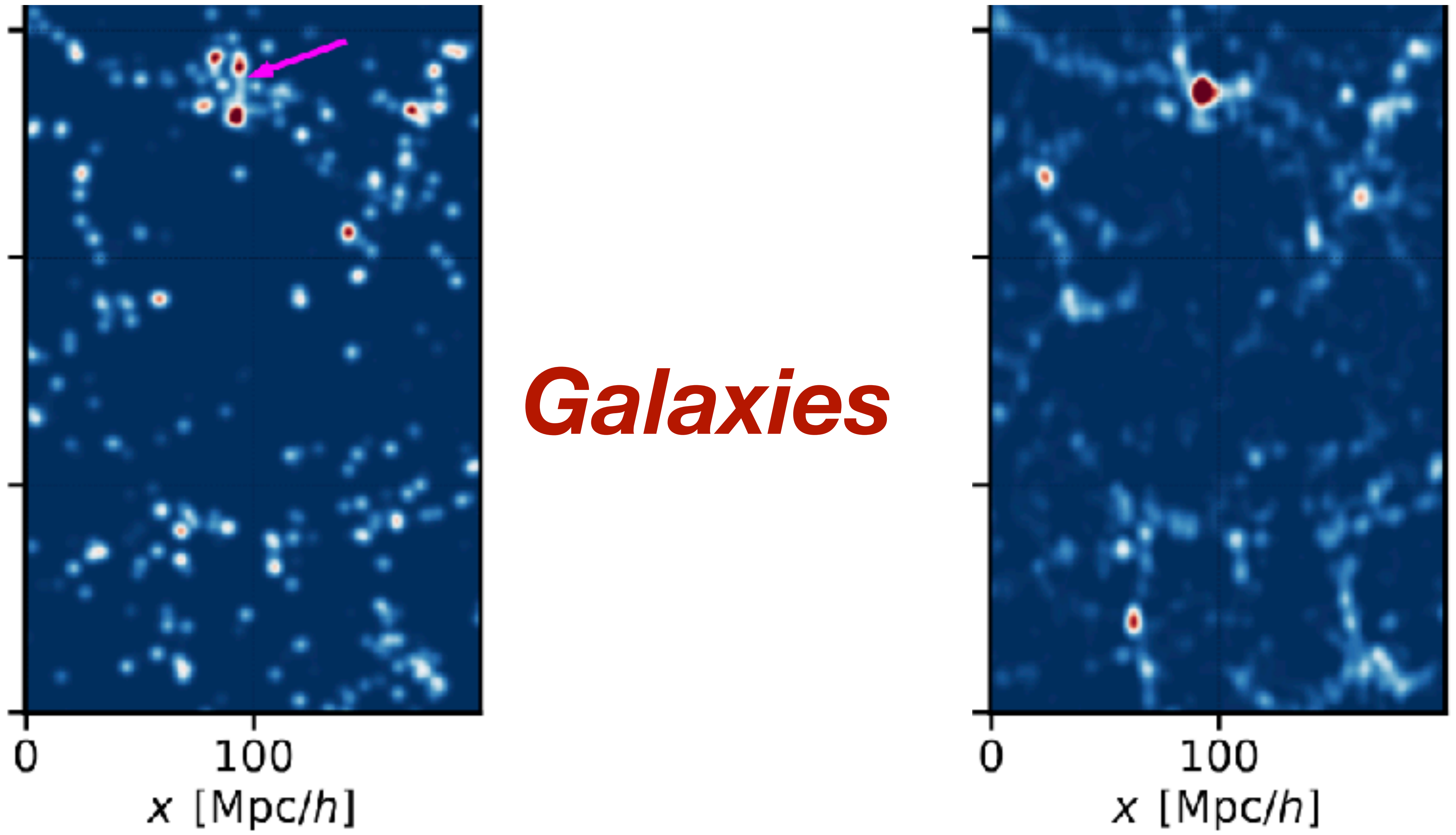
Trispectra

Field-Level

- EFT can **directly** model the field itself, e.g.,
  - HI **intensity mapping**
  - Galaxy **density fields**
- We **don't** need to compress to summary statistics!
- There's much work doing **field-level inference** with perturbation theory likelihoods!

