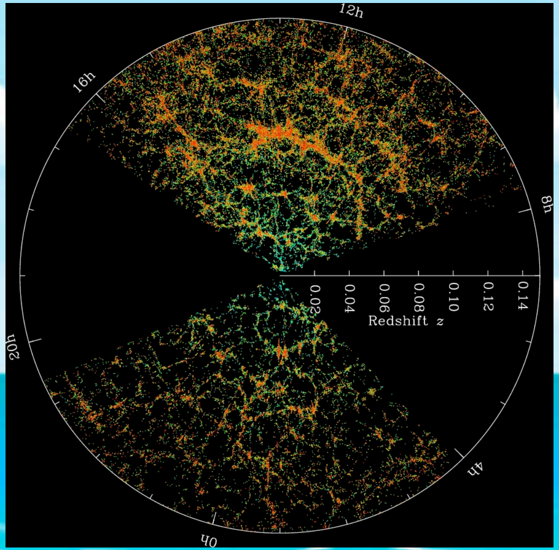
SIMONS FOUNDATION

COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK



+

$$
\begin{aligned}
Z_1(\boldsymbol{q}_1) &= K_1 + f\mu_1^2, \qquad\qquad \text{(A.3)}\\
Z_2(\boldsymbol{q}_1,\boldsymbol{q}_2) &= K_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + f\mu_{12}^2 G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{f\mu_{12}q_{12}}{2}K_1\left[\frac{\mu_1}{q_1}+\frac{\mu_2}{q_2}\right] + \frac{(f\mu_{12}q_{12})^2}{2}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2},\\
Z_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) &= K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + f\mu_{123}^2 G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\\
&\quad + (f\mu_{123}q_{123})\left[\frac{\mu_{12}}{q_{12}}K_1 G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_3}{q_3}K_2(\boldsymbol{q}_1,\boldsymbol{q}_2)\right]\\
&\quad + \frac{(f\mu_{123}q_{123})^2}{2}\left[2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_1\right] + \frac{(f\mu_{123}q_{123})^3}{6}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3},\\
Z_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) &= K_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) + f\mu_{1234}^2 G_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4)\\
&\quad + (f\mu_{1234}q_{1234})\left[\frac{\mu_{123}}{q_{123}}K_1 G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_4}{q_4}K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\right.\\
&\qquad\qquad \left. + \frac{\mu_{12}}{q_{12}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^2}{2}\left[2\frac{\mu_{123}}{q_{123}}\frac{\mu_4}{q_4}G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_{12}}{q_{12}}\frac{\mu_{34}}{q_{34}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)G_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right.\\
&\qquad\qquad \left. + 2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}K_1 G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^3}{6}\left[3\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}K_1\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^4}{24}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4},
\end{aligned}
$$

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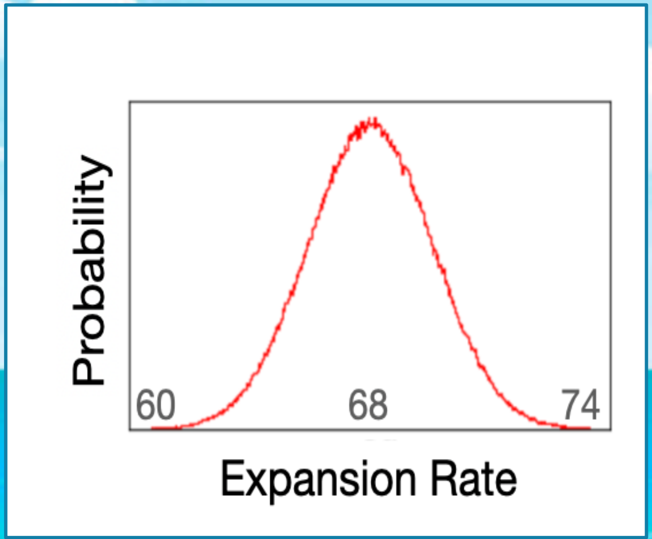


The Effective Field Theory of Large Scale Structure:
A User's Guide

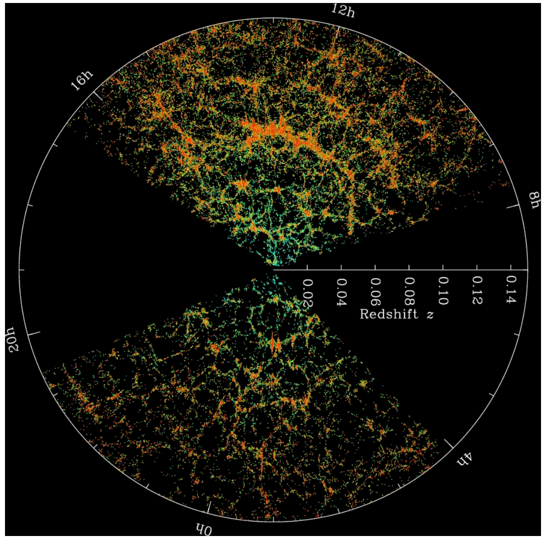
Oliver Philcox (Columbia / Simons Foundation)

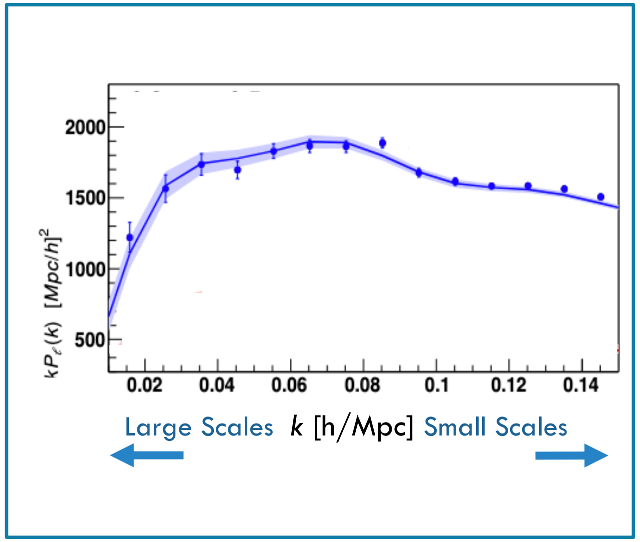
Cosmology on the Beach, *December 2022*

Collaborators: *Mikhail Ivanov, Giovanni Cabass, Marko Simonovic, Matias Zaldarriaga*

See also: *Guido d'Amico, Leonardo Senatore, Pierre Zhang, Matt Lewandowski, Martin White, Zvonimir Vlah, Stephen Chen*

LARGE SCALE STRUCTURE ROADMAP





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&+ (f\mu_{123}q_{123})\left[\frac{\mu_{12}}{q_{12}}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_3}{q_3}K_2(\boldsymbol{q}_1,\boldsymbol{q}_2)\right]\\
&+ \frac{(f\mu_{123}q_{123})^2}{2}\left[2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_1\right] + \frac{(f\mu_{123}q_{123})^3}{6}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3},\\
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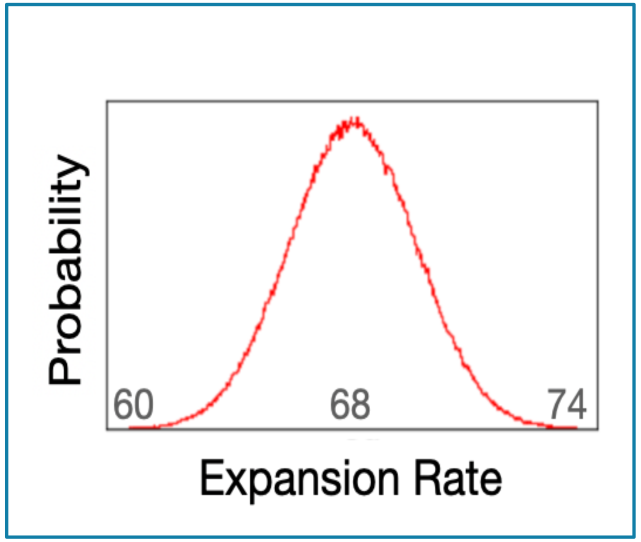
&+ (f\mu_{1234}q_{1234})\left[\frac{\mu_{123}}{q_{123}}K_1G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_4}{q_4}K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\right.\\
&\qquad\left. + \frac{\mu_{12}}{q_{12}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&+ \frac{(f\mu_{1234}q_{1234})^2}{2}\left[2\frac{\mu_{123}}{q_{123}}\frac{\mu_4}{q_4}G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_{12}}{q_{12}}\frac{\mu_{34}}{q_{34}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)G_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right.\\
&\qquad\left. + 2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&+ \frac{(f\mu_{1234}q_{1234})^3}{6}\left[3\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}K_1\right]\\
&+ \frac{(f\mu_{1234}q_{1234})^4}{24}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4},
\end{aligned}
$$



Galaxy map → Summary Statistics → Theoretical Model → Parameters

Galaxy map
- BOSS
- DESI
- Euclid
- SPHEREx

Summary Statistics
- Power Spectrum
- Bispectrum
- CNNs
- Wavelets

Theoretical Model
- Perturbation Theory
- Emulators

Parameters
- Expansion rate
- Matter density
- Neutrino Mass

SDSS-III, Philcox+22

THE ROLE OF THEORETICAL MODELS:

Classical approach:

See also ShapeFit!

- Extract key features from the power spectrum
- Model these directly

"Full Shape" approach:

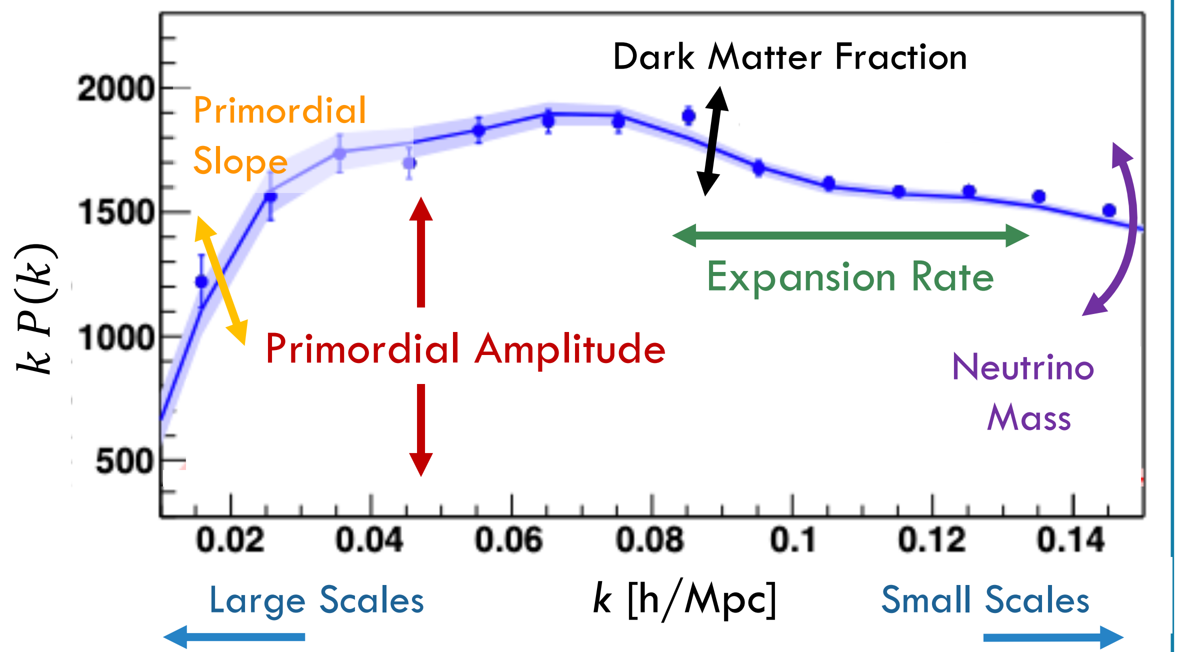
Just like the CMB!

- Fit the **whole** statistic with some model, $P_{\mathrm{theory}}(k, \theta)$
- Directly extract cosmological parameters, θ

We need a good theory model!

(+systematics treatment)

Galaxy Power Spectrum



BOSS DR12 Galaxy Survey, Beutler+17

e.g. Ivanov+19,20, d'Amico+19,20, Philcox+20ab, Chen+21, Kobayashi+21

TWO TYPES OF THEORETICAL MODEL

Perturbation Theory

- **Pen-and-paper** model
- Compute prediction **analytically** based on underlying cosmological model
- Numerically integrate to find $P_{\text{theory}}(k, \theta)$

Usually **cheaper** (no simulations) with **controlled** assumptions

Assumes **underlying equations** are valid!

Emulator Model

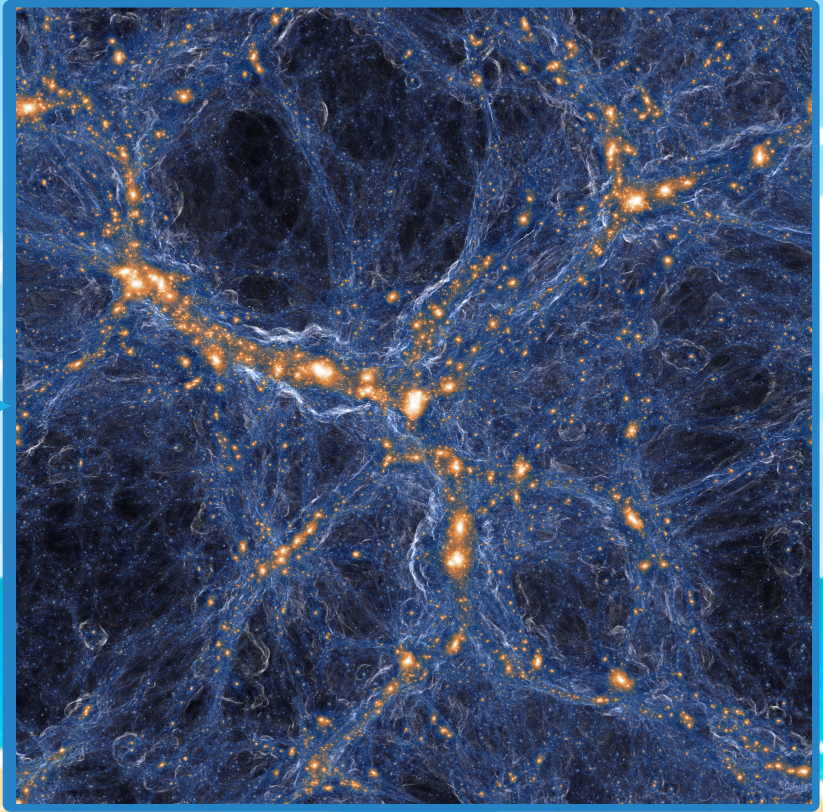
- **Simulation-based** model
- Run simulations for a range of values of θ
- Interpolate to obtain $P_{\text{theory}}(k, \theta)$

Can extend to **non-perturbative** regimes

Assumes **simulations** are accurate!

$$
\begin{aligned}
P_{\mathrm{gg}}(z,k) = {} & b_1^2(z)\left(P_{\mathrm{lin}}(z,k) + P_{\text{1-loop, SPT}}(z,k)\right) + b_1(z)b_2(z)\mathcal{I}_{\delta^2}(z,k) \\
& + 2b_1(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\mathcal{G}_2}(z,k) + \left(2b_1(z)b_{\mathcal{G}_2}(z) + \frac{4}{5}b_1(z)b_{\Gamma_3}(z)\right)\mathcal{F}_{\mathcal{G}_2}(z,k) \\
& + \frac{1}{4}b_2^2(z)\mathcal{I}_{\delta^2\delta^2}(z,k) + b_{\mathcal{G}_2}^2(z)\mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z,k) + b_2(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\delta^2\mathcal{G}_2}(z,k) \\
& + P_{\nabla^2\delta}(z,k) + P_{\epsilon\epsilon}(z,k)\,,
\end{aligned}
$$

???



Part I: What is the Effective Field Theory of LSS?

IDEAL THEORY CHECKLIST

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

STANDARD PERTURBATION THEORY (SPT)

- Basic assumption: the Universe is a **perfect fluid**

$\dot{\delta} + \nabla \cdot [(1+\delta)\boldsymbol{v}] = 0,$	Continuity Equation
$\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\mathcal{H}\boldsymbol{v} - \nabla\phi,$	Euler Equation
$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta,$	Poisson Equation

for density δ, velocity $\mathbf{v}$, potential ϕ

- Solve the equations by expanding in powers of δ

$$\delta(\mathbf{k},\tau) = D(\tau)\delta^{(1)}(\mathbf{k}) + D^2(\tau)\delta^{(2)}(\mathbf{k}) + D^3(\tau)\delta^{(3)}(\mathbf{k}) + \ldots$$

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Bernardeau+02, Ma & Fry, Scoccimarro, etc.

