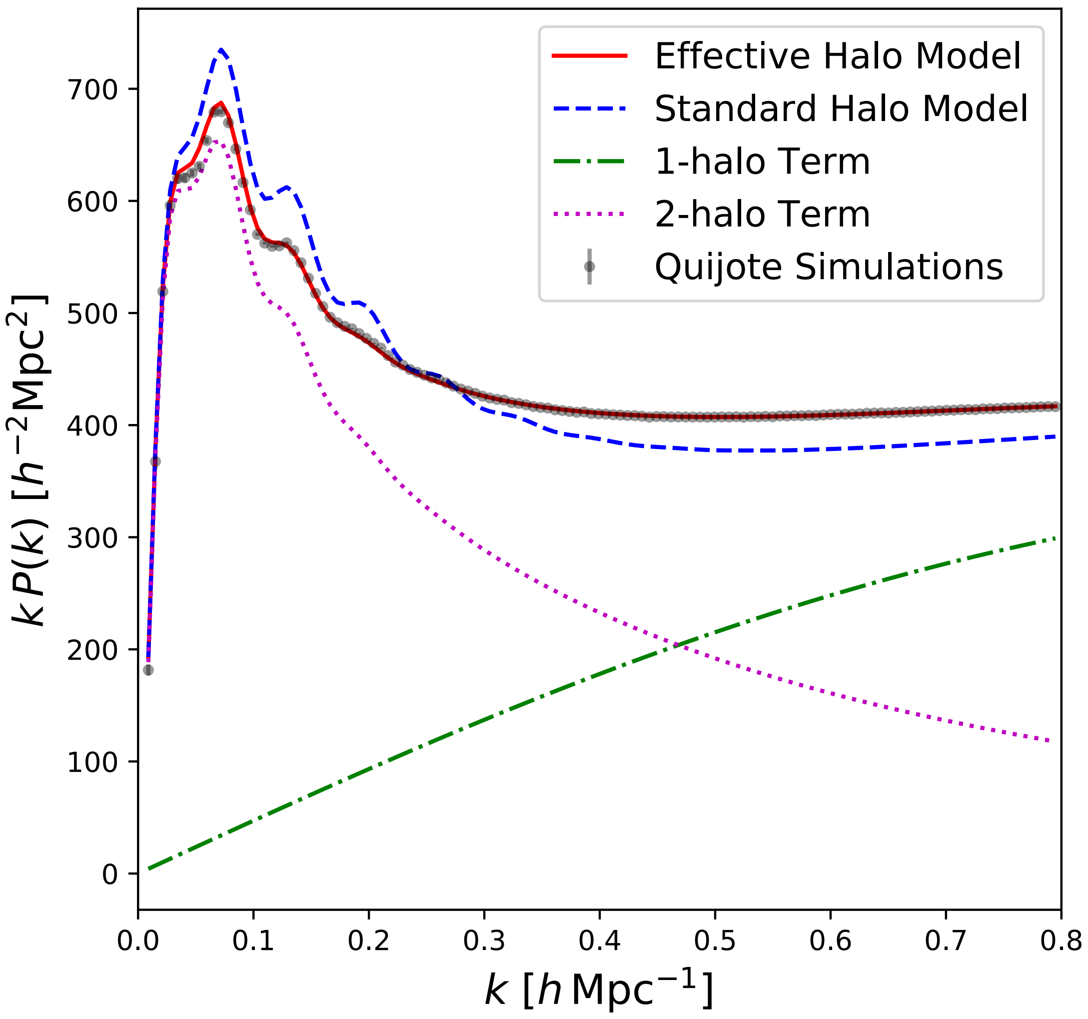


The Effective Halo Model:

Accurate Models for the Power Spectrum and Cluster Count Covariances

OLIVER PHILCOX (PRINCETON)