$$\log \mathscr{L}(\hat{\delta} \mid \theta) = \cdots$$

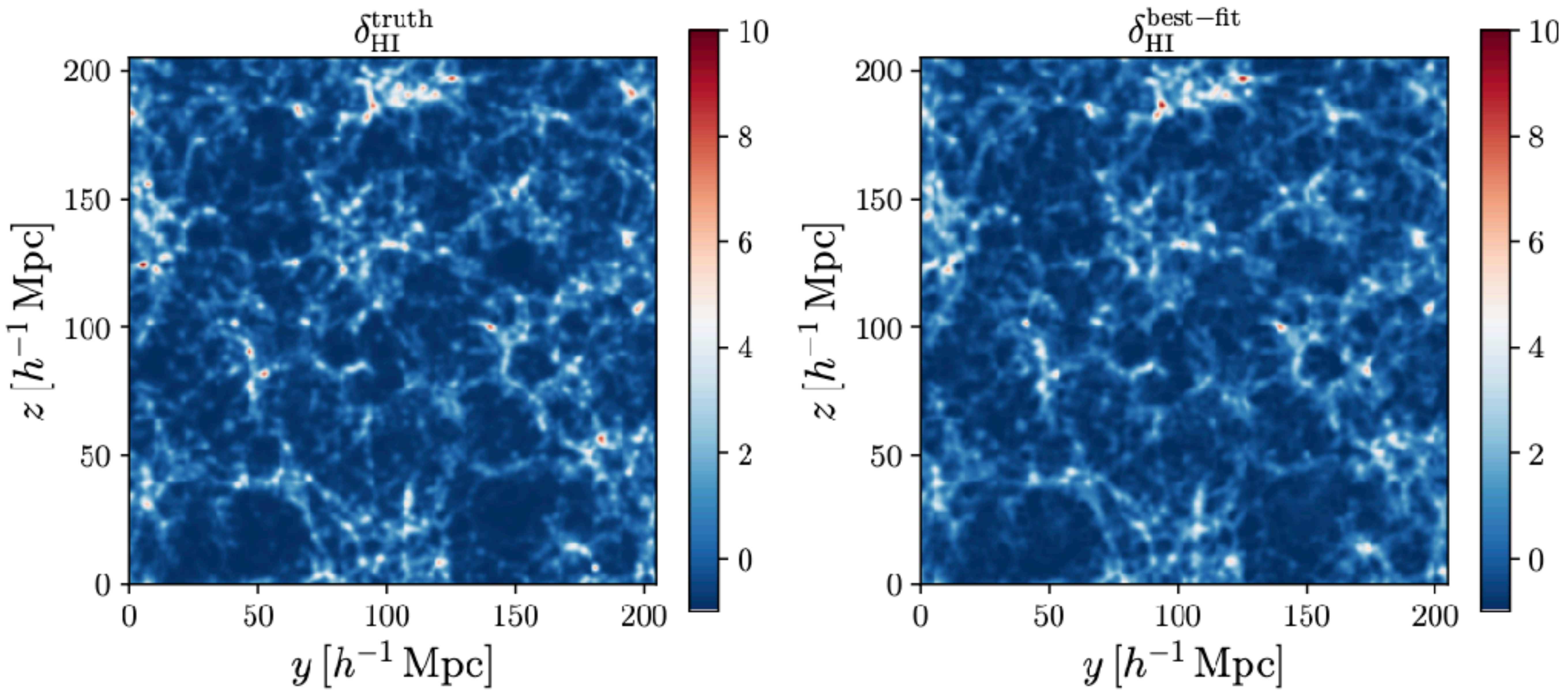
— COTB 2024



Galaxies

True Field

EFT Field



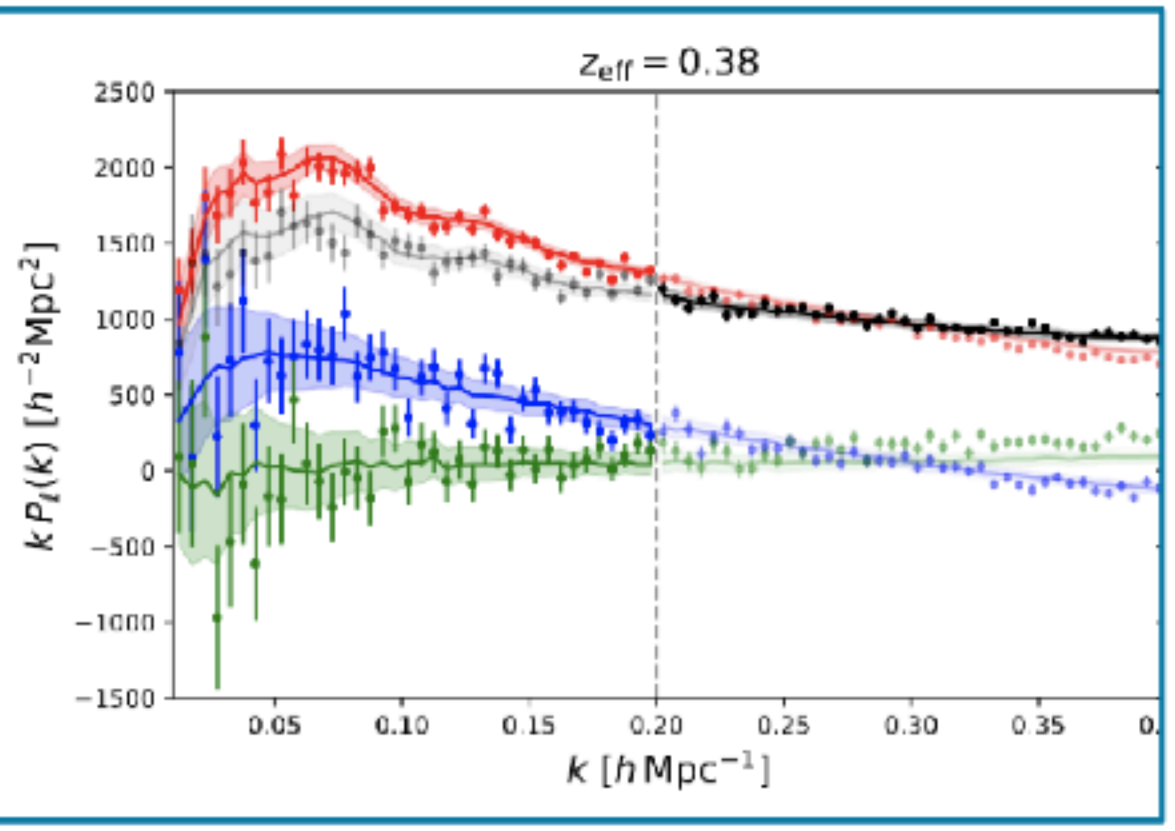
Neutral Hydrogen

Ivanov, Simonovic, Obuljen, Schmittful, Schmidt, Stadler, Tucci…

Cosmological Constraints

Building a Cosmological Likelihood — I

Summary Statistics



EFTofLSS Model

$$
\begin{aligned}
Z_1(\boldsymbol{q}_1) &= K_1 + f\mu_1^2, \qquad\qquad \text{(A.3)}\\
Z_2(\boldsymbol{q}_1,\boldsymbol{q}_2) &= K_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + f\mu_{12}^2 G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{f\mu_{12}q_{12}}{2}K_1\left[\frac{\mu_1}{q_1}+\frac{\mu_2}{q_2}\right] + \frac{(f\mu_{12}q_{12})^2}{2}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2},\\
Z_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) &= K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + f\mu_{123}^2 G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\\
&\quad + (f\mu_{123}q_{123})\left[\frac{\mu_{12}}{q_{12}}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_3}{q_3}K_2(\boldsymbol{q}_1,\boldsymbol{q}_2)\right]\\
&\quad + \frac{(f\mu_{123}q_{123})^2}{2}\left[2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_1\right] + \frac{(f\mu_{123}q_{123})^3}{6}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3},\\
Z_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) &= K_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) + f\mu_{1234}^2 G_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4)\\
&\quad + (f\mu_{1234}q_{1234})\left[\frac{\mu_{123}}{q_{123}}K_1G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_4}{q_4}K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\right.\\
&\qquad\qquad \left. + \frac{\mu_{12}}{q_{12}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^2}{2}\left[2\frac{\mu_{123}}{q_{123}}\frac{\mu_4}{q_4}G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_{12}}{q_{12}}\frac{\mu_{34}}{q_{34}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)G_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right.\\
&\qquad\qquad \left. + 2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^3}{6}\left[3\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}K_1\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^4}{24}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4},
\end{aligned}
$$