STANDARD PERTURBATION THEORY (SPT)

- At second order:

$$\boxed{\delta(\mathbf{k},\tau) = D(\tau)\delta^{(1)}(\mathbf{k}) + D^2(\tau)\int \frac{d^3\mathbf{q}}{(2\pi)^3} F_2(\mathbf{q},\mathbf{k}-\mathbf{q})\delta^{(1)}(\mathbf{q})\delta^{(1)}(\mathbf{k}-\mathbf{q})}$$

Physics enters here

- The late-time density field δ depends on:
  - **Kernels**, F_n, (set by the fluid equations, giving mode coupling)
  - **Initial conditions**, $\delta^{(1)}$ (set by inflation)

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Bernardeau+02, Ma & Fry, Scoccimarro, etc.

STANDARD PERTURBATION THEORY (SPT)

- SPT predicts the matter power spectrum:

$$P(\mathbf{k},\tau) = \underbrace{P_L(\mathbf{k},\tau)}_{\text{Linear}} + P_{22}(\mathbf{k},\tau) + 2P_{13}(\mathbf{k},\tau) + \ldots$$

Linear *One-loop*

N-loop = (N-1) Fourier-space integrals!

- The **loop corrections** are integrals over the linear power spectrum

$$\begin{aligned} P_{22}(\mathbf{k}) &= \int \frac{d^3q}{(2\pi)^3} P_L(\mathbf{q}) P_L(\mathbf{k}-\mathbf{q}) |F_2(\mathbf{q},\mathbf{k}-\mathbf{q})|^2 \\ P_{13}(\mathbf{k}) &= 3P_L(\mathbf{k}) \int \frac{d^3q}{(2\pi)^3} P_L(\mathbf{q}) F_3(\mathbf{k},\mathbf{q},-\mathbf{q}) \end{aligned}$$

? **Convergence**
- Need a **small** expansion parameter

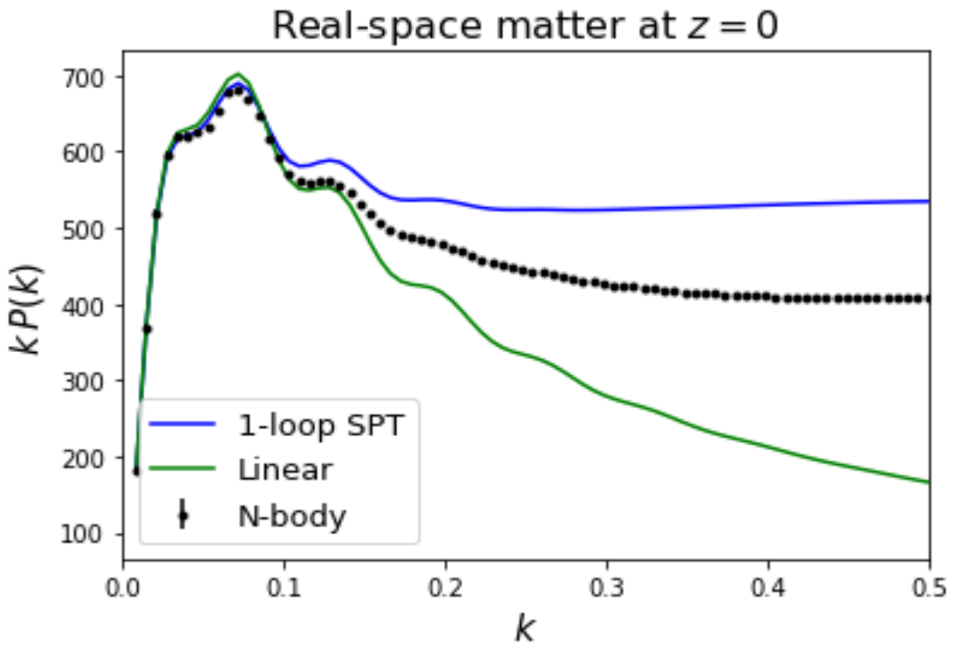
? **Accuracy**
- Should be **arbitrarily** accurate

? **Behavior**
- No **divergences**!

e.g. Bernardeau+02, Ma & Fry, Scoccimarro, etc.

STANDARD PERTURBATION THEORY (SPT)

- How does SPT compare to simulations?



SPT is no better than linear theory!

Convergence

- Need a **small** expansion parameter

Accuracy

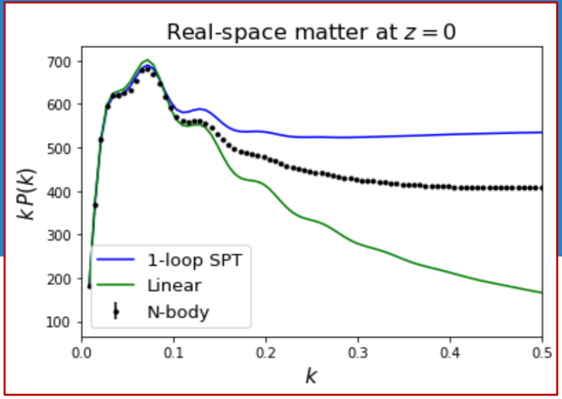
- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

Using Quijote simulations (Villaescusa-Navarro+19) and CLASS-PT (Ivanov+20)

STANDARD PERTURBATION THEORY (SPT)



Problems with SPT

- There is no well-defined **expansion parameter**

  δ can be arbitrarily large! ($\sigma = \mathrm{rms}(\delta) \to \infty$)

- Adding more loops does not improve **convergence**

  (NB: lots of knobs + whistles added to help with this)

- The predictions can **diverge** for certain inputs

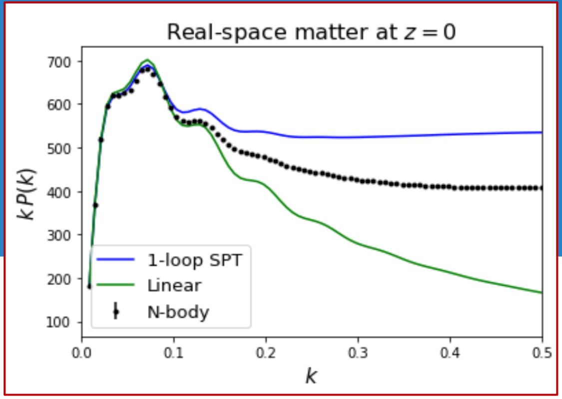
Convergence
- Need a **small** expansion parameter

Accuracy
- Should be **arbitrarily** accurate

Behavior
- No **divergences**!

e.g. Baldauf, Carrasco, Senatore, Baumann

STANDARD PERTURBATION THEORY (SPT)



What's going wrong?

- ▷ The density **doesn't** have to be small!
- ▷ The Universe is **not** an ideal fluid!
- ▷ We are integrating over UV modes in the **non-linear** regime!

$$P_{13}(\mathbf{k}) \quad = \quad 3P_L(\mathbf{k}) \int^{\infty} \frac{d^3 q}{(2\pi)^3} P_L(\mathbf{q}) F_3(\mathbf{k}, \mathbf{q}, -\mathbf{q})$$

Convergence
- Need a **small** expansion parameter

Accuracy
- Should be **arbitrarily** accurate

Behavior
- No **divergences**!

e.g. Baldauf, Carrasco, Senatore, Baumann

SPT → EFFECTIVE FIELD THEORY

What's going wrong?

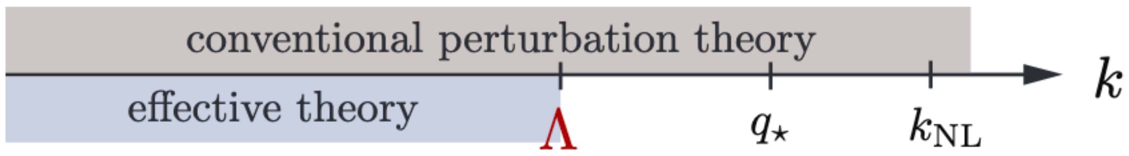
- The Universe is **not** an ideal fluid! → Use **non-ideal** fluid equations
- The density **doesn't** have to be small! → **Smooth** the density field
- We are integrating over UV modes in the **non-linear** regime! → Only integrate where theory is **valid**

$$P_{13}(\mathbf{k}) \quad = \quad 3P_L(\mathbf{k}) \int^{\Lambda} \frac{d^3q}{(2\pi)^3} P_L(\mathbf{q}) F_3(\mathbf{k}, \mathbf{q}, -\mathbf{q})$$

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: FORMULATION

▷ The **Effective Field Theory** of LSS explicitly restricts the theory to scales $k < \Lambda < k_{\mathrm{NL}}$



▷ The relevant expansion parameter is the **smoothed** density δ_Λ: this is **always** small

▷ **Small-scale** (UV) physics impacts the **large-scale** (IR) modes – this can be parametrized by **symmetry**

Convergence
- Need a **small** expansion parameter

Accuracy
- Should be **arbitrarily** accurate

Behavior
- No **divergences!**

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: IMPLEMENTATION

- Basic assumption: the Universe is an **imperfect fluid**

$$\dot{\delta}_\Lambda + \nabla\cdot[(1+\delta_\Lambda \mathbf{v}_\Lambda] = 0$$

Continuity Equation

$$\dot{\mathbf{v}}_\Lambda + (\mathbf{v}_\Lambda\cdot\nabla)\,\mathbf{v}_\Lambda = -\mathcal{H}\mathbf{v}_\Lambda - \nabla\phi_\Lambda - \frac{1}{\rho_\Lambda}\boxed{\nabla\underline{\underline{\tau}}}$$

Euler Equation

$$\nabla^2\phi = 4\pi G a^2\bar{\rho}\delta,$$

Poisson Equation

for **smoothed** density δ_Λ, velocity $\mathbf{v}_\Lambda$, potential ϕ

This involves a **stress tensor** (even for a perfect fluid)

$$\boxed{\tau^{ji}} = -\boxed{c_s^2}\rho\delta^{ij} + \boxed{\eta}\left(\partial^j v^i + \partial^i v^j\right) + \dots$$

Sound-speed

Viscosity

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: IMPLEMENTATION

▷ Expanding perturbatively:

$$\delta_\Lambda(\mathbf{k},\tau) = \delta_\Lambda^{\mathrm{SPT}}(\mathbf{k},\tau) - \boxed{c_{s,\Lambda}^2(\tau)}\, k^2 \delta_\Lambda^{(1)}(\mathbf{k}) + \cdots$$

▷ There is a new **counterterm** from the stress tensor, encoding **small-scale (UV)** physics

Power spectrum:

$$P(k) = P_{\mathrm{lin}}(k) + P_{22,\Lambda}(k) + 2P_{13,\Lambda}(k) - \boxed{2c_{s,\Lambda}^2}\, k^2 P_{\mathrm{lin}}(k)$$

Linear *One-loop* *Counterterm*

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: RENORMALIZATION

▷ The one-loop power spectra are integrated up to $q_{\max} = \Lambda$

$$P_{13}(\mathbf{k},\Lambda) \sim P_L(\mathbf{k}) \int_{q<\Lambda} \frac{d^3\mathbf{q}}{(2\pi)^3} P(q) k^2/q^2$$

This avoids any **divergent** behavior!

▷ The theory depends explicitly on the cut-off Λ ?

$$P_{13}(\mathbf{k},\Lambda) = P_{13}(\mathbf{k},\infty) - \boxed{f(\Lambda) k^2 P_L(\mathbf{k})}$$

This dependence be **absorbed** (= renormalized) by the counterterm

$$P(k) = P_{\rm lin}(k) + P_{22,\Lambda}(k) + 2P_{13,\Lambda}(k) - \boxed{2c^2_{s,\Lambda} k^2 P_{\rm lin}(k)}$$

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: COUNTERTERMS

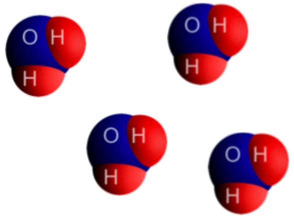
- At one-loop order, we have **one** relevant counterterm, c_s^2

$$P_{\mathrm{EFT}}(k) = P_{\mathrm{EFT}}(k;\ \theta_{\mathrm{cosmology}}, c_s^2)$$

- This depends on **UV physics** so cannot be predicted by EFT
- **Solution**: marginalize over it!

Analogy: viscocity in fluid flow

$$\boxed{\dot{v}^i + H v^i + v^j \partial_j v^i = \frac{1}{\rho}\partial_j \tau^{ij}}$$



large scales

Convergence
- Need a **small** expansion parameter

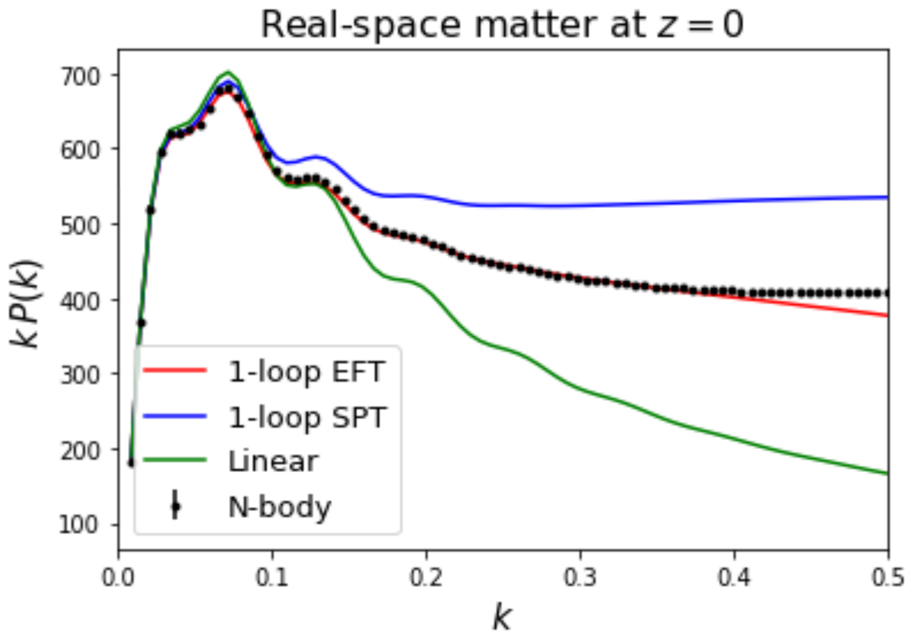
Accuracy
- Should be **arbitrarily** accurate

Behavior
- No **divergences!**

e.g. Carrasco+12, Baumann+12

THE EFT OF LSS: RESULTS

How does EFT compare to simulations?



▷ One-loop does **much** better than linear theory

▷ Two-loops does even better!

Convergence

- Need a **small** expansion parameter

Accuracy

- Should be **arbitrarily** accurate

Behavior

- No **divergences**!

e.g. Carrasco+12, Baumann+12, Senatore+, CLASS-PT, QUIJOTE

BIASED TRACERS

- How do we model galaxy distributions?