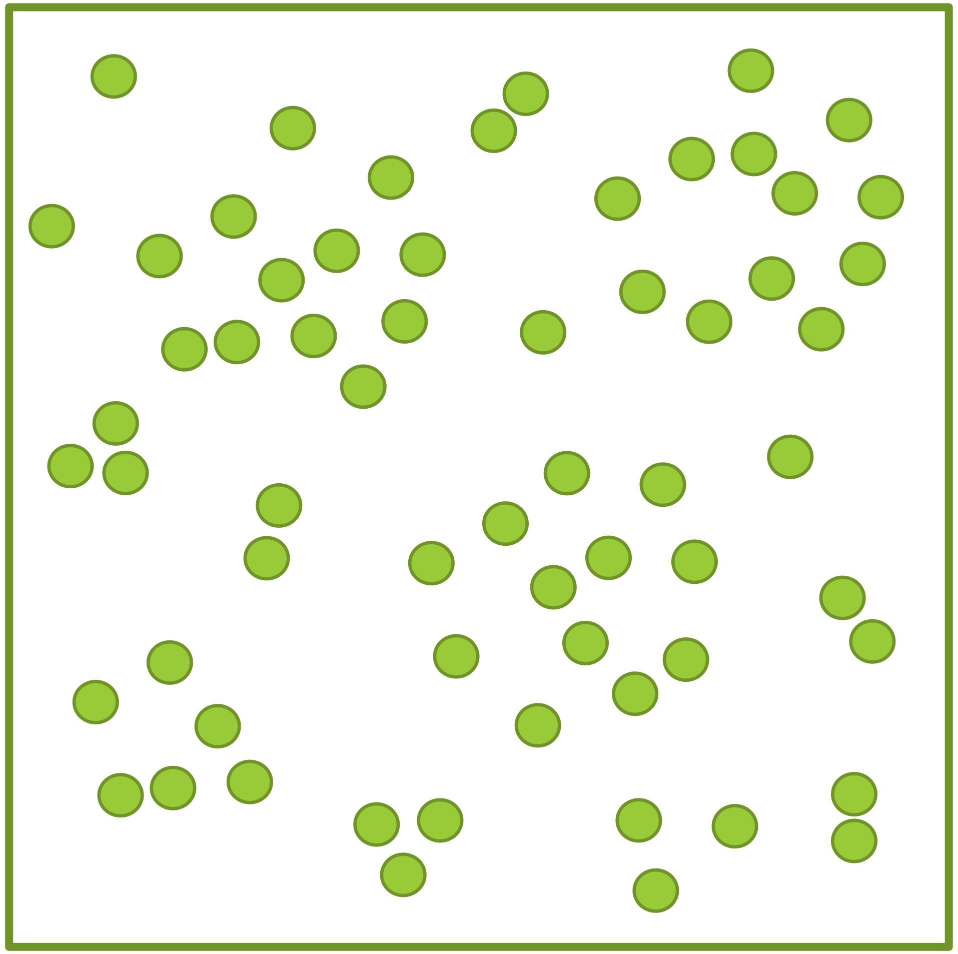
- David Spergel (Princeton / CCA)
- Francisco Villaescusa-Navarro (Princeton / CCA)

May 5, 2020

The Halo Model

- Introduced in **Peebles, 1980**

Distribution of Matter Particles



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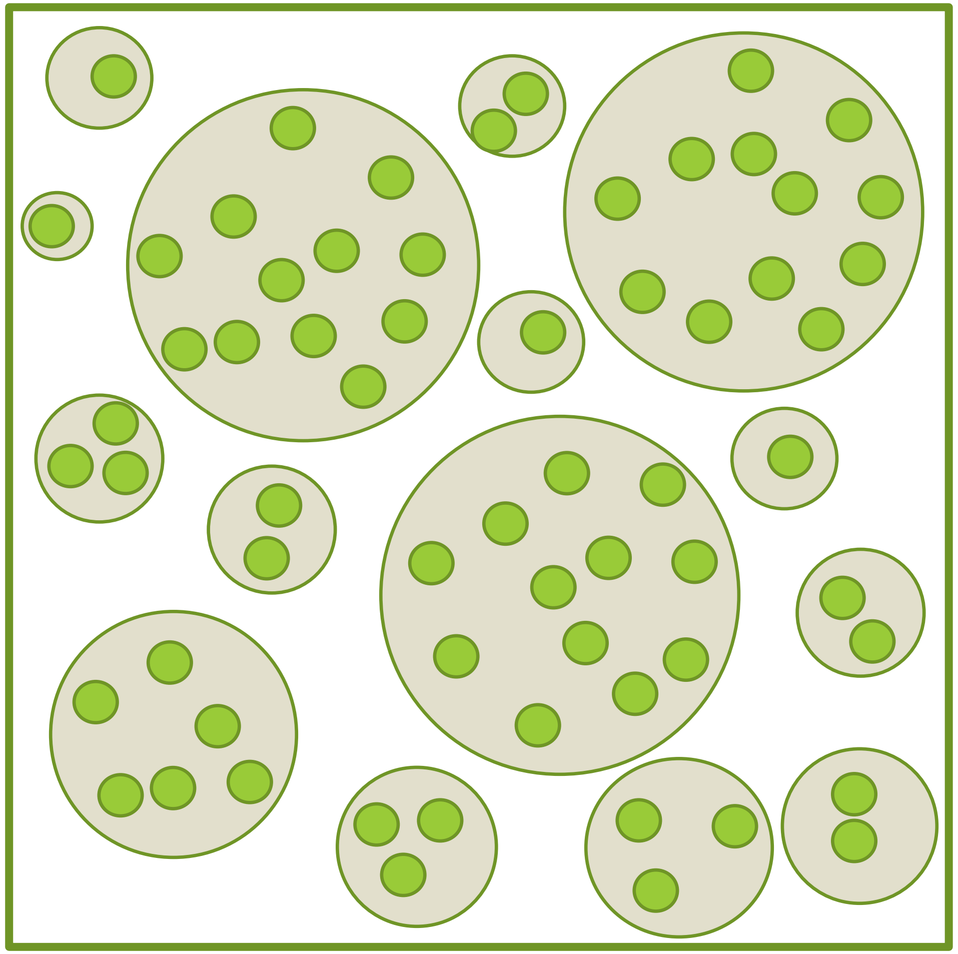