GitHub.com/oliverphilcox/full_shape_likelihoods

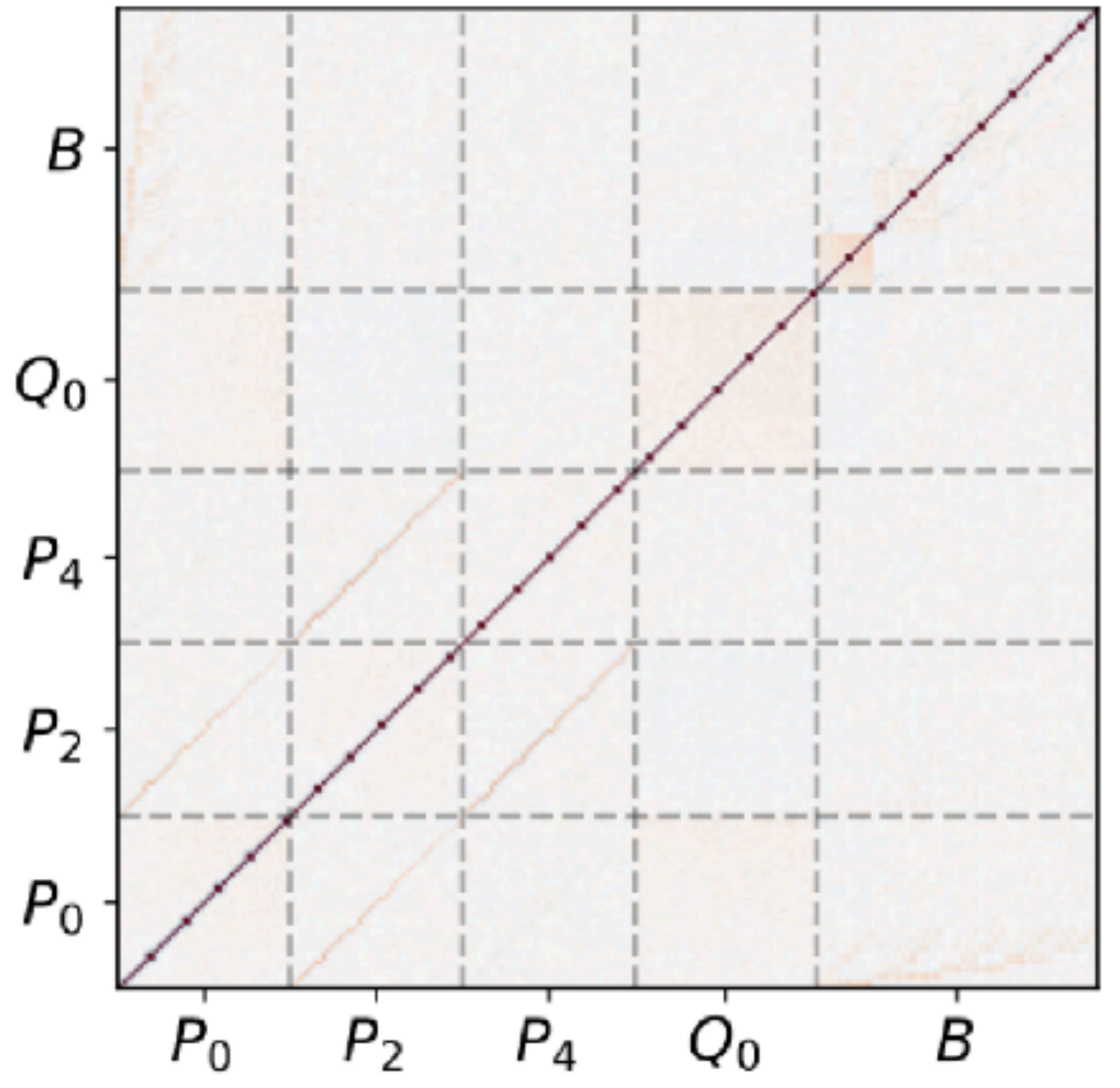
Gaussian likelihood

$$-2\log L = (\hat{P} - P_{\rm theory})C^{-1}(\hat{P} - P_{\rm theory})$$

MCMC

Constraints on $H_0, \Omega_m, \sigma_8, b_1, P_{\rm shot}, \ldots$

Covariance Matrices



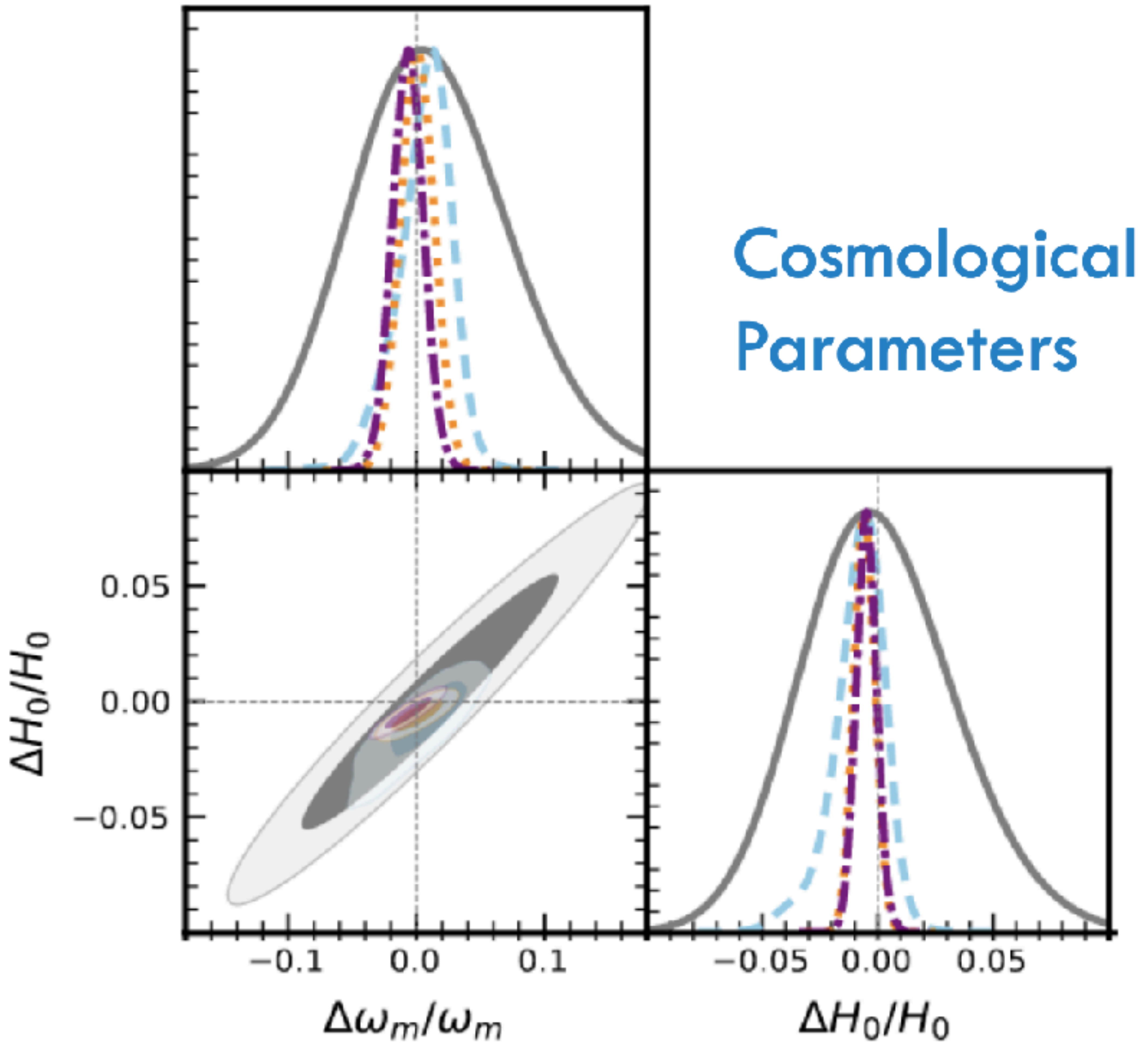
Analysis takes O(10) CPU-hours!

Ivanov, d'Amico, Philcox, Chen, Chudaykin, Simonovic, d'Amico, Zhang…

Building a Cosmological Likelihood — II

- In principle, we can constrain **any** parameter that enters:
  - The growth factor $D(z)$
  - The growth rate $f(z)$
  - The power spectrum $P_L(k)$!

- The perturbative model has been **heavily validated** with N-body simulations
- For example, the **PT-Challenge** simulations test EFT in a $566\,h^{-3}\mathrm{Gpc}^3$ box
- Running **MCMC** analyses on simulated data finds **unbiased** results