1. (SPT) Expand the galaxy overdensity in powers of δ:

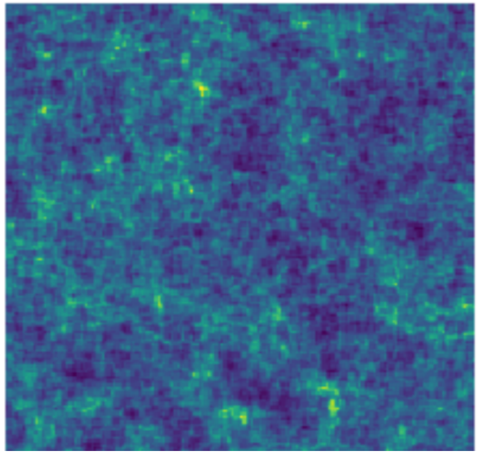
$$\delta_g(\mathbf{x}) = b_1\delta(\mathbf{x}) + \frac{b_2}{2}\delta^2(\mathbf{x}) + \frac{b_3}{6}\delta^3(\mathbf{x}) + \ldots$$

2. (EFT) Include all possible parameters allowed by symmetry

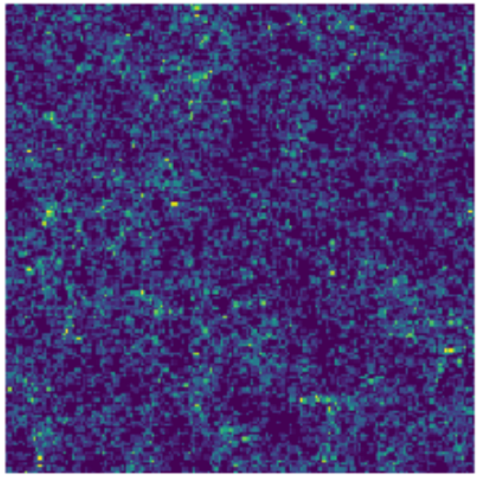
$$\delta_g = b_1\delta + \epsilon + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + \frac{b_3}{6}\delta^3 + b_{\delta\mathcal{G}_2}\delta\mathcal{G}_2 + b_{\mathcal{G}_3}\mathcal{G}_3 + b_{\Gamma_3}\Gamma_3 + R_*^2\partial^2\delta$$

with density operators, tidal operators, stochastic operators, and non-local operators (all integrated over a lightcone)

Dark Matter, δ_m



Galaxies, δ_g



QUIJOTE Simulations, Senatore+, Ivanov+19, de la Bella+18

BIASED TRACERS

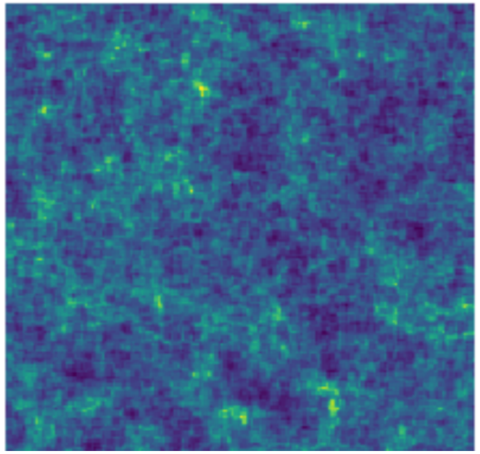
$$\delta_g = b_1\delta + \epsilon + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + \frac{b_3}{6}\delta^3 + b_{\delta\mathcal{G}_2}\delta\mathcal{G}_2 + b_{\mathcal{G}_3}\mathcal{G}_3 + b_{\Gamma_3}\Gamma_3 + R_*^2\partial^2\delta$$

- Matter EFT is a **Taylor expansion** in $k/k_{\rm NL}$
- Galaxy EFT is a **Taylor expansion** in $k/k_{\rm NL}$ and $kR_{\rm Halo}$

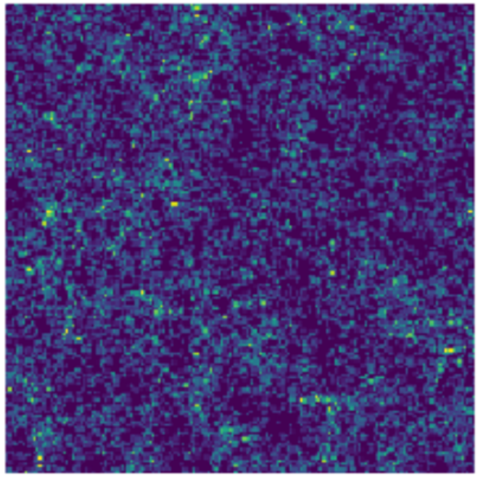
$$\boxed{P_{gg,\rm EFT}(k) = P_{gg,\rm EFT}(k;\ \theta_{\rm cosmology}, c_s^2, b_1, b_2, P_{\rm shot}, \cdots)}$$

If $R_{\rm Halo}^{-1} > k_{\rm NL}$, we can do better by computing **matter** power spectrum from simulations,
$\Rightarrow$ Hybrid EFT (Kokron+21)

Dark Matter, δ_m



Galaxies, δ_g

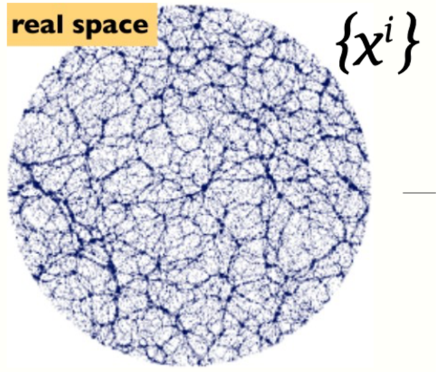


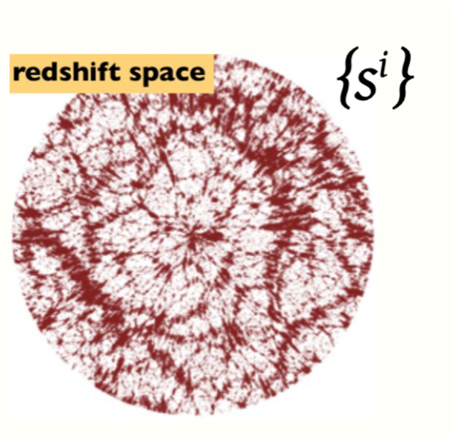
QUIJOTE Simulations, Senatore+, Ivanov+19, de la Bella+18

REDSHIFT-SPACE DISTORTIONS

We observe **spectroscopic** surveys in redshift-space!

$$\mathbf{s} = \mathbf{x} + \frac{\hat{z}\cdot\mathbf{v}}{aH}\hat{z}$$





- There is an **exact map** between real- and redshift-space

$$\delta_{g,s}(\mathbf{k}) \quad = \quad \delta_g(\mathbf{k}) - i\frac{k_z}{aH}v_z(\mathbf{k}) + \dots$$

Velocity field

- Expand perturbatively in δ_g **and** v_z
- **Taylor expand** non-perturbative fingers-of-God

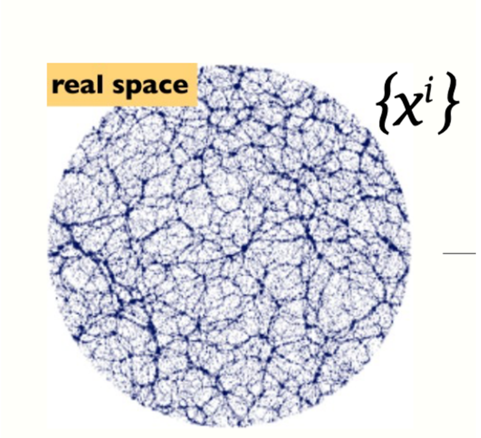
Senatore+14, Perko+16, Fonseca de la Bella

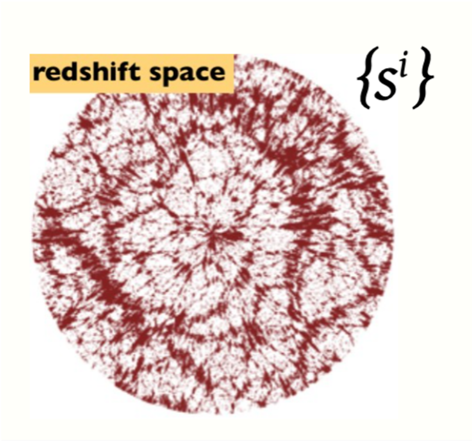
REDSHIFT-SPACE DISTORTIONS

Full expansion includes **new counterterms** from **velocity effects** and **Fingers-of-God**

▷ Redshift-space galaxy EFT is a **Taylor expansion** in $k/k_{\rm NL}$, $kR_{\rm Halo}$, $k_{\|}\sigma_{\rm FoG}$

If FoG dominates, we can do better by adding in real-space power spectrum proxies (Ivanov+21, d'Amico+21)





Senatore+14, Perko+16, Fonseca de la Bella

INFRARED RESUMMATION

- The basic EFT formalism incorrectly treats **long-wavelength** (IR) displacements, Ψ

$$\delta(\mathbf{k}) \sim \int d\mathbf{q}\, e^{i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} \neq \int d\mathbf{q}(1 + i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q}) + \cdots)$$

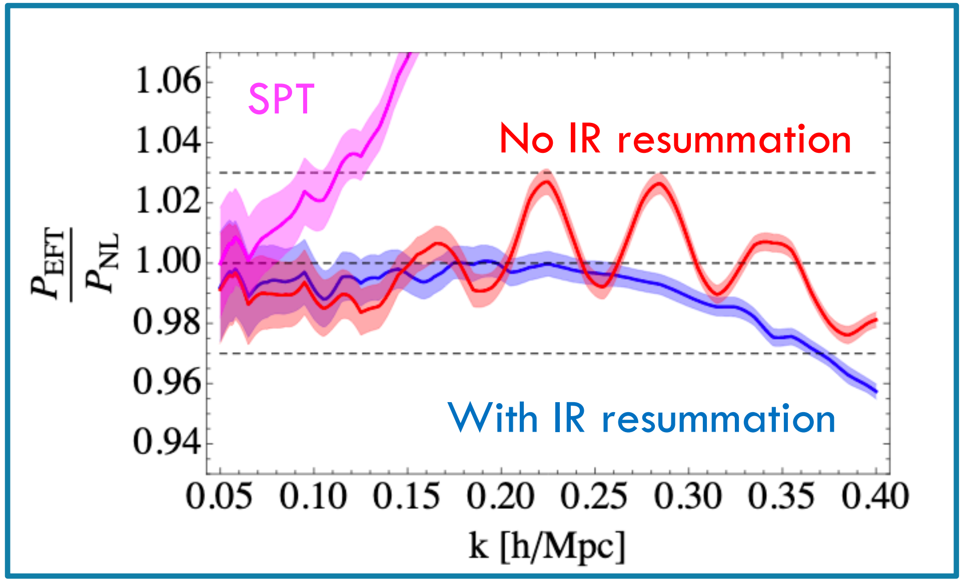
- These cannot be expanded perturbatively!

- This **damps out** the BAO wiggles

Correction is possible using **IR Resummation**

$$P_L(k) \to P_{nw}(k) + P_w(k)e^{-k^2\Sigma^2}$$

Naturally solved using Lagrangian PT!



Senatore+14, Baldauf+15, Blas+16, Ivanov+19, etc.

THE EFT OF LSS: A SUMMARY

▷ **Perturbative** solution of the **non-ideal** fluid equations

▷ A **controlled** Taylor series in k/k_{NL}, kR_{Halo}, $k_{\parallel}\sigma_{\mathrm{FoG}}$

▷ **Agnostic** to UV physics

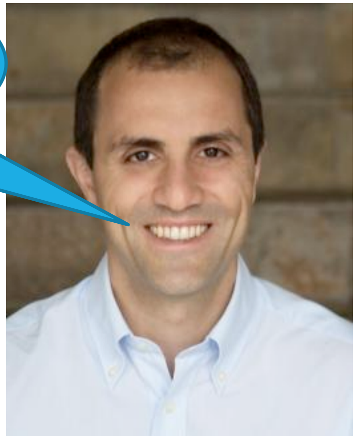
- Includes all effects relevant to symmetry

- Naturally includes **baryonic** effects

▷ Maximally **conservative**

- Can do better with knowledge of biases etc.!

This is manifestly correct!



L. Senatore

POWER SPECTRA

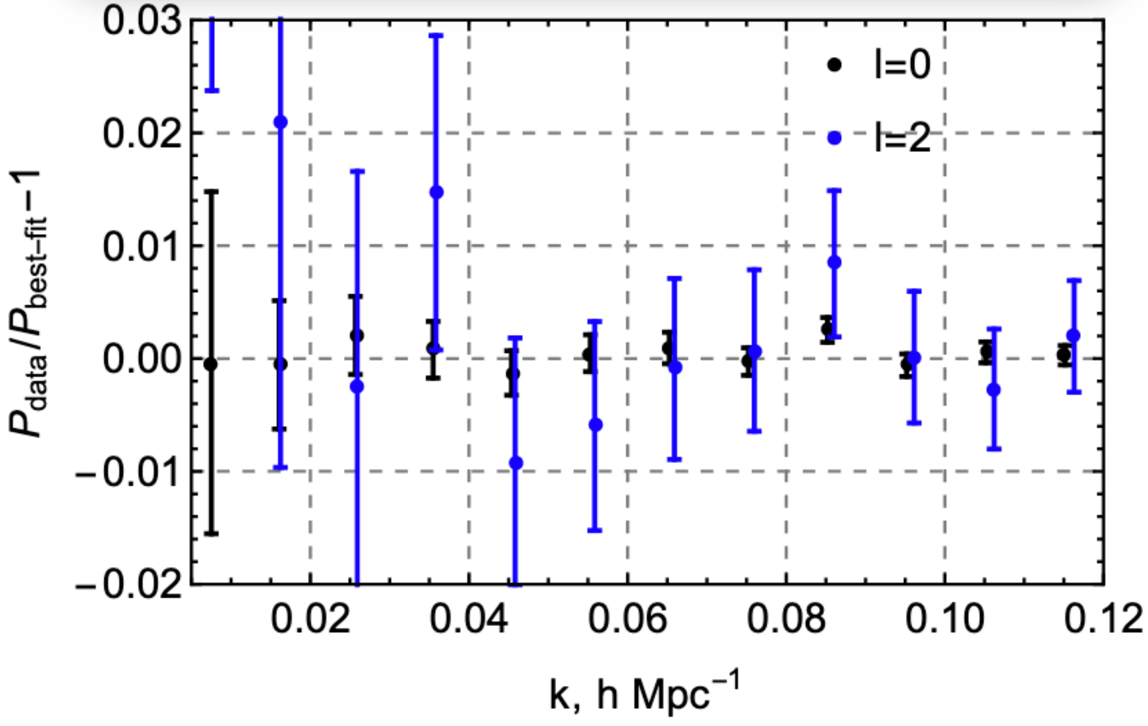
EFT can predict the **galaxy power spectrum** in **redshift-space**

$$
\begin{aligned}
P_{\mathrm{gg}}(z,k) = {} & b_1^2(z)(P_{\mathrm{lin}}(z,k) + P_{\text{1-loop, SPT}}(z,k)) + b_1(z)b_2(z)\mathcal{I}_{\delta^2}(z,k) \\
& + 2b_1(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\mathcal{G}_2}(z,k) + (2b_1(z)b_{\mathcal{G}_2}(z) + \frac{4}{5}b_1(z)b_{\Gamma_3}(z))\mathcal{F}_{\mathcal{G}_2}(z,k) \\
& + \frac{1}{4}b_2^2(z)\mathcal{I}_{\delta^2\delta^2}(z,k) + b_{\mathcal{G}_2}^2(z)\mathcal{I}_{\mathcal{G}_2\mathcal{G}_2}(z,k) + b_2(z)b_{\mathcal{G}_2}(z)\mathcal{I}_{\delta^2\mathcal{G}_2}(z,k) \\
& + P_{\nabla^2\delta}(z,k) + P_{\epsilon\epsilon}(z,k)\,,
\end{aligned}
$$

At **one-loop**, this requires:

- **Third**-order galaxy bias
- **Counterterms**
- Large-scale displacements
- Coordinate transformations
- Fingers-of-God
- Stochasticity

7 physical parameters



Accurate up to $k_{\max} \approx \mathbf{0.15}\ h/\mathrm{Mpc}$

Ivanov+20, Nishimichi+20, see also d'Amico++

BISPECTRA: O(1)

EFT also predicts **higher-order** statistics, including **bispectra**

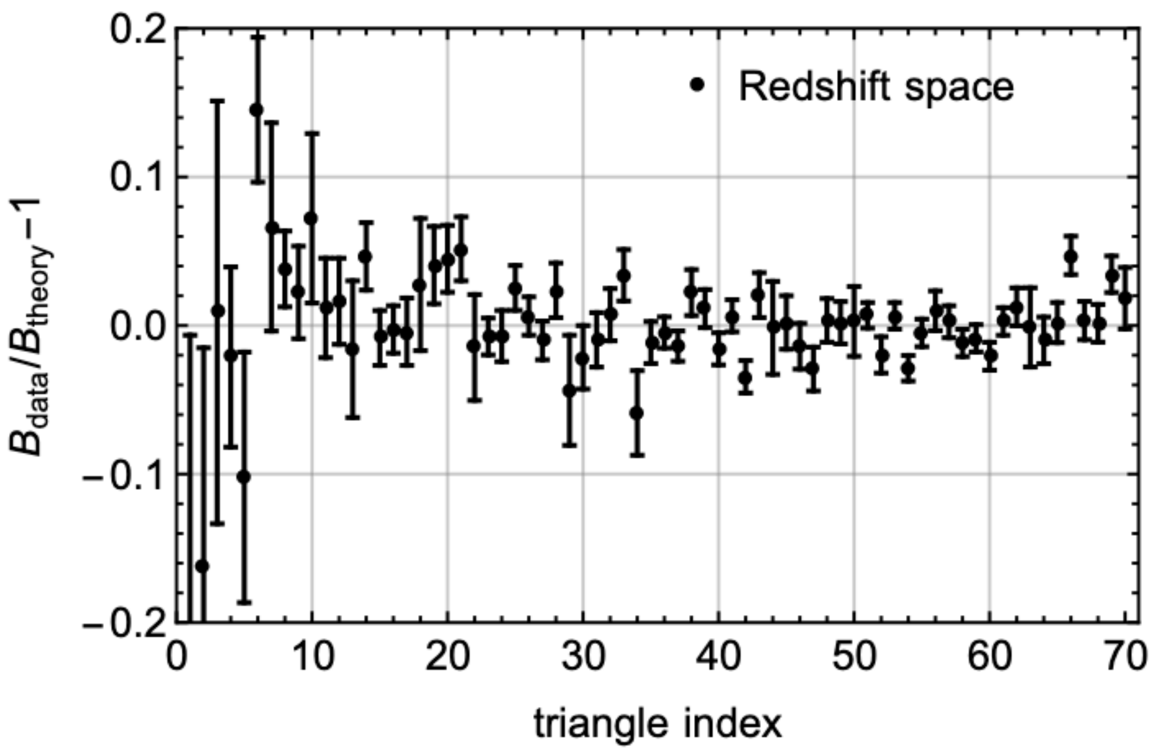
At **tree-level**, this requires:

- ▷ **Second-order** galaxy bias
- ▷ All the other power spectrum effects…

12 physical parameters

Accurate up to $k_{\max} = \mathbf{0.08}\ h/\mathrm{Mpc}$

$$B_{\rm ggg}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2Z_2(\mathbf{k}_1,\mathbf{k}_2)Z_1(\mathbf{k}_1)Z_1(\mathbf{k}_2)P_{\rm lin}(k_1)P_{\rm lin}(k_2) + P_\epsilon(k_2)2d_1\left(d_2b_1 + d_1 f\mu_1^2\right)Z_1(\mathbf{k}_1)P_{\rm lin}(k_1) + \text{cycl.} + d_1^3 B_\epsilon(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$



Ivanov, Philcox+21, see also d'Amico++

BISPECTRA: O(2)

EFT also predicts **higher-order** statistics, including **bispectra**

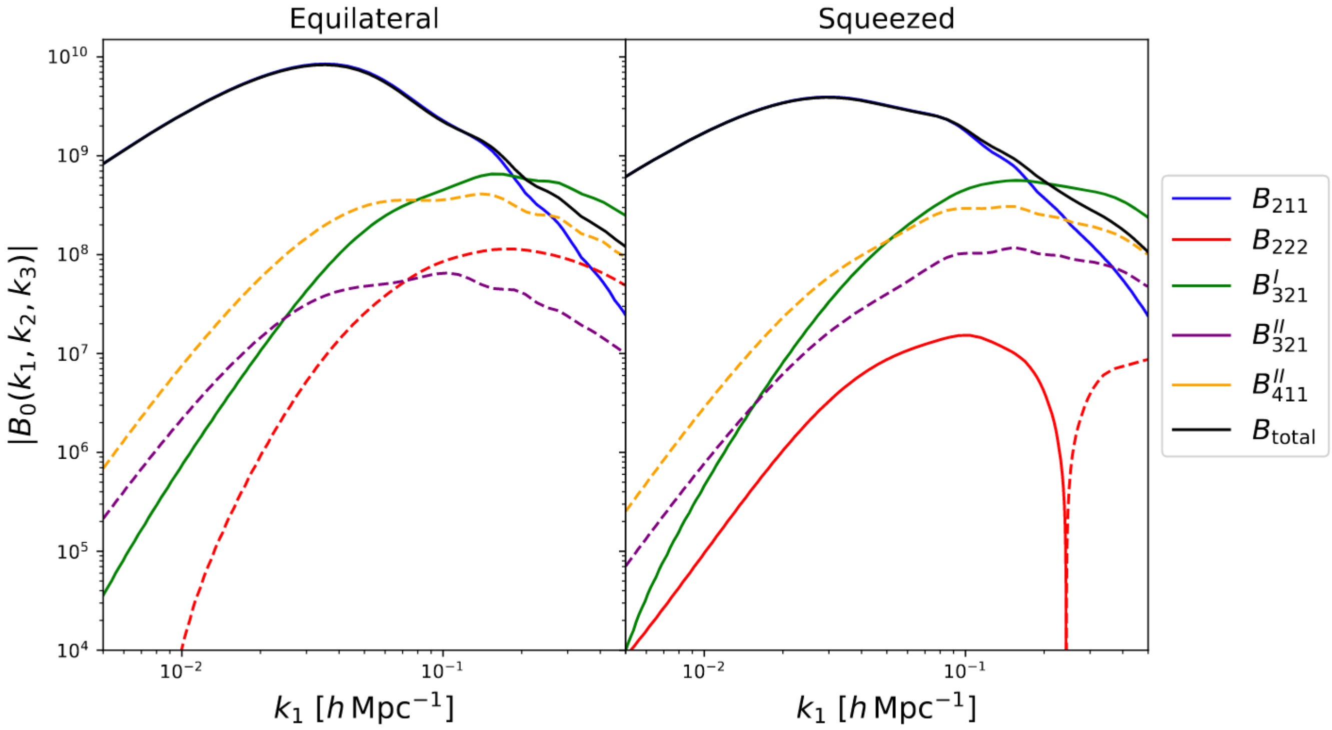
$$B_{1-\text{loop}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = B_{211} + \left[B_{222} + B^{I}_{321} + B^{II}_{321} + B_{411}\right] + B_{\text{ct}} + B_{\text{stoch}},$$

At **one-loop,** this requires:

- **Fourth-order** galaxy bias
- New **counterterms**
- All the other power spectrum effects…

44 (highly correlated) physical parameters

Accurate up to $k_{\max} = \mathbf{0.15}\ h/\text{Mpc}$



See GitHub.com/oliverphilcox/OneLoopBispectrum

Philcox+22, see also d'Amico++

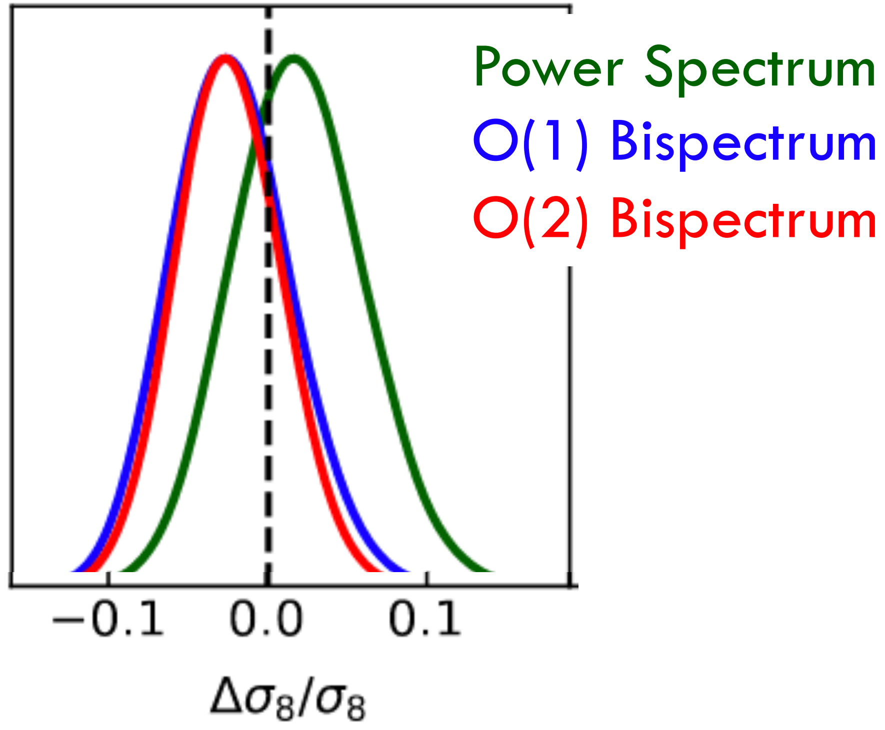
EFT BISPECTRA: O(2)

- More loops → **many** more parameters
- More loops → **little** increase in cosmological parameter constraints

Is this a problem?

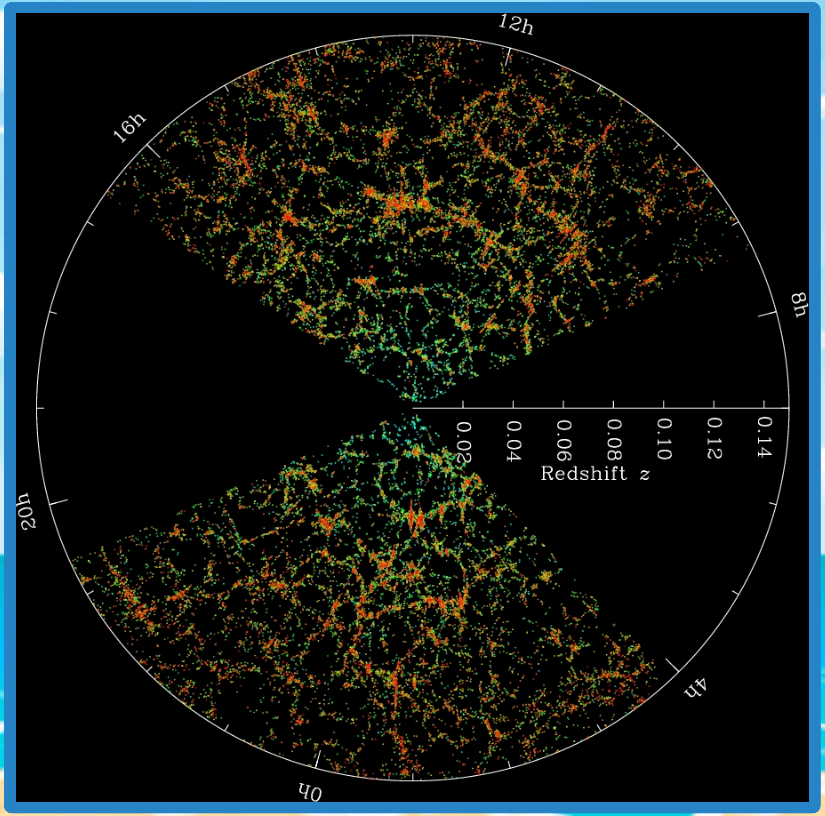
To make better use of loop corrections we need:

- Better **priors** on higher-order parameters
- Better **statistics**, e.g., bispectrum multipoles

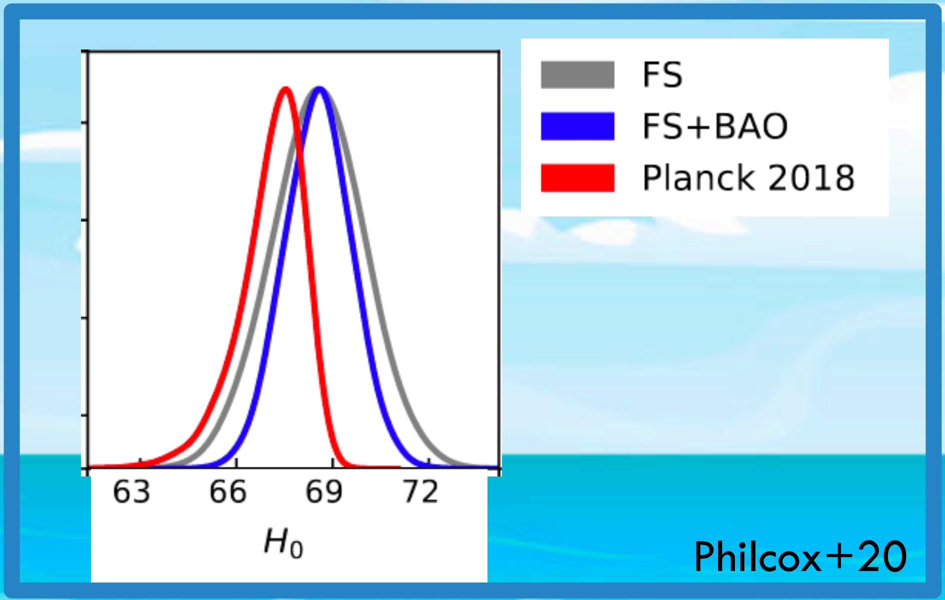


See GitHub.com/oliverphilcox/OneLoopBispectrum

Philcox+22, see also d'Amico++



???



Part II: What have we learnt using the EFTofLSS?

EFT OF LSS IMPLEMENTATIONS

- Several **public codes** implement EFT

1. *CLASS-PT* [Eulerian]
2. *PyBird* [Eulerian]
3. *Velocileptors* [Lagrangian]

Also includes f_{NL}+ bispectra!

Michalychforever / **CLASS-PT** Public

Nonlinear perturbation theory extension of the Boltzmann code CLASS

17 stars **10** forks

pierrexyz / **pybird** Public

Python code for Biased tracers in redshift space

pybird.readthedocs.io/en/latest/

MIT license

17 stars **12** forks

sfschen / **velocileptors** Public

A code for velocity-based Lagrangian and Eulerian PT expansions of redshift-space distortions.

MIT license

12 stars **3** forks

Ivanov+, d'Amico+, Chen+

EFT OF LSS IMPLEMENTATIONS

Michalychforever / **CLASS-PT** Public

Nonlinear perturbation theory extension of the Boltzmann code CLASS

17 stars 10 forks

▷ Several **public codes** implement EFT

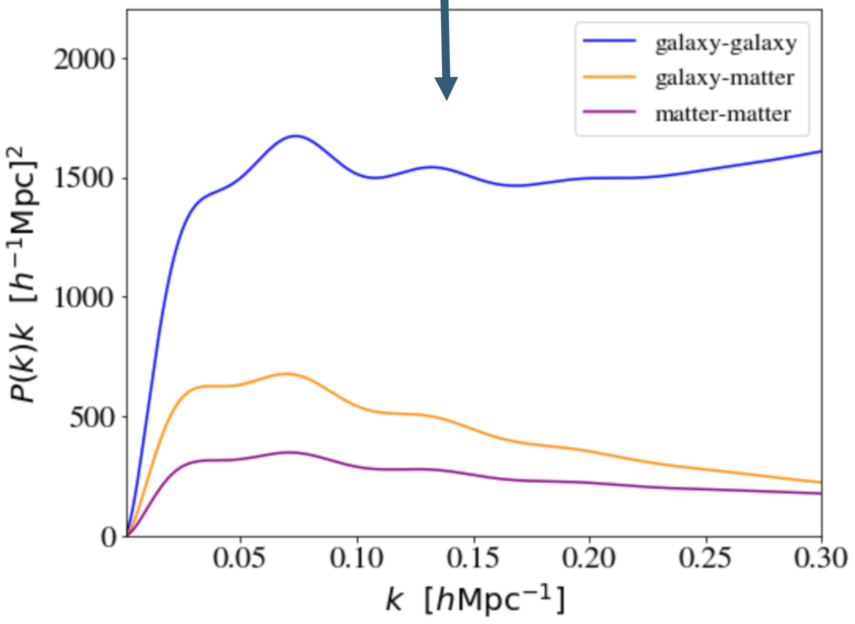
1. *CLASS-PT* [Eulerian]
2. *PyBird* [Eulerian]
3. *Velocileptors* [Lagrangian]

Also includes $f_{\rm NL}$ + bispectra!

```
# real space matter power spectrum
pk_full_ir = M1.pk_mm_real(cs)

# real space galaxy-galaxy power spectrum
pk_gg = M1.pk_gg_real(b1, b2, bG2, bGamma3, cs, cs0, Pshot)

# real space galaxy-matter power spectrum
pk_gm = M1.pk_gm_real(b1, b2, bG2, bGamma3, cs, cs0)
```



Example: CLASS-PT

▷ Computes the 1-loop PT integrals in < 1 s

▷ Includes **power spectra** + **bispectra** for matter + galaxies

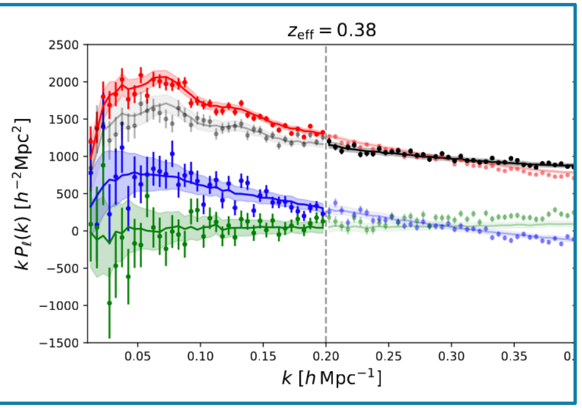
▷ Can be interfaced with MontePython for MCMC sampling

https://github.com/michalychforever/class-pt

Ivanov+, d'Amico+, Chen+

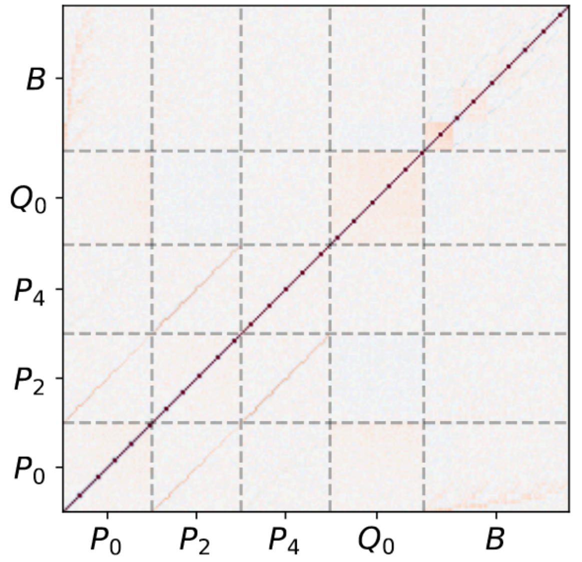
THE COSMOLOGICAL LIKELIHOOD

Summary Statistics



GitHub.com/oliverphilcox/full_shape_likelihoods

Covariance Matrices



Gaussian likelihood

$$-2\log L = (\hat{P} - P_{\mathrm{theory}})C^{-1}(\hat{P} - P_{\mathrm{theory}})$$