The Halo Model

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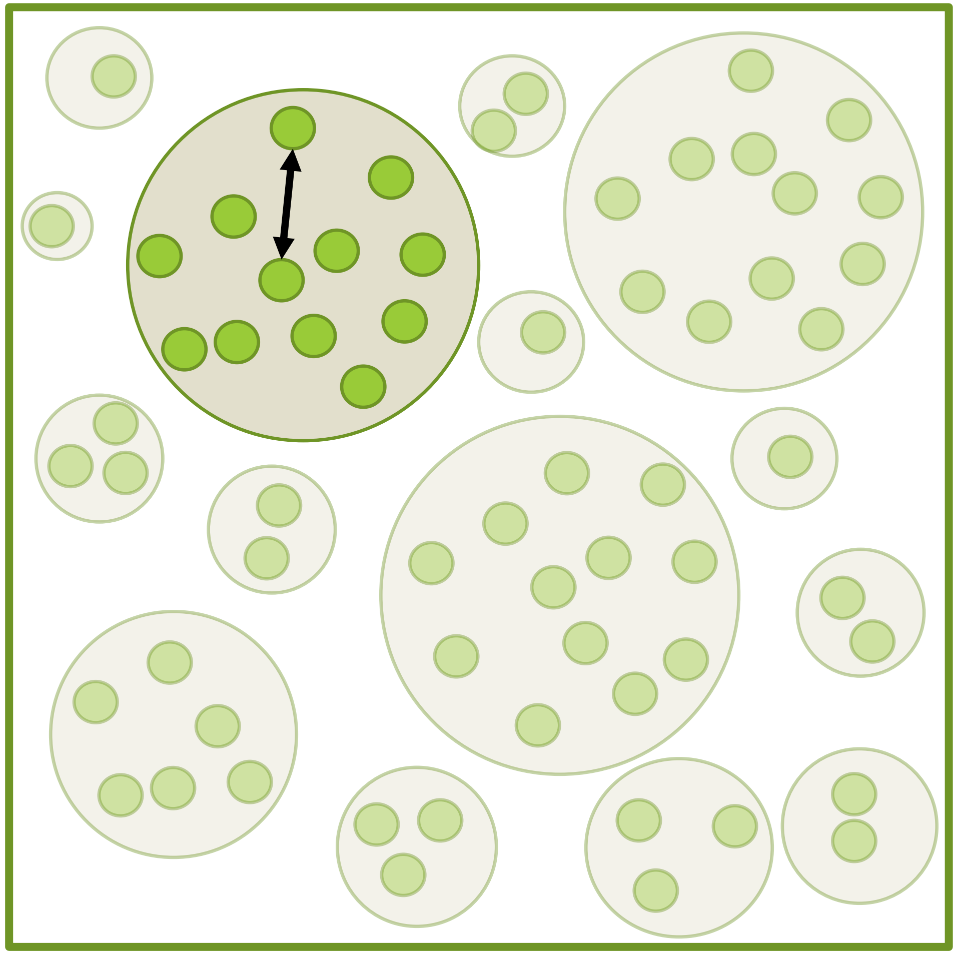
- Assumes that all matter is in **halos**
- Power spectrum composed of **one-halo** and two-halo terms