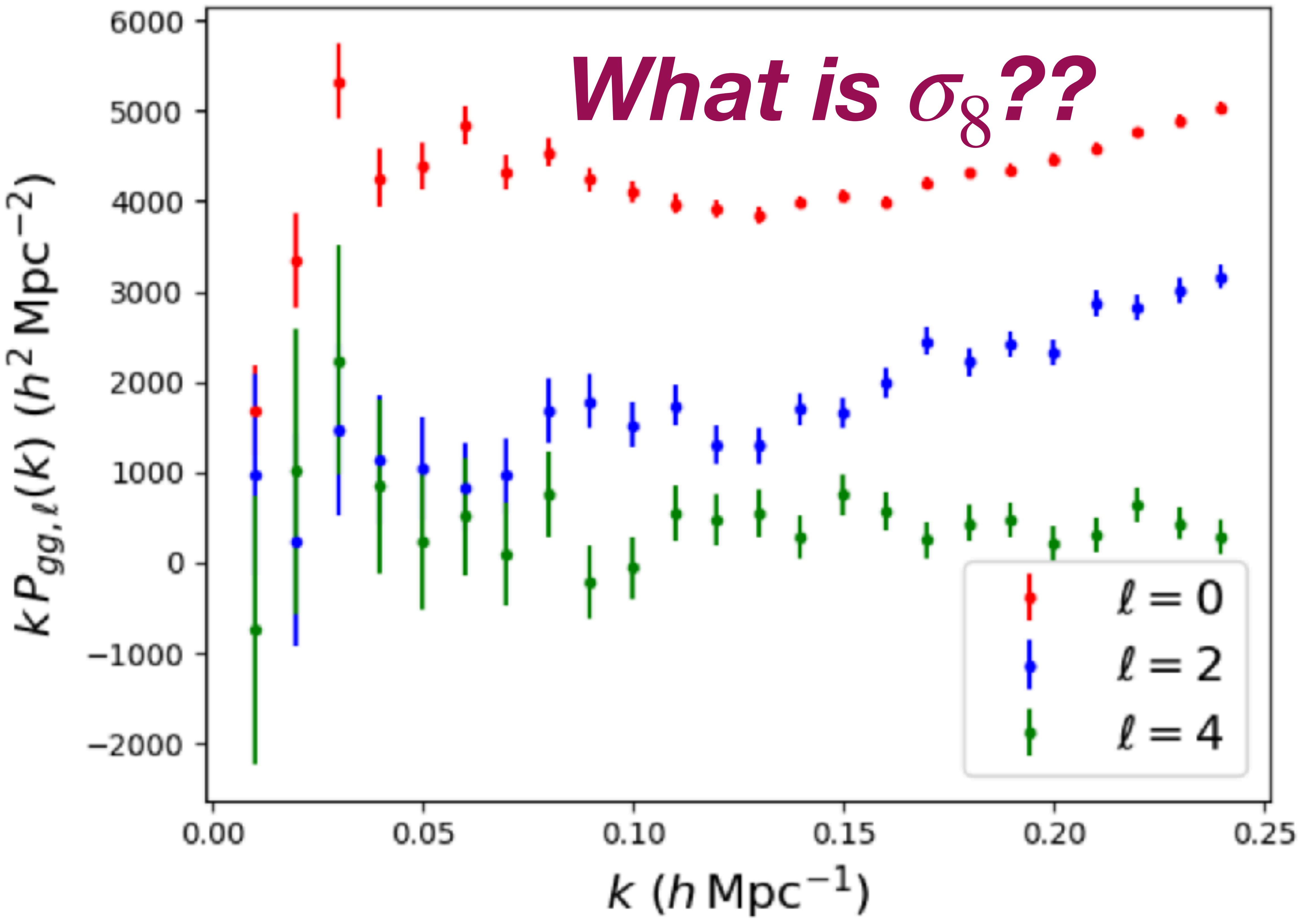


Ivanov, Nishimichi, Senatore, Zaldarriaga…

TIME FOR A BREAK!

https://tinyurl.com/myfirstsigma8

- How does this work in practice?



SCAN ME

Run your own σ_8 analysis here!

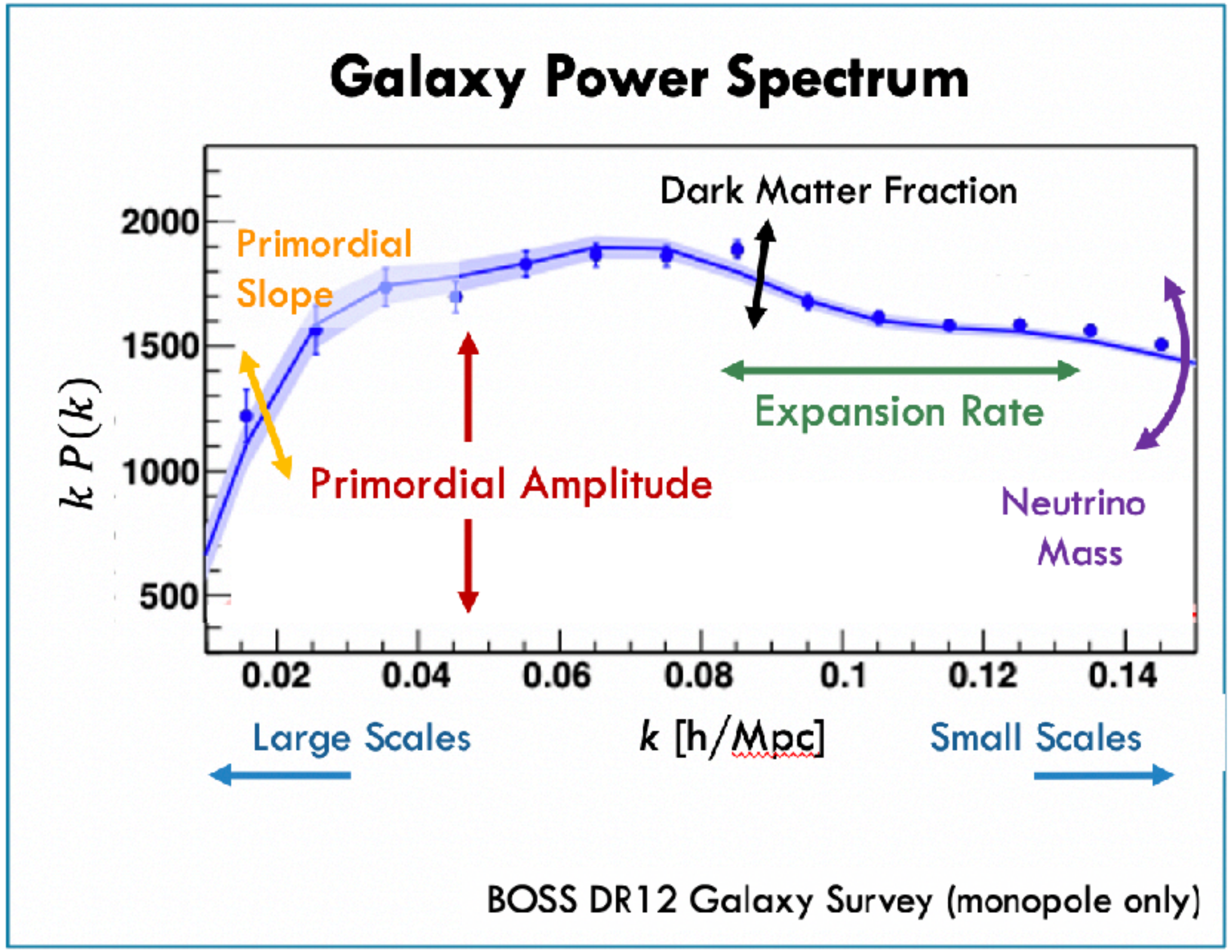
What have we learnt about cosmology?

- Boring ΛCDM (**DESI 2024**):
  - H_0: 68.6 ± 0.8 [from BAO] *or* 71 ± 4 [from k_{eq}] km/s/Mpc
  - σ_8 : 0.84 ± 0.03
  - Ω_m: 0.30 ± 0.01
  - $\sum m_\nu < 0.4\,\mathrm{eV}$

All agrees with *Planck* CMB!

No *strong* evidence for anything weird!