MCMC

Constraints on $H_0, \Omega_m, \sigma_8, b_1, P_{\mathrm{shot}}, \ldots$

EFTofLSS Model

$$\begin{aligned}
Z_1(\boldsymbol{q}_1) &= K_1 + f\mu_1^2, \qquad\qquad (A.3)\\
Z_2(\boldsymbol{q}_1,\boldsymbol{q}_2) &= K_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + f\mu_{12}^2 G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{f\mu_{12}q_{12}}{2}K_1\left[\frac{\mu_1}{q_1}+\frac{\mu_2}{q_2}\right] + \frac{(f\mu_{12}q_{12})^2}{2}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2},\\
Z_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) &= K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + f\mu_{123}^2 G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\\
&\quad + (f\mu_{123}q_{123})\left[\frac{\mu_{12}}{q_{12}}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_3}{q_3}K_2(\boldsymbol{q}_1,\boldsymbol{q}_2)\right]\\
&\quad + \frac{(f\mu_{123}q_{123})^2}{2}\left[2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_1\right] + \frac{(f\mu_{123}q_{123})^3}{6}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3},\\
Z_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) &= K_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4) + f\mu_{1234}^2 G_4(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3,\boldsymbol{q}_4)\\
&\quad + (f\mu_{1234}q_{1234})\left[\frac{\mu_{123}}{q_{123}}K_1G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_4}{q_4}K_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\right.\\
&\qquad\qquad \left. + \frac{\mu_{12}}{q_{12}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^2}{2}\left[2\frac{\mu_{123}}{q_{123}}\frac{\mu_4}{q_4}G_3(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3) + \frac{\mu_{12}}{q_{12}}\frac{\mu_{34}}{q_{34}}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2)G_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right.\\
&\qquad\qquad \left. + 2\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}K_1G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}K_2(\boldsymbol{q}_3,\boldsymbol{q}_4)\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^3}{6}\left[3\frac{\mu_{12}}{q_{12}}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4}G_2(\boldsymbol{q}_1,\boldsymbol{q}_2) + \frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}K_1\right]\\
&\quad + \frac{(f\mu_{1234}q_{1234})^4}{24}\frac{\mu_1}{q_1}\frac{\mu_2}{q_2}\frac{\mu_3}{q_3}\frac{\mu_4}{q_4},
\end{aligned}$$

+FFTLog (Simonovic)

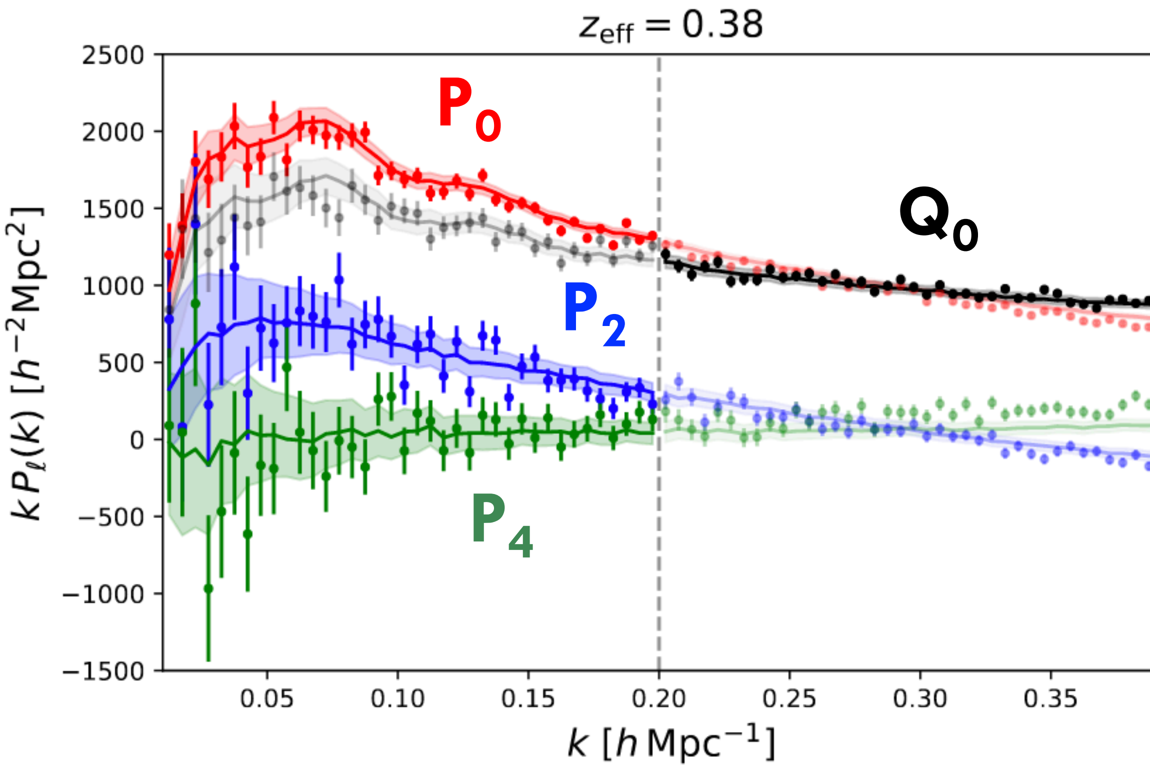
Analysis takes O(10) CPU-hours!

Philcox+, Ivanov+, etc.

Q0 STATISTIC

Compute the **real-space** power spectrum

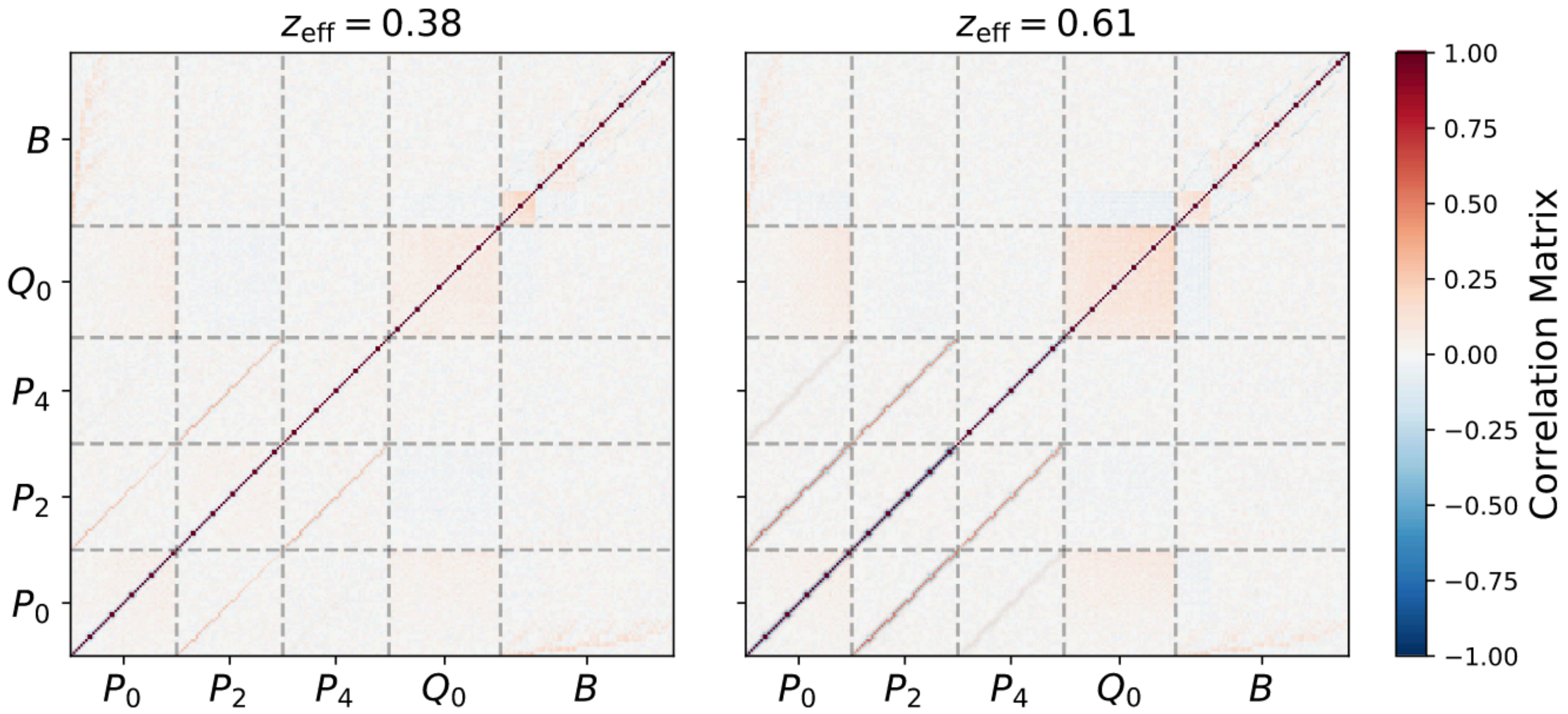
$$P_0(k) + P_2(k) + P_4(k) \rightarrow \begin{matrix} Q_0(k) \\ \approx \\ P(k, \mu = 0) \end{matrix}$$



- No Fingers-of-God!
- Push to $k_{\max} = 0.4h/\mathrm{Mpc}$
- Constraints improve by (10 – 100)%

Ivanov+21 (see also Tegmark, d'Amico+21)

CORRELATION MATRICES



MODEL VALIDATION

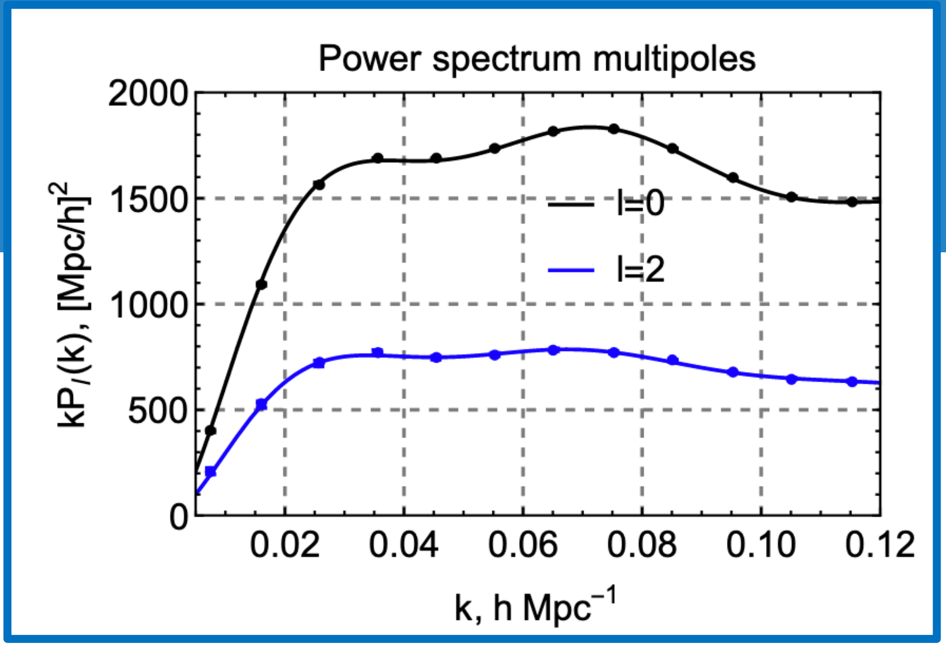
- Validate with high-resolution N-body simulations

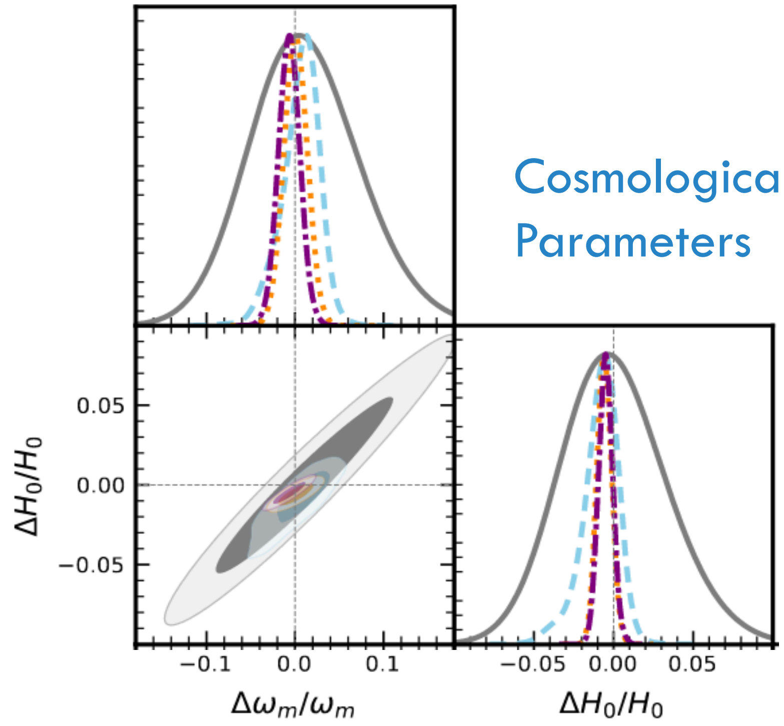
Total volume: $566\ (h^{-1}\mathrm{Gpc})^3$ *Larger than DESI / Euclid!*

- Fully **blind** analysis

- **Unbiased** cosmological parameters from the **power spectrum** and **bispectrum**!