- Fun stuff (**BOSS**):
  - Curvature $\Omega_k \approx 0$
  - Early dark energy, $f_{\mathrm{EDE}} \approx 0$
  - Axion dark matter, $f_{\mathrm{axion}} \approx 0$
  - Massive relics, modified gravity, interacting neutrinos, dark sector, friction, …



All analysis is public: github.com/oliverphilcox/full_shape_likelihoods

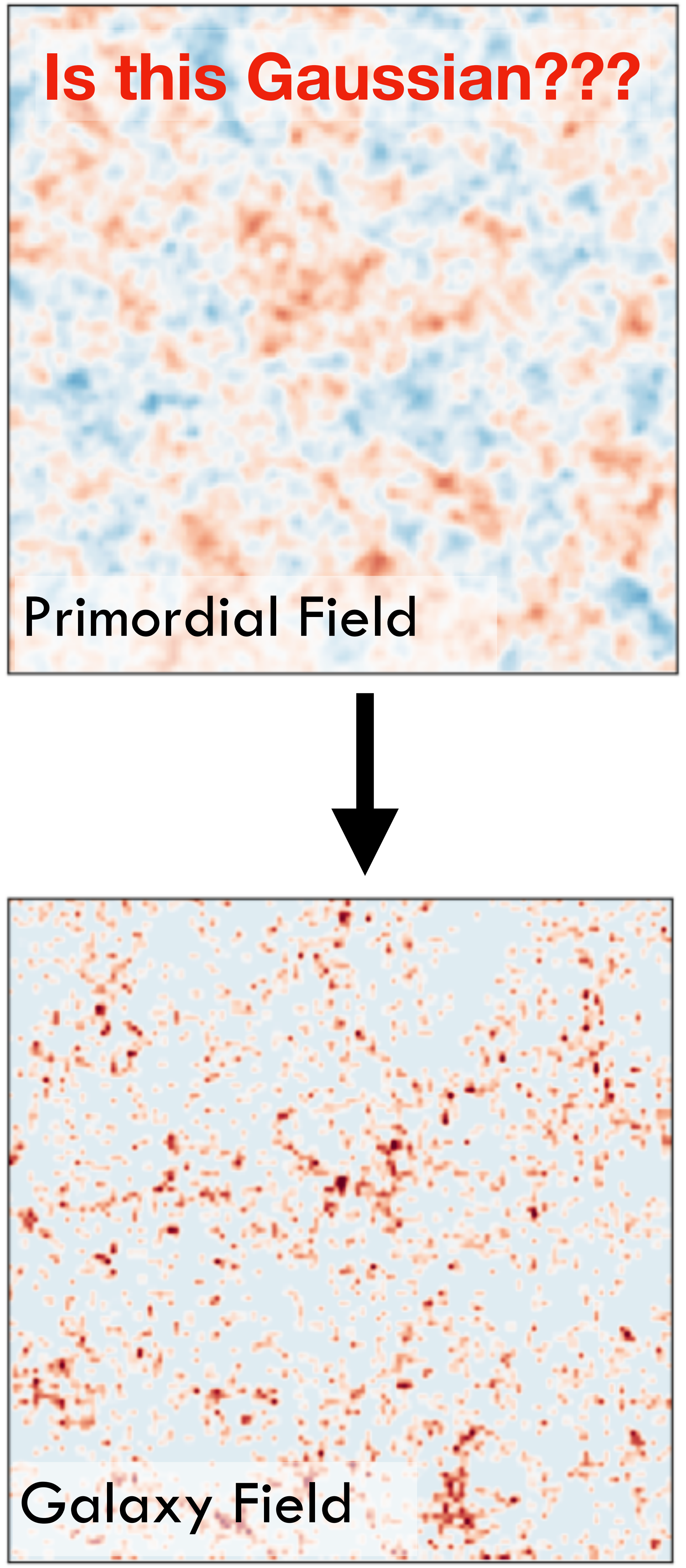
Ivanov, d'Amico, Philcox, Chen, Chudaykin, Simonovic, d'Amico, Zhang…

Primordial Non-Gaussianity — I

- Up to now, we have assumed $B_L \sim \langle \delta_L(\mathbf{k}_1)\delta_L(\mathbf{k}_2)\delta_L(\mathbf{k}_3)\rangle = 0$.
- **New physics can change this!**
- Examples:
  - $f_{\rm NL}^{\rm loc}$: Extra **light-fields** in inflation
  - $f_{\rm NL}^{\rm eq,orth}$: **Self-interactions** in inflation
  - “Cosmological Collider Physics” in inflation
- We can **probe** these phenomena using LSS!

$$P_\ell = P_\ell(\text{bias}, {\color{blue}\text{cosmo}}, {\color{red}\text{PNG}})$$

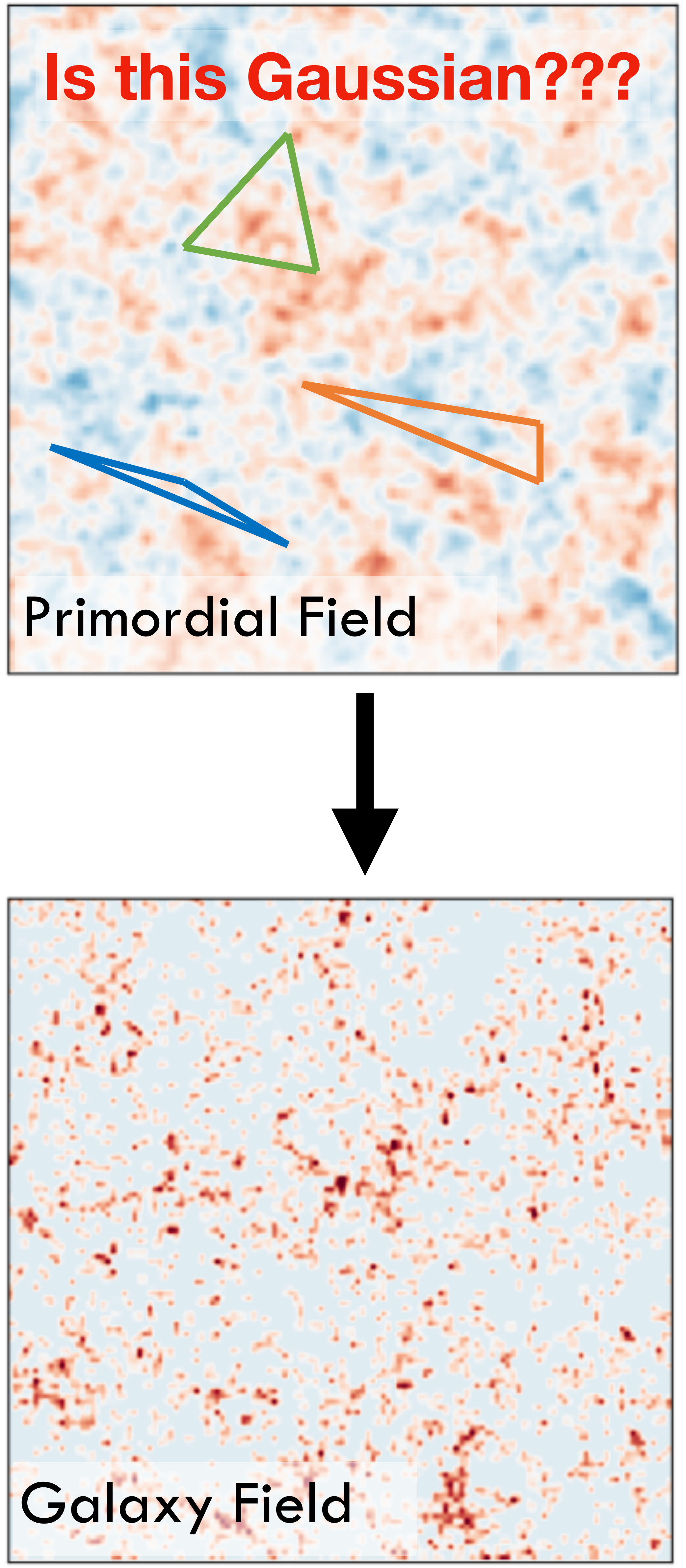
$$B_\ell = B_\ell(\text{bias}, {\color{blue}\text{cosmo}}, {\color{red}\text{PNG}})$$



Senatore, Assassi, Zaldarriaga, Schmidt…

Primordial Non-Gaussianity — Ib

- **Light fields** are produced from the vacuum:
  - These act as **isocurvature modes** ⇒ **local shape**
- **Heavy fields** are produced from the vacuum:
  - These **decay** but can **couple** to the inflaton ⇒ **local-like shape**
- **Very heavy** fields are resonantly produced from the vacuum:
  - These have **oscillatory** signals ⇒ **local-like shape + oscillations**
- **Spinning** massive fields are produced from the vacuum:
  - These have peculiar **spin-dependence** ⇒ **local-like shape + angular features**
- The Lagrangian can be **non-linear**
  - e.g. inflaton has a **sound-speed** $c_s < 1$ ⇒ **equilateral & orthogonal shapes**
- The inflationary **vacuum** can be non-Bunch Davies
  - e.g. alpha-vacua: ⇒ **folded shape**



Senatore, Assassi, Zaldarriaga, Schmidt…

Primordial Non-Gaussianity — II

- How does PNG change the theory?