- Also validated on BOSS-like Nseries mocks

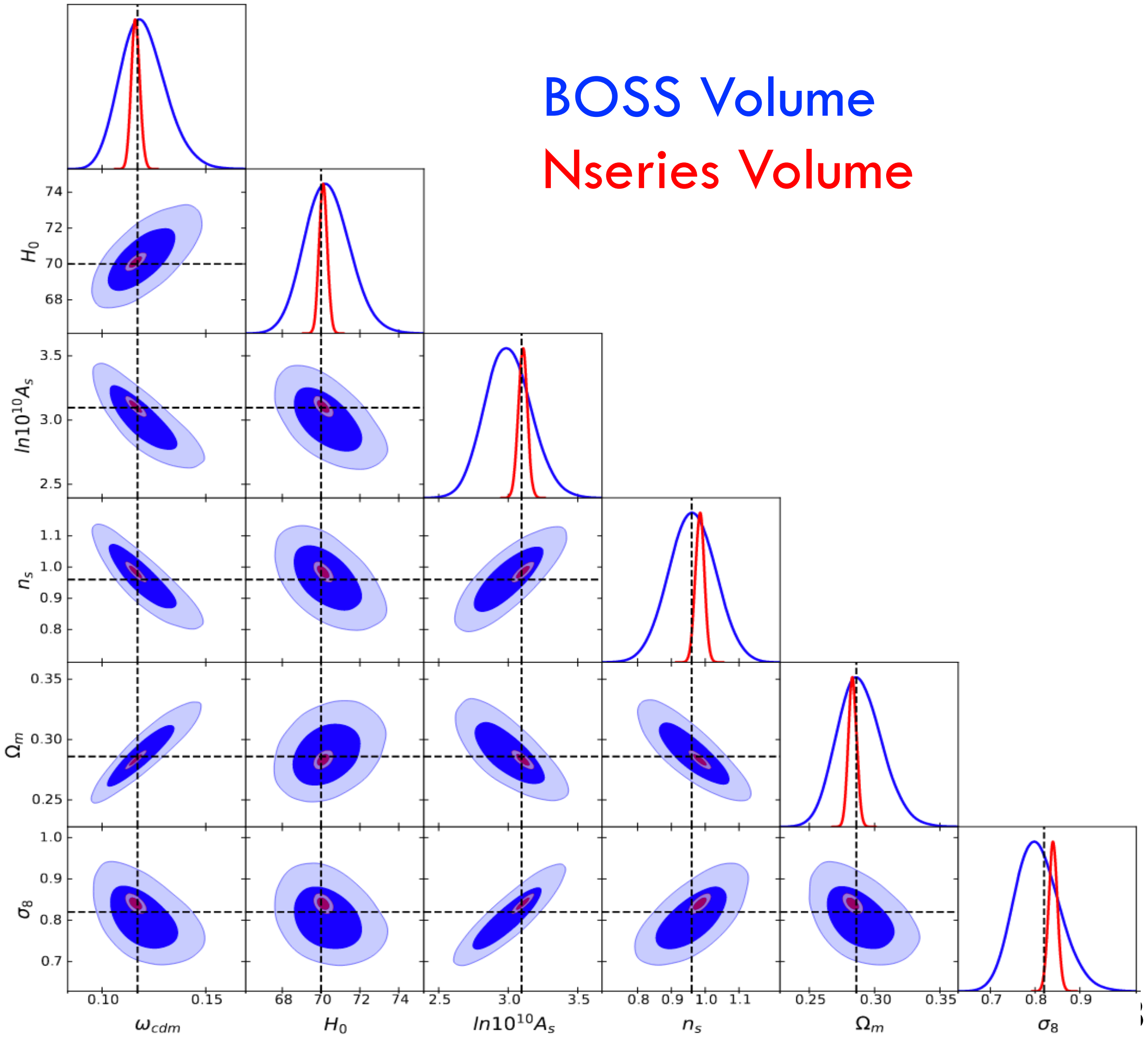




Cosmological Parameters

Nishimichi+21, Ivanov+21, Philcox+22

MODEL VALIDATION



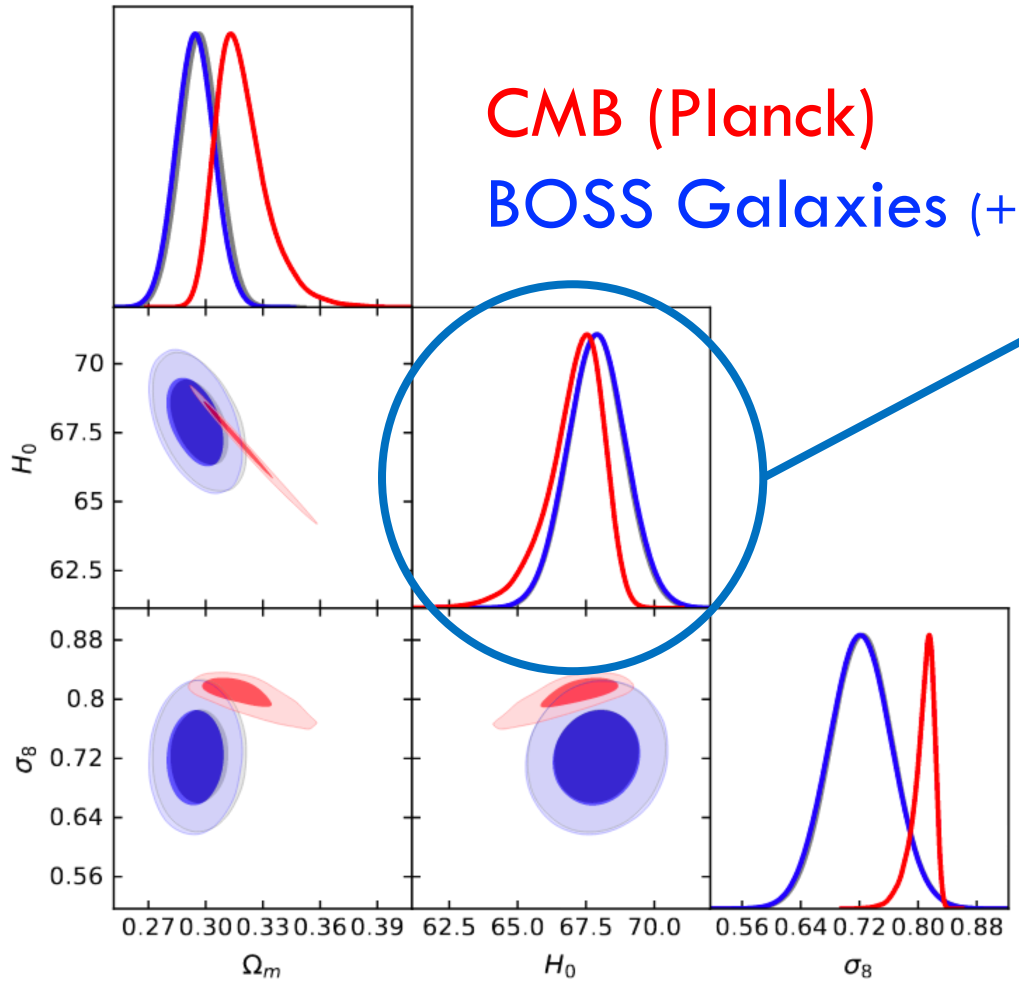
Validate with high-resolution **Nseries** mocks

- All parameters recovered at $\ll 1\sigma$
- Theory model works!
- Window function works!
- Fiber collisions work!

See [GitHub.com/oliverphilcox/full_shape_likelihoods]

Philcox+21

CONSTRAINING ΛCDM: H_0



CMB (Planck)

BOSS Galaxies (+ BBN)

BOSS Power Spectrum + Bispectrum:

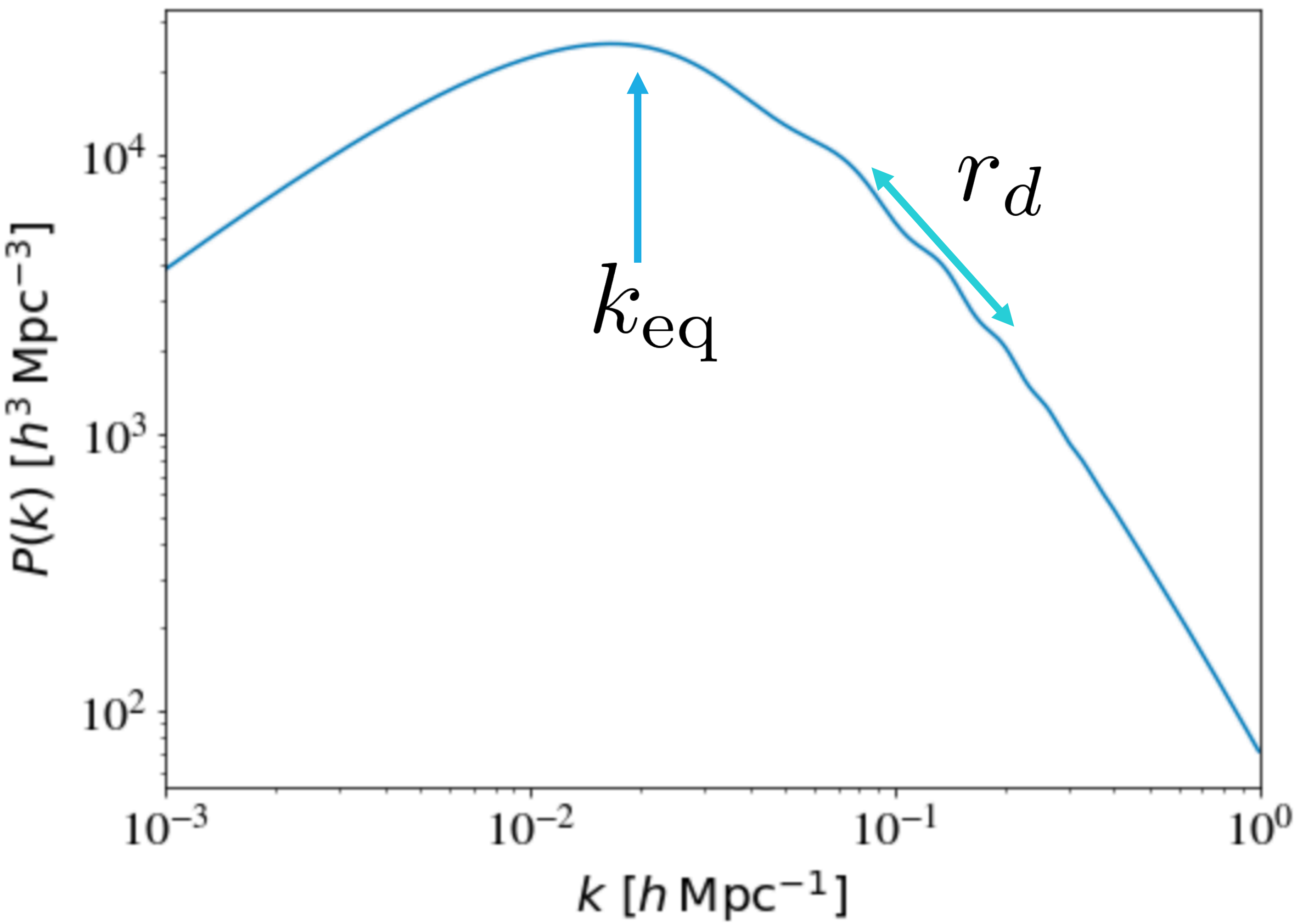
$$H_0 = 68.3 \pm 0.8\,\mathrm{km\,s^{-1}Mpc^{-1}}$$

- H_0 agrees with *Planck*
- *3.7σ discrepant with SH0ES!*

Where does this information come from?

Philcox+21,22 (also Chen+21, d'Amico+21)

TWO STANDARD RULERS FOR H_0



1. <u>The Sound Horizon</u>: r_d
   - The **sound horizon** at baryon drag ($z \sim 1100$)
2. <u>The Equality Scale</u>: $k_{\rm eq}^{-1}$
   - The **horizon** at radiation-matter equality ($z \sim 3600$)

***Both** can be used to extract H_0*

Ivanov+21, Philcox+21

THE EQUALITY SCALE: AN (OLD) PROBE OF H_0?

See also Samuel's talk!

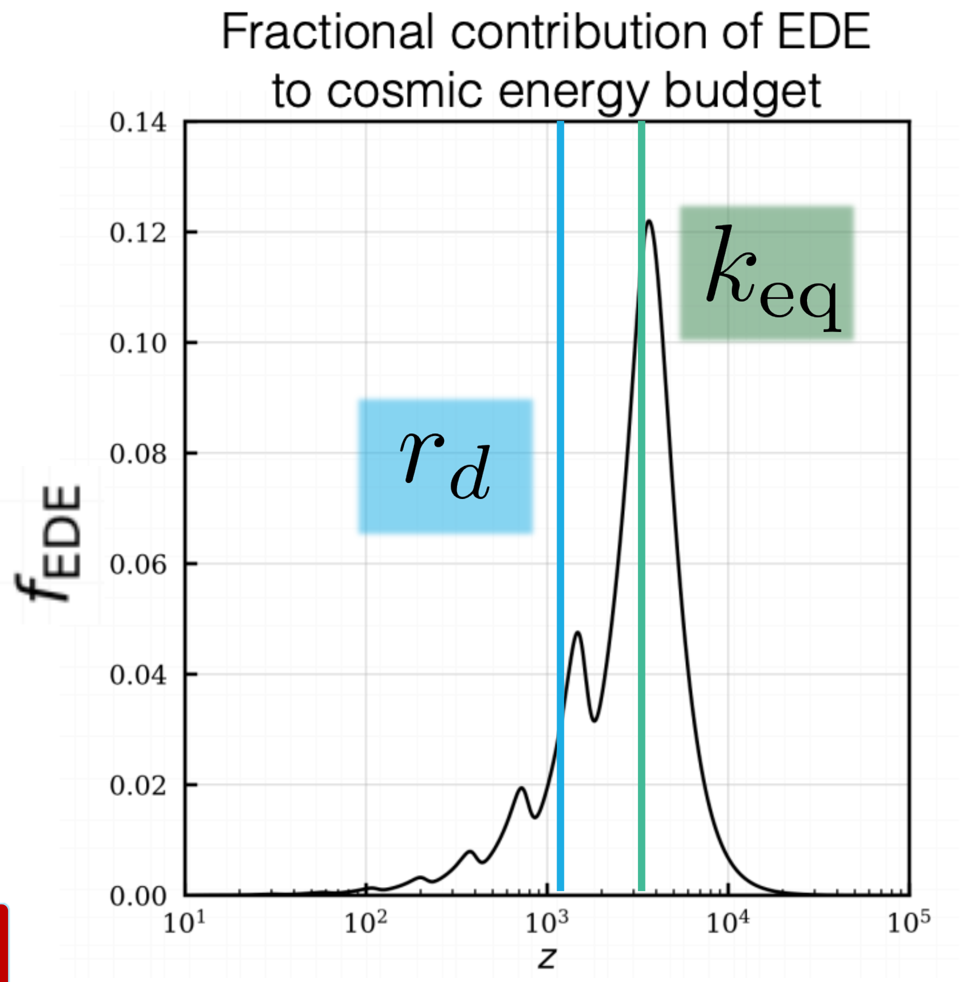
- The **equality scale** contains H_0 information

$$\theta_{\rm eq} \sim k_{\rm eq} \mathrm{D_A(z)} \propto H_0$$

- This is anchored at $z_{\rm eq} \sim 3600$, **much** before recombination at $z_d \sim 1100$

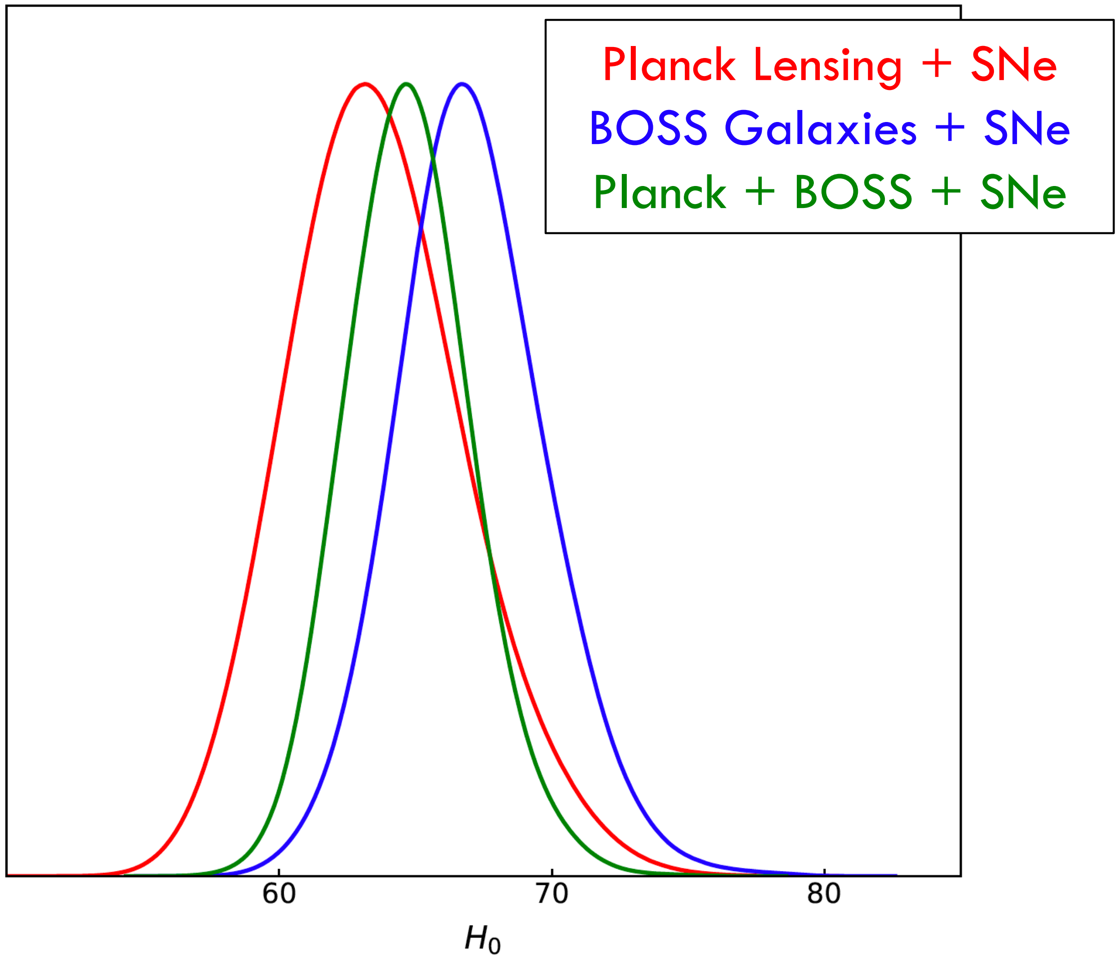
- New physics at $z \sim 10^3$ should affect **BAO** and **equality** H_0 measurements **differently**

$H_0(z_{eq}) - H_0(z_d)$ is a consistency test for ΛCDM



Baxter & Sherwin 2020, Hill+19,20

CONSTRAINTS ON H_0



Sound-Horizon Independent Constraints

BOSS Full Power Spectrum + Bispectrum:

$$(z \approx 1100\,)\quad H_0 \;= 68.3 \pm 0.8\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

BOSS-without-the-sound-horizon:

(using new r_d-marginalized pipeline)

$$(z \approx 3500)\quad H_0 \;= 67.1 \pm 2.7\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

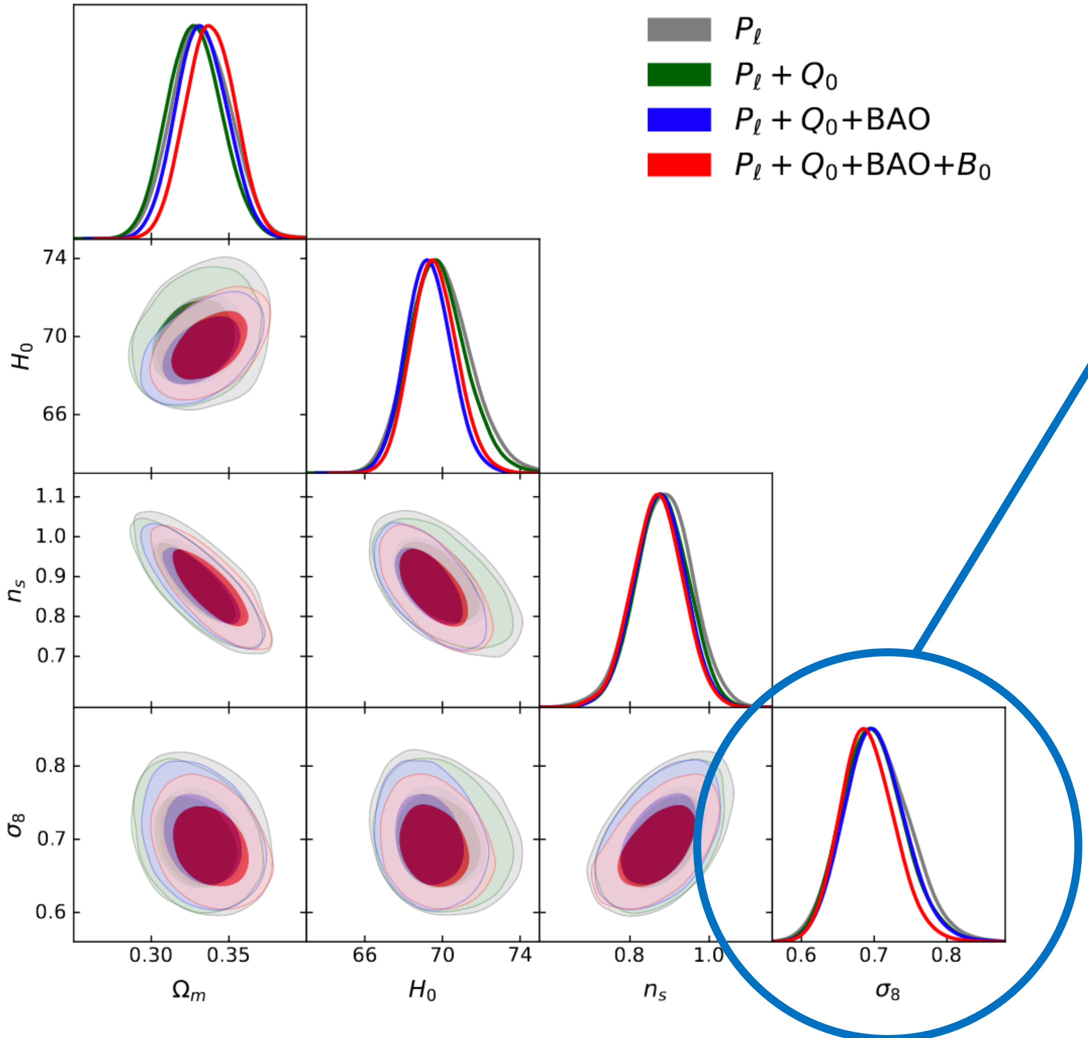
3.0σ *tension with SH0ES!*

No evidence for new physics from BOSS!

Philcox+21,22, Farren+21

CONSTRAINING ΛCDM: σ_8

BOSS (+ BBN) Constraints



BOSS Power Spectrum + Bispectrum:

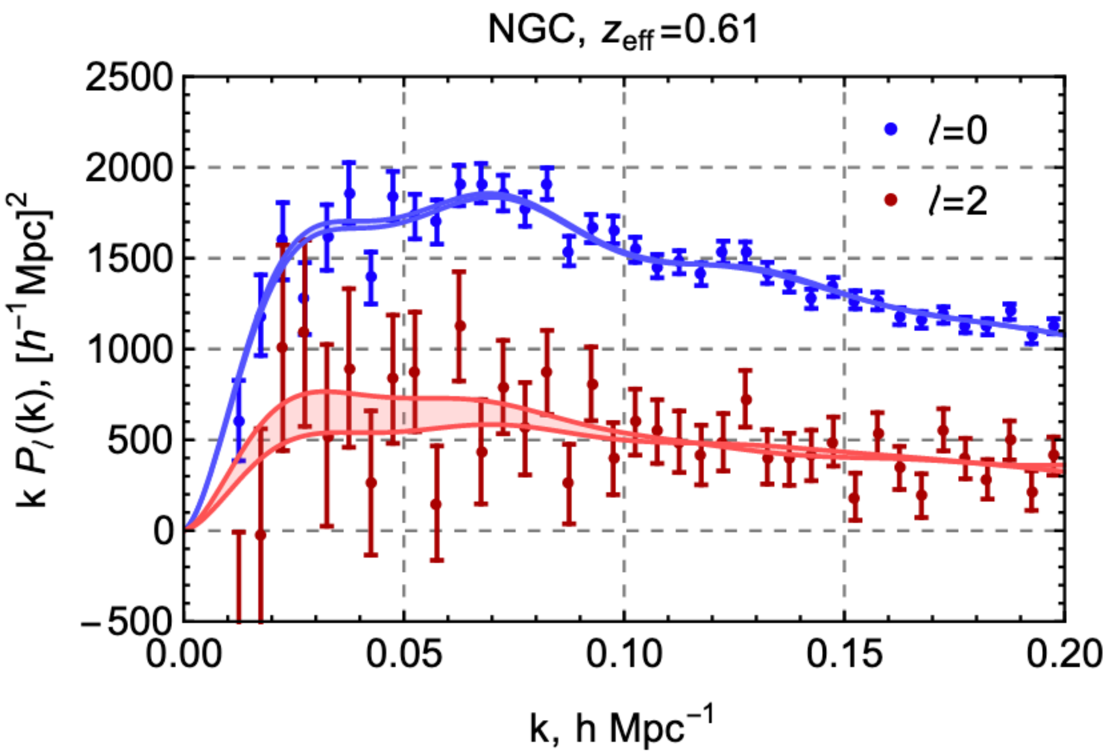
$\boldsymbol{S_8 = 0.73 \pm 0.04}$ (BOSS, with *Planck* n_s)

This is consistent with weak lensing, but somewhat lower than *Planck*:

$\boldsymbol{S_8 = 0.83 \pm 0.01}$ (*Planck*)

Philcox+21 (see also Chen+21, d'Amico+21)

WHERE DOES THE σ_8 INFORMATION COME FROM?



$\boldsymbol{\sigma_8}$ is set by the **large-scale** ($k < 0.1h/\mathrm{Mpc}$) quadrupole

This is hard to change!

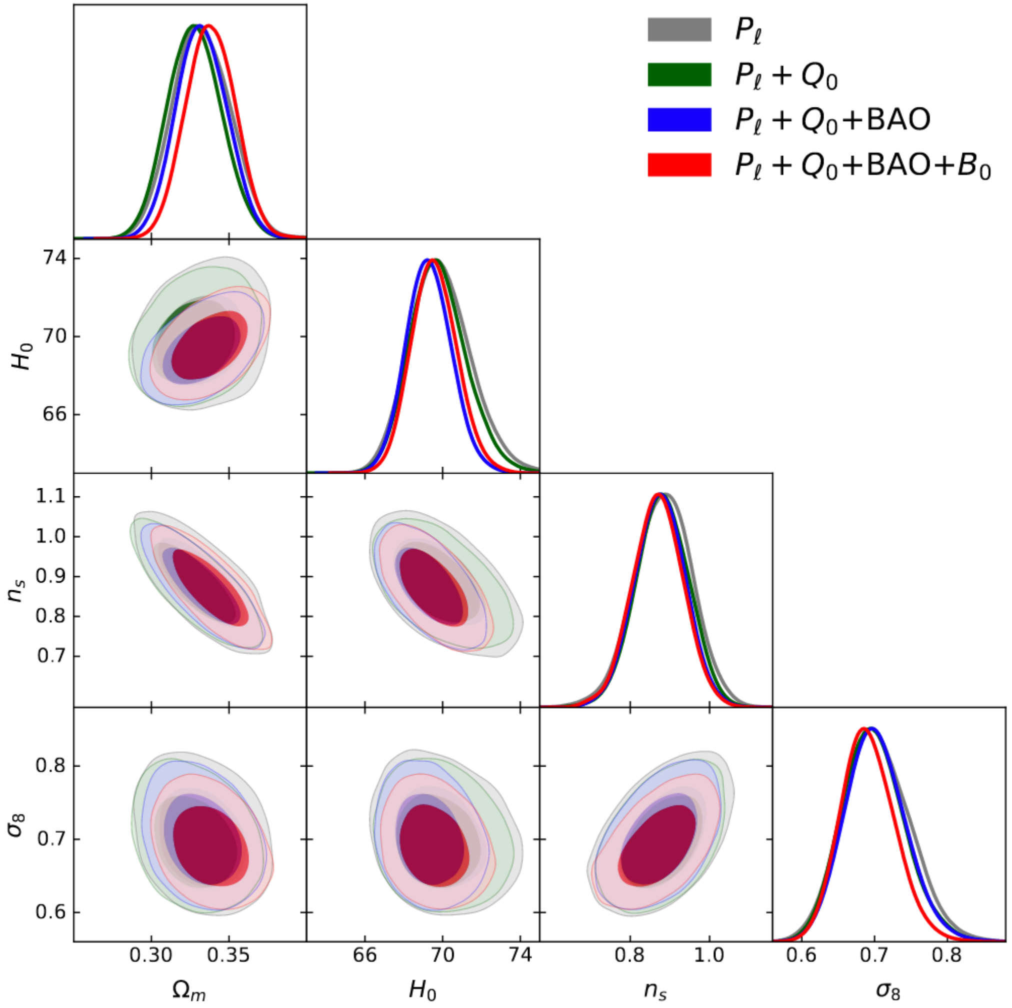
- ▷ Mostly linear scales
- ▷ Bias well understood
- ▷ Fingers-of-God suppressed

<u>But</u> priors are 1σ effect! [Simon+22]

Philcox+21 (see also Chen+21, d'Amico+21)

CONSTRAINTS ON OTHER PARAMETERS

BOSS (+ BBN) Constraints



Matter Density:

$$\Omega_m = 0.34 \pm 0.02$$

Consistent with Pantheon+ supernovae!

Spectral Slope:

$$n_s = 0.87 \pm 0.07$$

Consistent with *Planck*

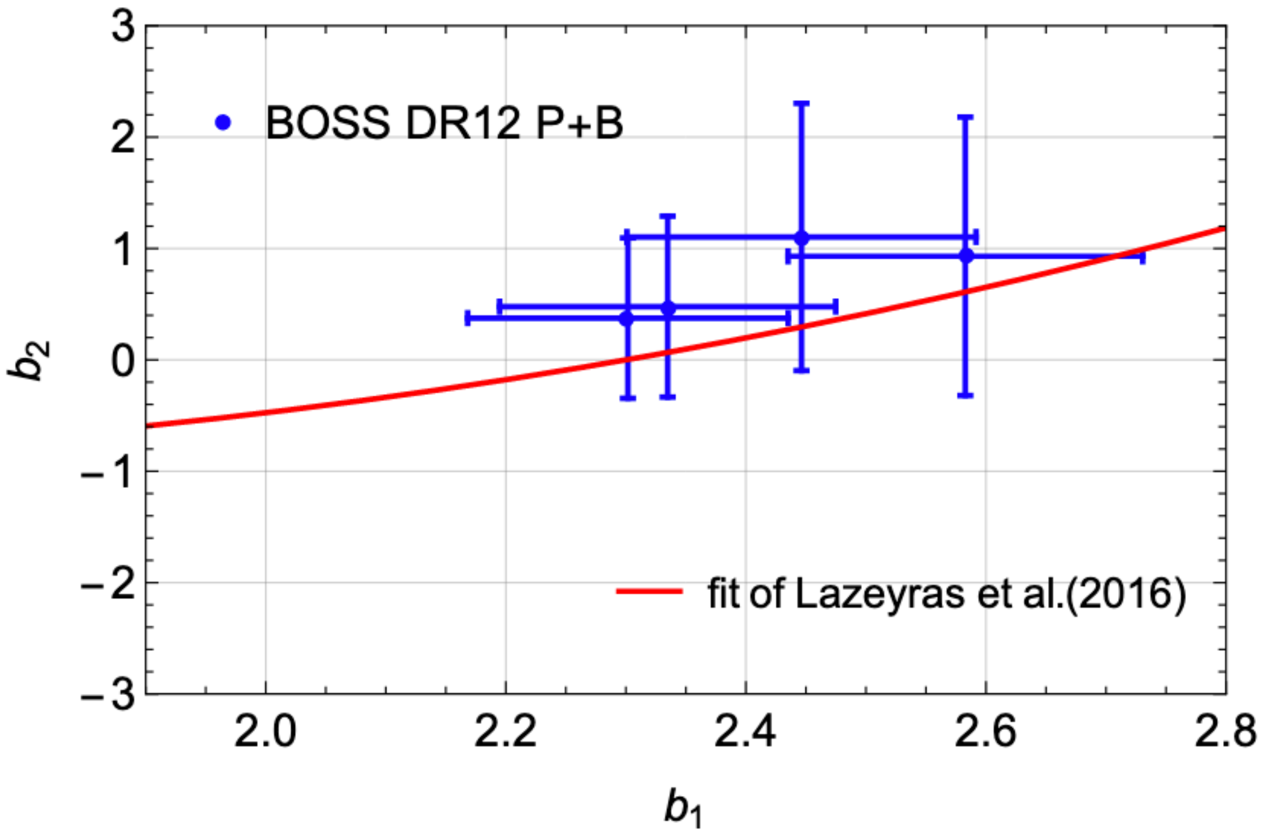
Neutrino Mass:

$$\sum m_\nu < 0.14\text{ eV (95\% CL)}$$

Philcox+20,21 (see also Chen+21, d'Amico+21)

CONSTRAINTS ON ASTROPHYSICS

- Analysis also measures **bias parameters** (especially the bispectrum)
- These encode the physics of galaxy formation
- Consistent with simulation results so far, though small deviations **expected**



Philcox+21 (see also Chen+21, d'Amico+21)

NON-GAUSSIAN INFLATION

*Are the primordial perturbations **Gaussian** and **adiabatic**?*

In Single-Field Slow-Roll Inflation:

$$f_{\rm NL} \sim (1-n_s) \ll 1$$

Non-standard inflation can beat this:

- Multifield Inflation [Local Bispectrum]
- New Kinetic Terms [Equilateral Bispectrum]
- New Vacuum States [Folded Bispectrum]

$$\boxed{B_\zeta(\mathbf{k}_1,\mathbf{k}_2) \approx \frac{6}{5} f_{\rm NL} P_\zeta(k_1) P_\zeta(k_2) + 2\text{ perms.}}$$

Search for in the galaxy bispectrum!

Maldacena 03, Creminelli+04

CONSTRAINING INFLATION

Need to include PNG in EFTofLSS modelling!