1. New **tree-level** contributions:

$$B_{ggg}(\mathbf{k}_1, \mathbf{k}_2, z) \supset B_{111}(\mathbf{k}_1, \mathbf{k}_2, z) \equiv Z_1(\mathbf{k}_1, z) Z_1(\mathbf{k}_2, z) Z_1(\mathbf{k}_3, z) B_L(\mathbf{k}_1, \mathbf{k}_2) \propto f_{\mathrm{NL}}$$

2. New **loop corrections**:

$$P_{gg}(\mathbf{k}, z) \supset P_{12}(\mathbf{k}, z) \equiv 2 \int_{\mathbf{p}} Z_1(\mathbf{k}) Z_2(\mathbf{p}, -\mathbf{k} - \mathbf{p}) B_L(\mathbf{p}, \mathbf{k}) \propto f_{\mathrm{NL}}$$

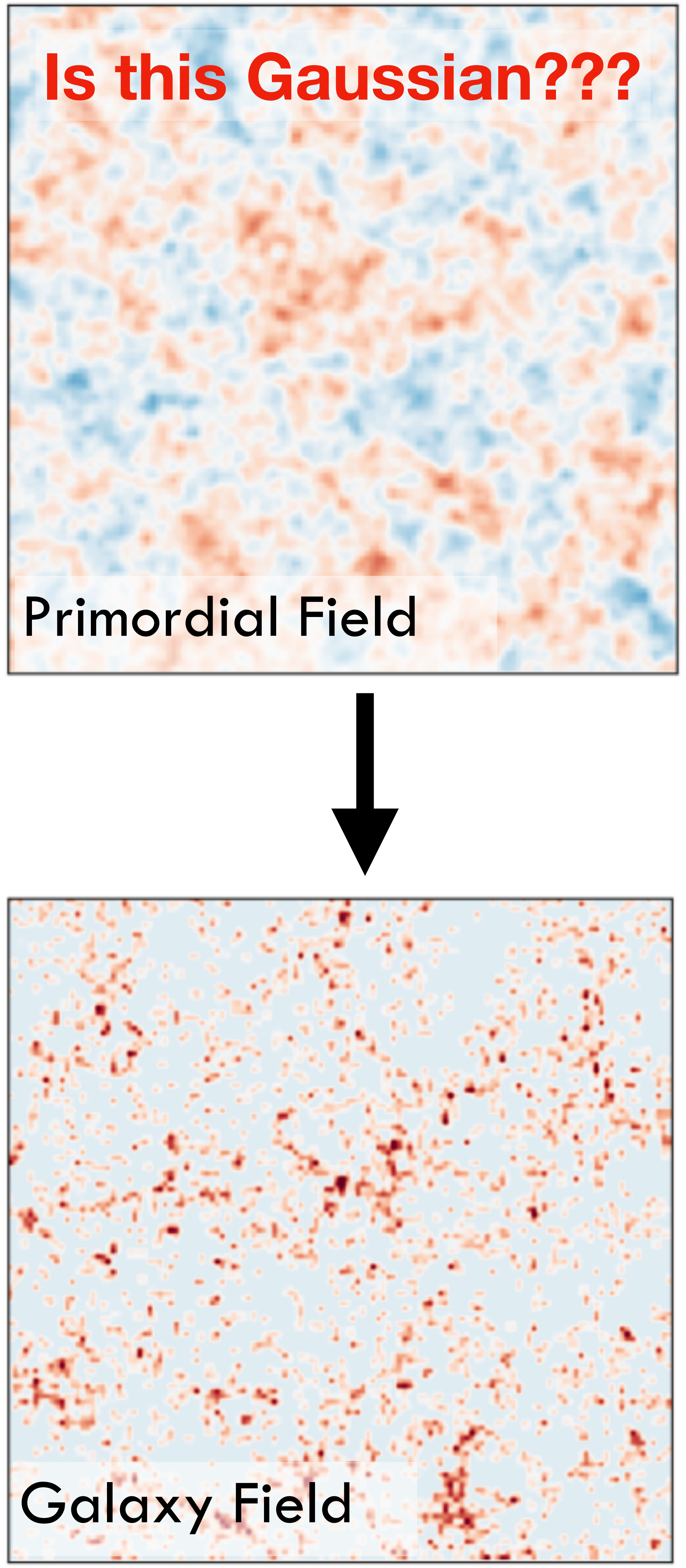
3. New **bias terms** / **counterterms**:

$$\delta_g(\mathbf{k}, z) \supset b_1 \delta + b_\phi f_{\mathrm{NL}}^{\mathrm{loc}} \delta / k^2 + \cdots$$

(This accounts for the cut-off dependence of P_{12}!)

- We must take **all** of these into account!

(Already done in CLASS-PT!)

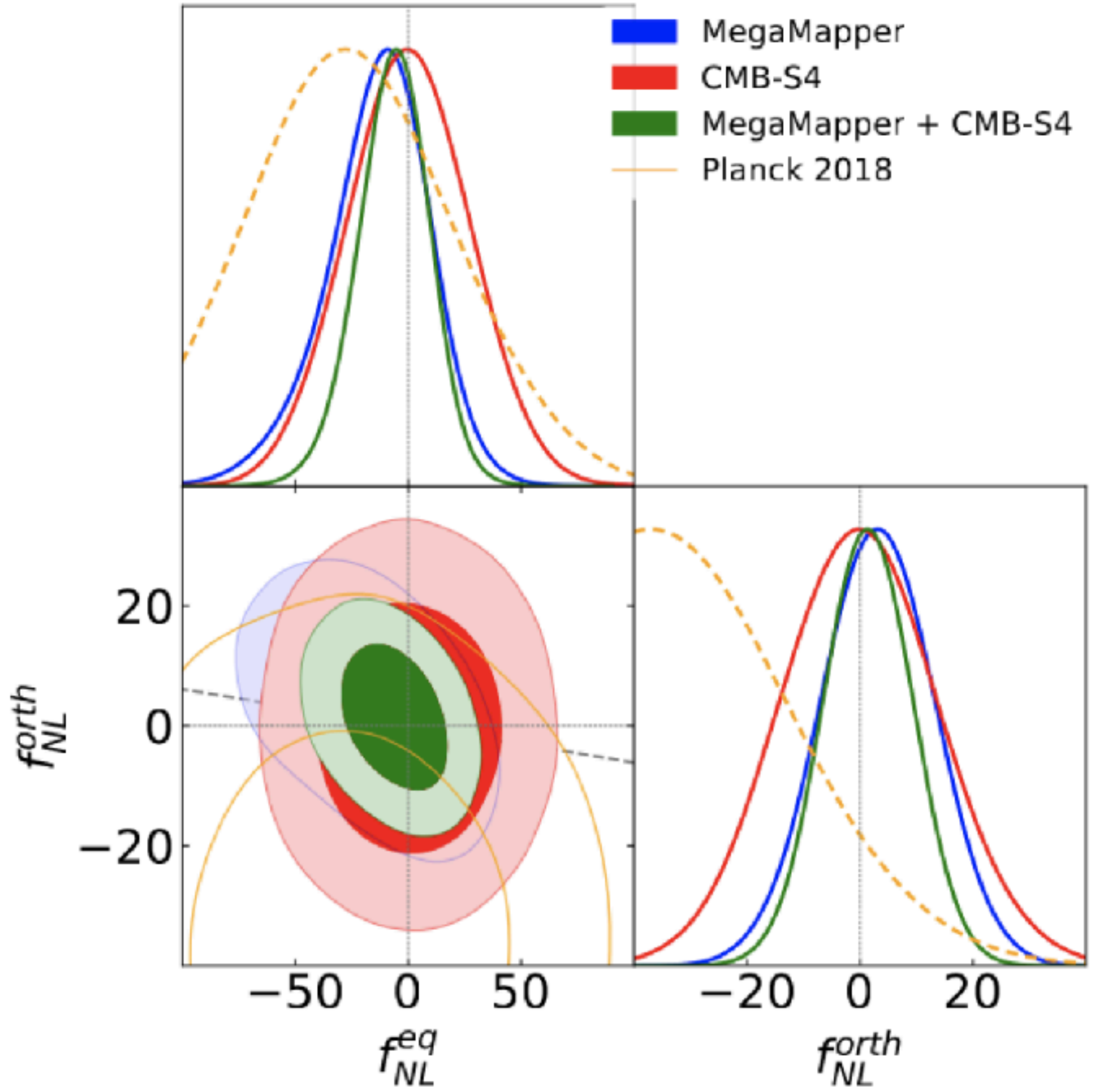


Senatore, Assassi, Zaldarriaga, Schmidt, Cabass, Ivanov, Philcox, d'Amico…

Primordial Non-Gaussianity — III

- This has been used in data to constrain PNG!
  1. $f_{\rm NL}^{\rm loc}: -4 \pm 9$ [DESI **P**], -33 ± 28 [BOSS **P+B**]
  2. $f_{\rm NL}^{\rm eq}: 260 \pm 300$ [BOSS **P+B**]
  3. $f_{\rm NL}^{\rm orth}: -23 \pm 120$ [BOSS **P+B**]
  4. Massive-particle $f_{\rm NL}$: **roughly zero** [BOSS]

- There's **many** more things to explore, e.g., $g_{\rm NL}, \tau_{\rm NL}$, spins, masses, …
- Some of these require **trispectra**!