- Primordial bispectrum:

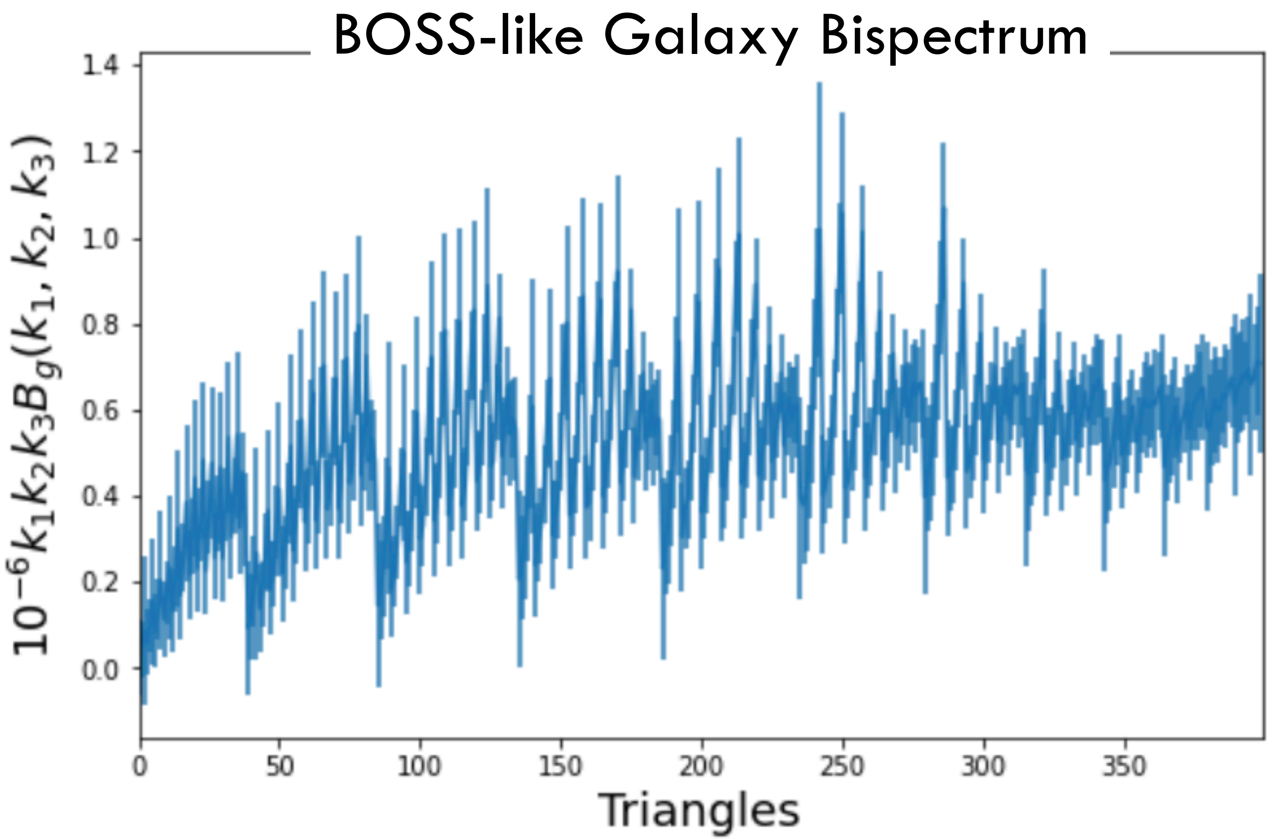
$$\left\langle \delta^{(1)}\delta^{(1)}\delta^{(1)} \right\rangle \sim f_{\rm NL} P^2(k)$$

- Scale dependent bias:

$$b_1(f_{\rm NL}) \to b_1 + (b_\phi f_{\rm NL})/k^2$$

- Loop corrections:

$$P_{gg}(\boldsymbol{k}) \to P_{gg}(\boldsymbol{k}) + f_{\rm NL}\int d\boldsymbol{q}\,\alpha\, P(\boldsymbol{q})P(\boldsymbol{k}-\boldsymbol{q})$$



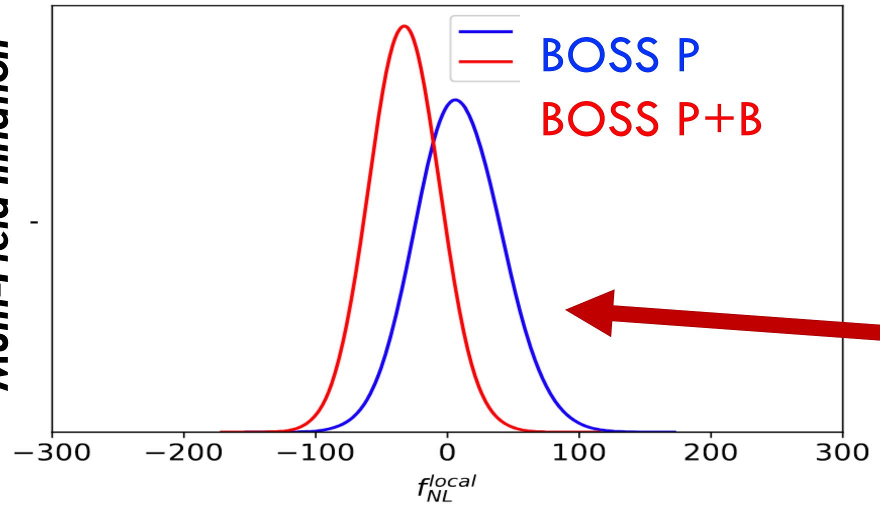
$$\boxed{B_g = B_g(f_{\rm NL}^{\rm eq}, f_{\rm NL}^{\rm orth}, f_{\rm NL}^{\rm loc})}$$

See GitHub.com/michalychforever/CLASS-PT

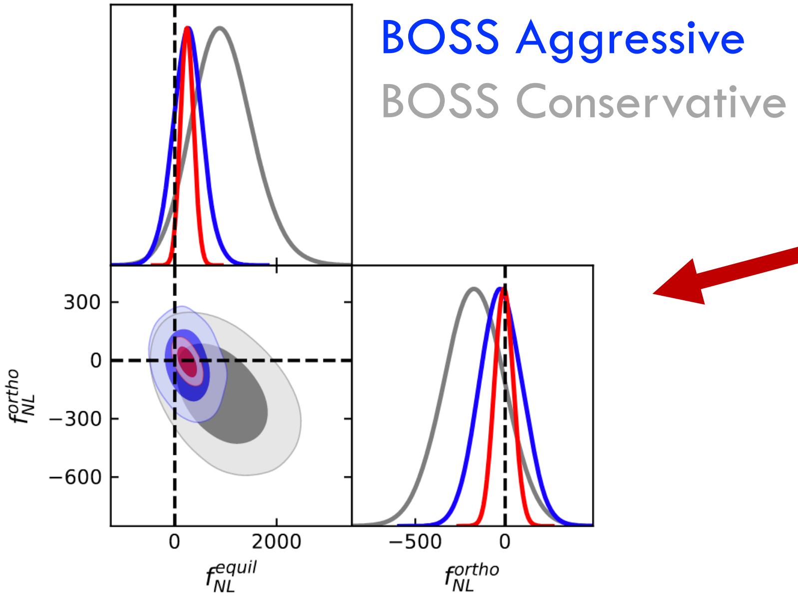
Cabass, Philcox+21,22 (see also d'Amico+22)

CONSTRAINING INFLATION

Multi-Field Inflation



Single-Field Inflation



BOSS Power Spectrum + Bispectrum + $O(f_{\rm NL})$ Theory Model

$$f_{\rm NL}^{\rm local} = -33 \pm 28$$

(Really measuring $b_\phi f_{\rm NL}$ - see Barreira+22)

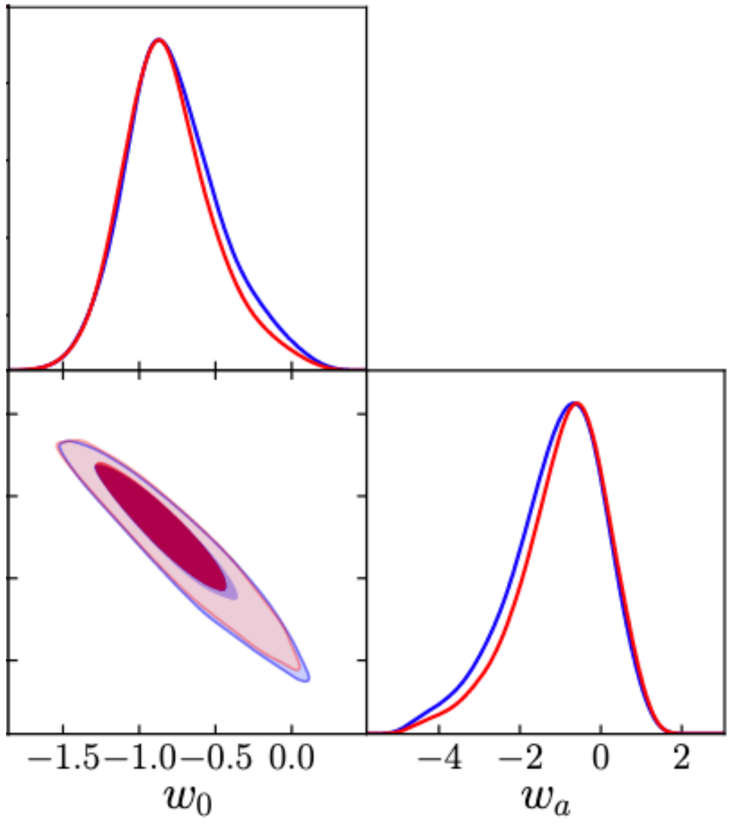
$$f_{\rm NL}^{\rm equil} = 260 \pm 300$$

$$f_{\rm NL}^{\rm orth} = -23 \pm 120$$

- First measurement without CMB
- Needs bispectrum

Cabass, Philcox+21,22 (see also d'Amico+22)

POST-ΛCDM CONSTRAINTS FROM THE COMMUNITY



$$w_0 = -0.98 \pm 0.01$$

$$w_a = -0.3 \pm 0.6$$

- w_0, w_a consistent with cosmological constant [Chudaykin+20]
- Curvature consistent with zero [Chudaykin+20]
- No evidence for early dark energy [Ivanov+20]
- Strong constraints on light massive relics [Xu+22]
- Strong constraints on axion dark matter [Lague+21, Rogers+ (in prep.)]
- Strong constraints on dark-sector interactions [Nunez+22]

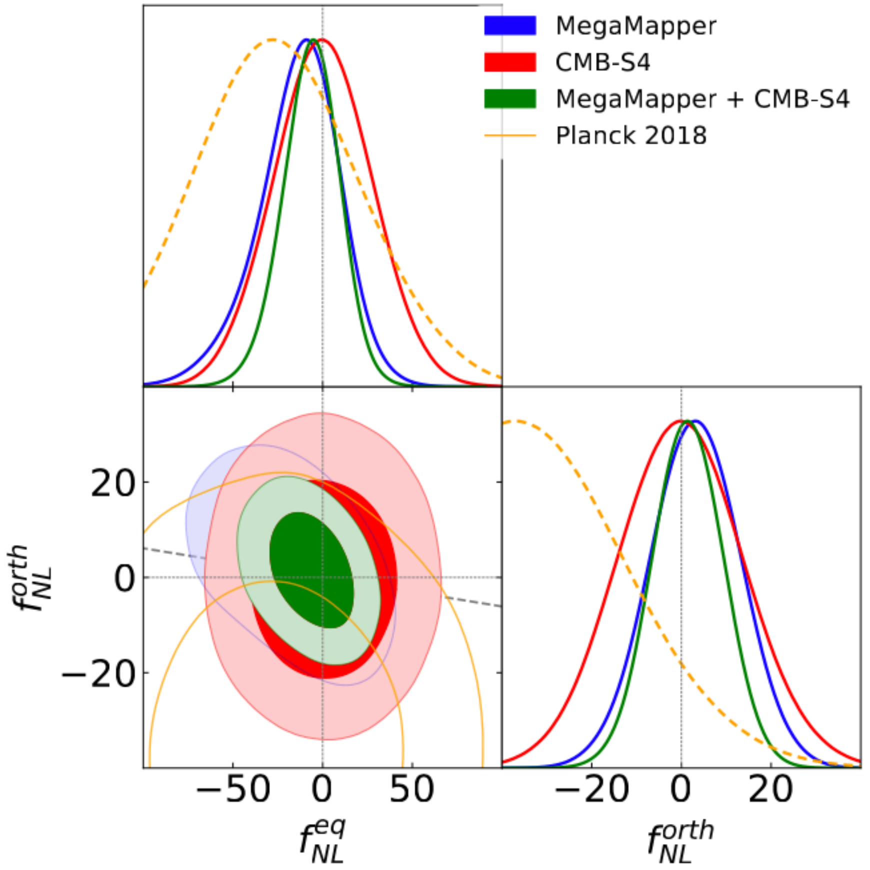
And many more…

All analysis is public:
github.com/oliverphilcox/full_shape_likelihoods

WHAT'S NEXT FOR THE EFT OF LSS?

- Compute **2-loop** power spectra?
- Compute the tree-level **trispectrum**?
- Explore other new physics?
- Apply to **DESI** / **Euclid** and beyond?

LSS constraints will (eventually) beat the CMB!



MegaMapper $f_{\rm NL}$ forecast

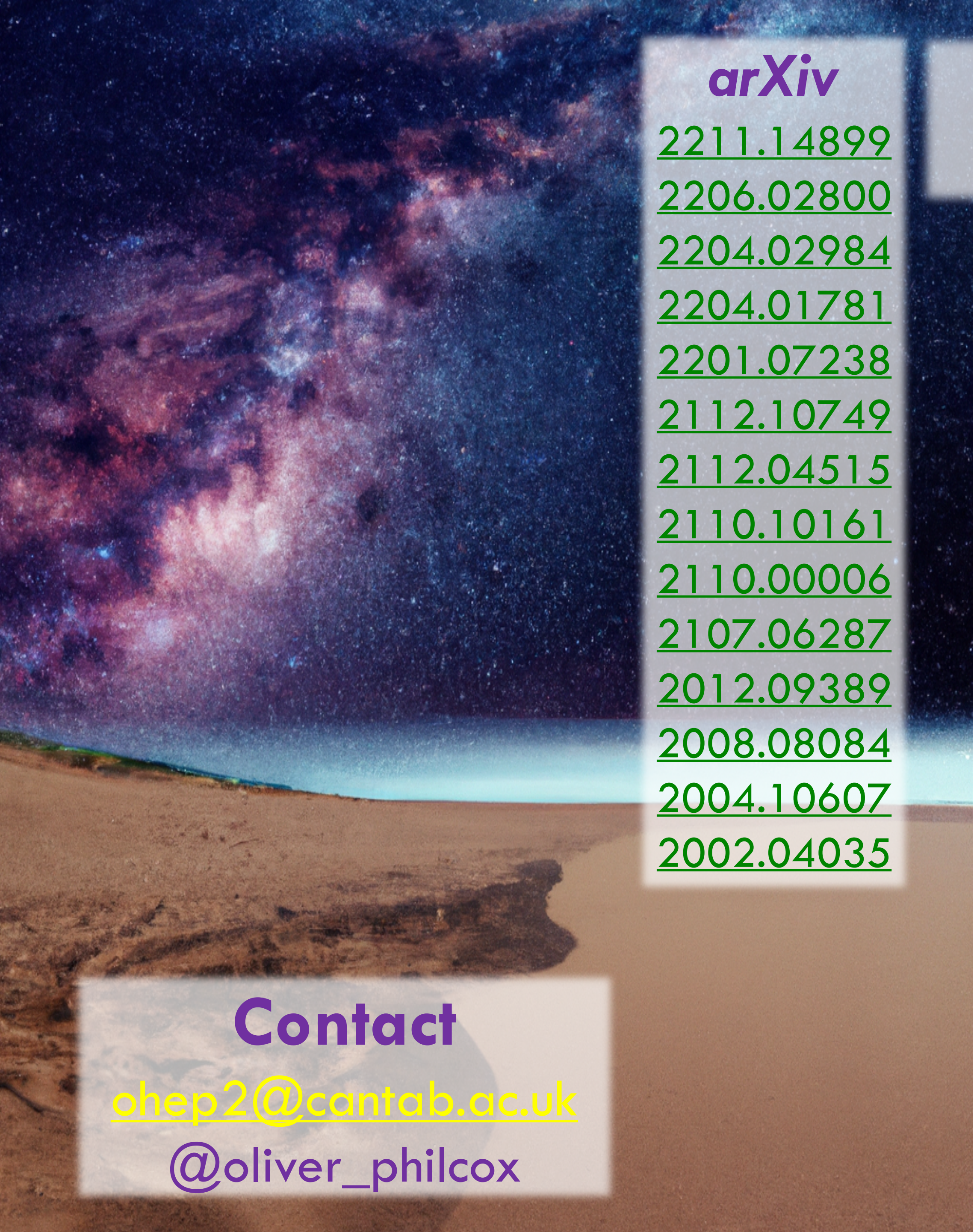
Cabass+22

CONCLUSIONS

- The **EFTofLSS** is a tool to **robustly** and **self-consistently** predict the galaxy power spectrum, bispectrum and beyond, *without* assuming UV physics

- This allows **direct** extraction of **cosmological parameters** including $H_0, \Omega_m, \sigma_8, f_{\rm NL}, w_0, \Omega_k, f_{\rm EDE}$

- BOSS data is already useful: this will get much better with **Euclid** / **DESI** and beyond



arXiv

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