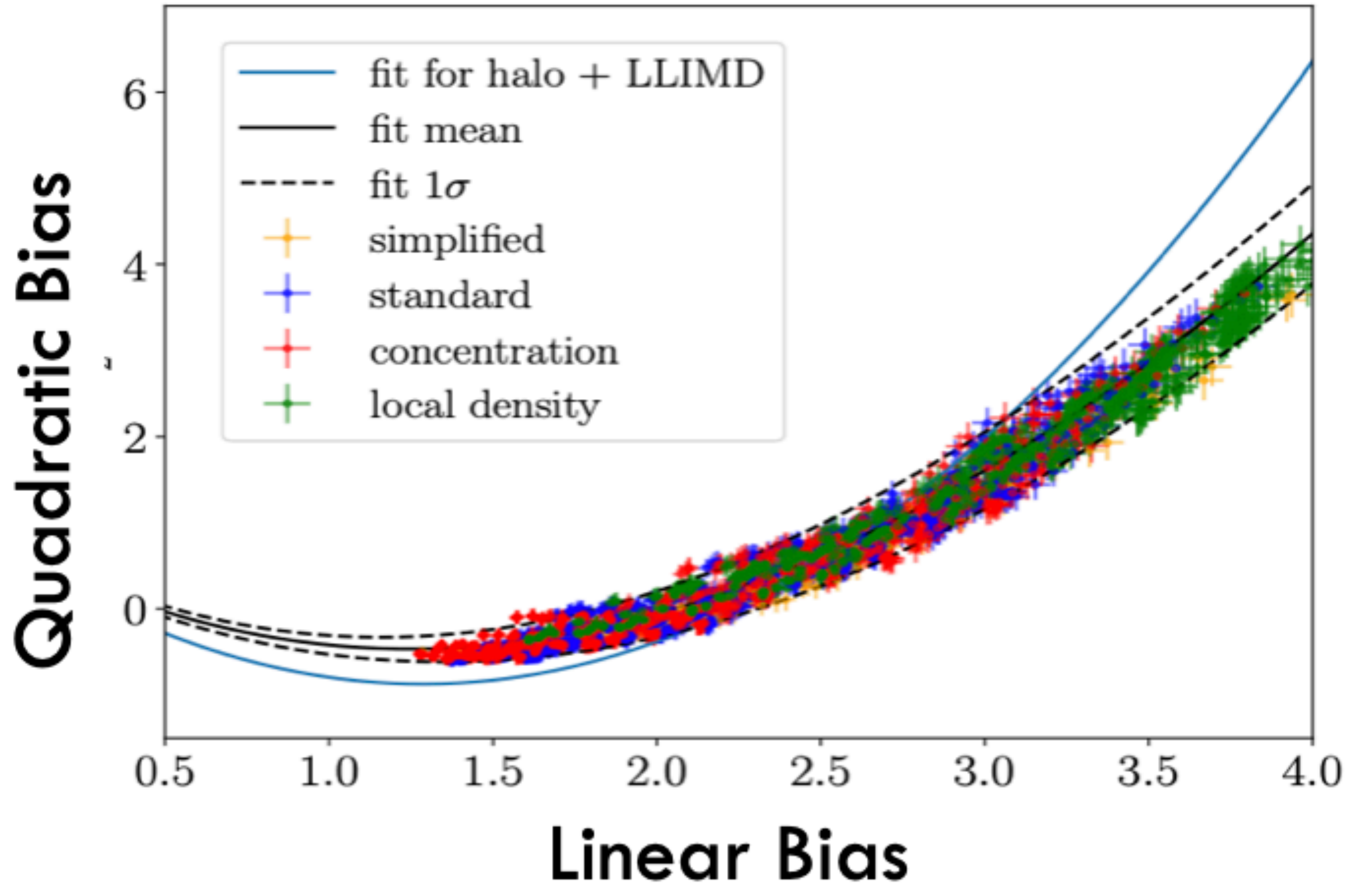


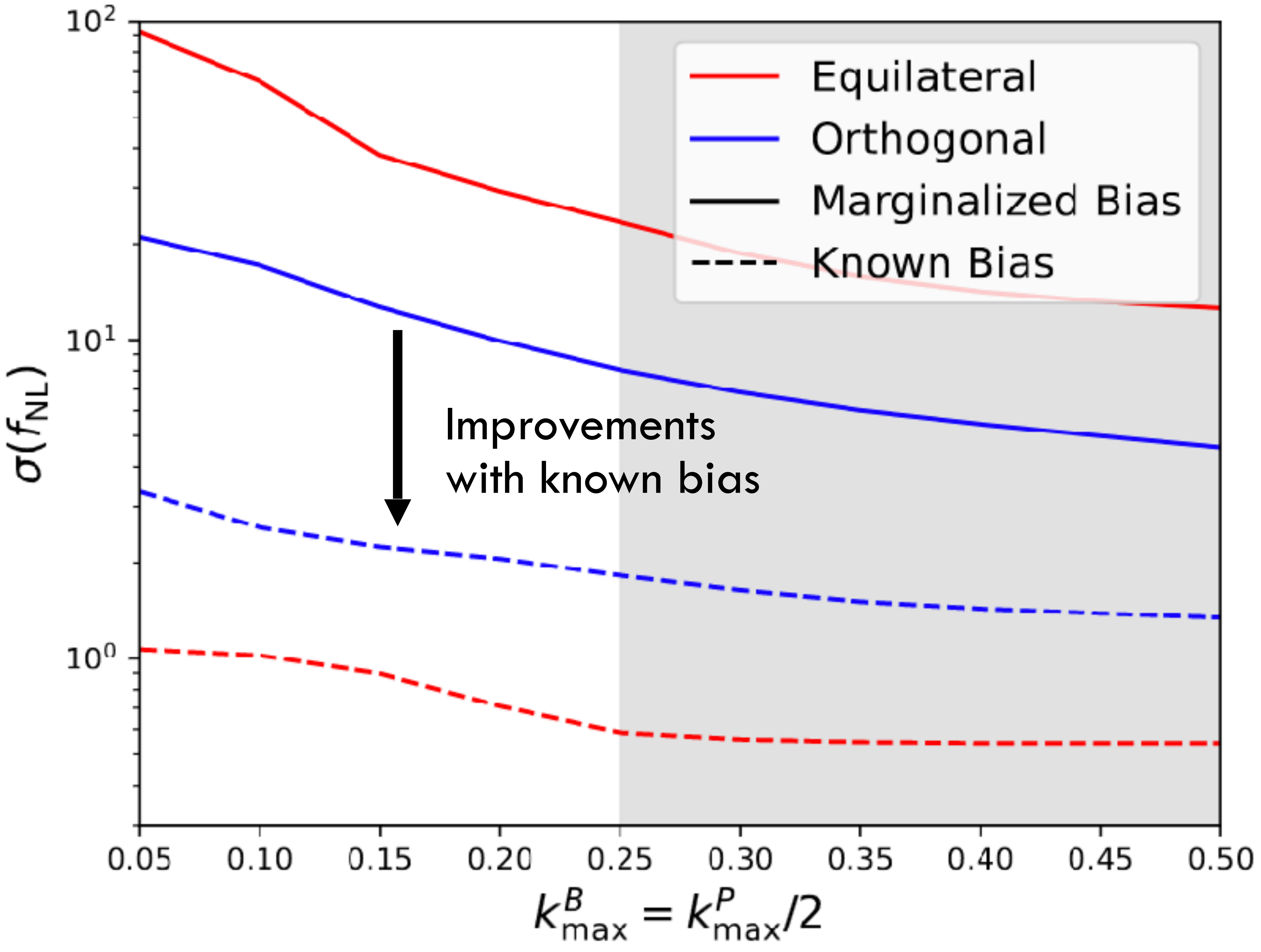
MegaMapper $f_{\rm NL}$ forecast

Senatore, Assassi, Zaldarriaga, Schmidt, Cabass, Ivanov, Philcox, d'Amico…

Biases & Priors

- EFT constraints are **limited** by bias parameters
- For $f_{\rm NL}$, constraints improve by $\gtrsim 10\times$ if we know bias!
- Better **priors** on bias parameters could help
  - ***However:*** we must be careful not to **bias** the result!



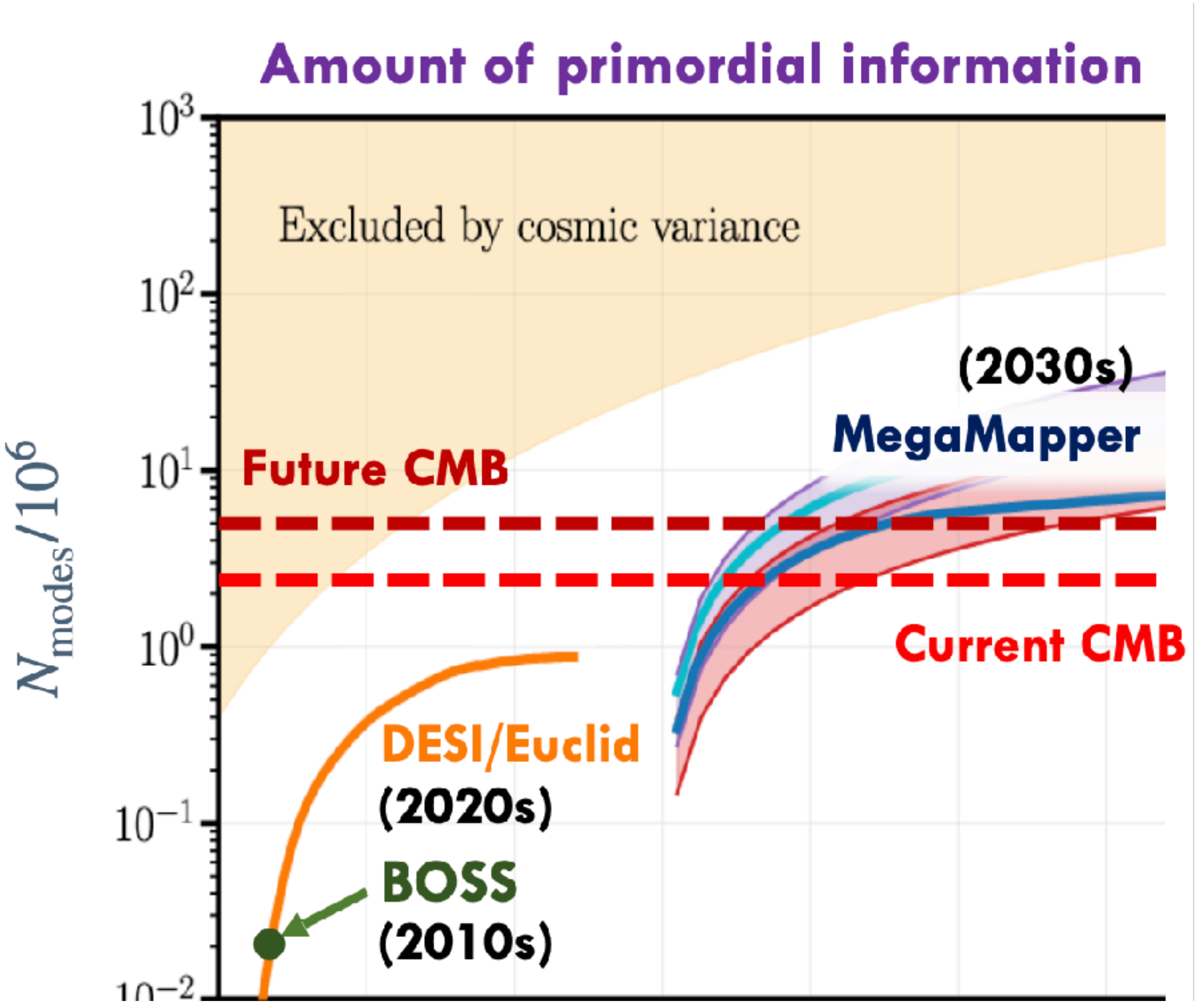


Akitsu, Ivanov, Toomey, Cuesta-Lazarro, Chen, Zhang, Modi…

What's next for PT?

- There are **many** more things to explore:
    - 2-loop power spectra
    - **Fast** 1-loop bispectra
    - Tree-level and 1-loop trispectra
    - **Field-level** inference
    - Many more types of **new physics**

- Whilst EFT is great, there's some **limitations**:
    - Limited to **large-scales** $k < k_{\rm NL}$
    - Limited by knowledge of **bias parameters**
    - Limited by **computation** at high-order!



Senatore, Assassi, Zaldarriaga, Schmidt, Cabass, Ivanov, Philcox, d'Amico, Ferraro, Sailer…

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Thanks!!