One-Halo: Particles distributed according to **halo profile**

Two-Halo: Particles distributed according to **halo distributions**

Distribution of Matter Particles



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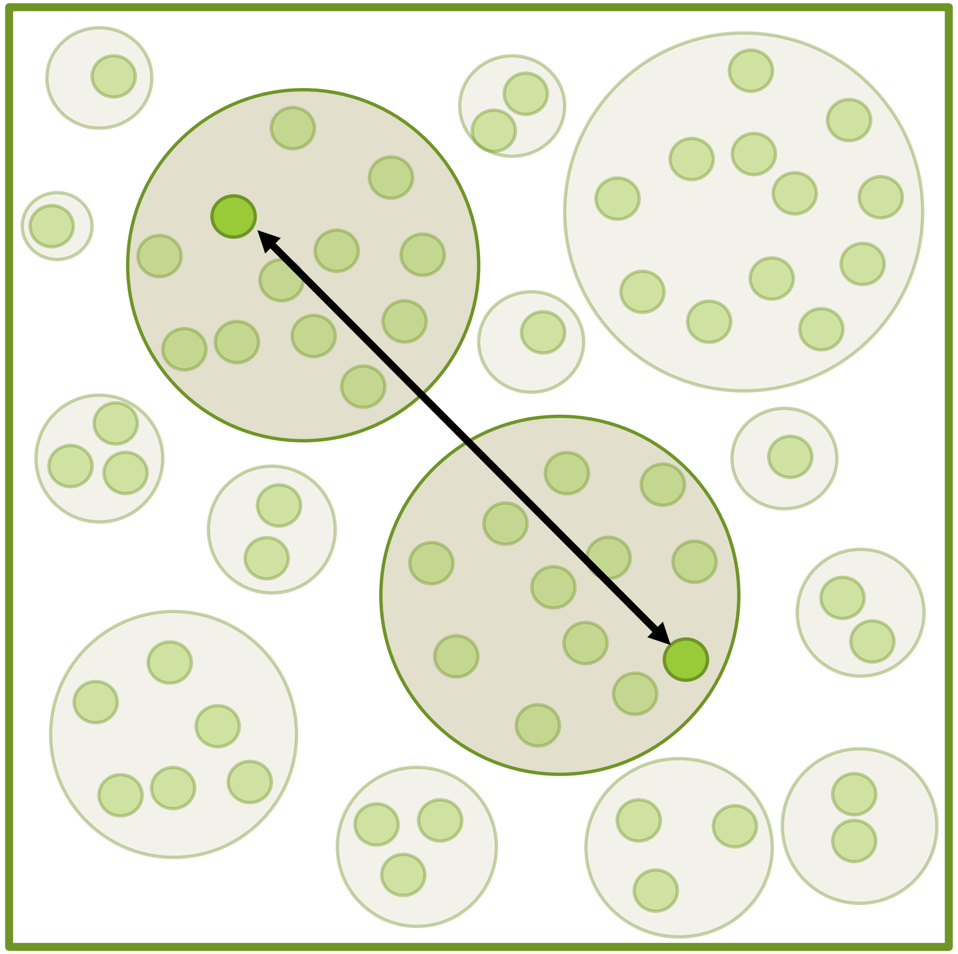

Standard Halo Model Assumptions

- Halos are **Poisson distributed**

$$\hat{n}(m_1,\mathbf{x}_1)\hat{n}(m_2,\mathbf{x}_2) = n(m_1,\mathbf{x}_1)n(m_2,\mathbf{x}_2) + n(m_1,\mathbf{x}_1)\delta_D(\mathbf{x}_1-\mathbf{x}_2)\delta_D(m_1-m_2)$$

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- Position is set by the **linear density field** $\delta_{\rm L}(\mathbf{x})$

$$n(m, \mathbf{x}) = n(m)\left[1 + b(m)\delta_{\rm L}(\mathbf{x})\right]$$

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$$n(m,\mathbf{x}) = n(m)\left[1 + b(m)\delta_{\rm L}(\mathbf{x})\right]$$

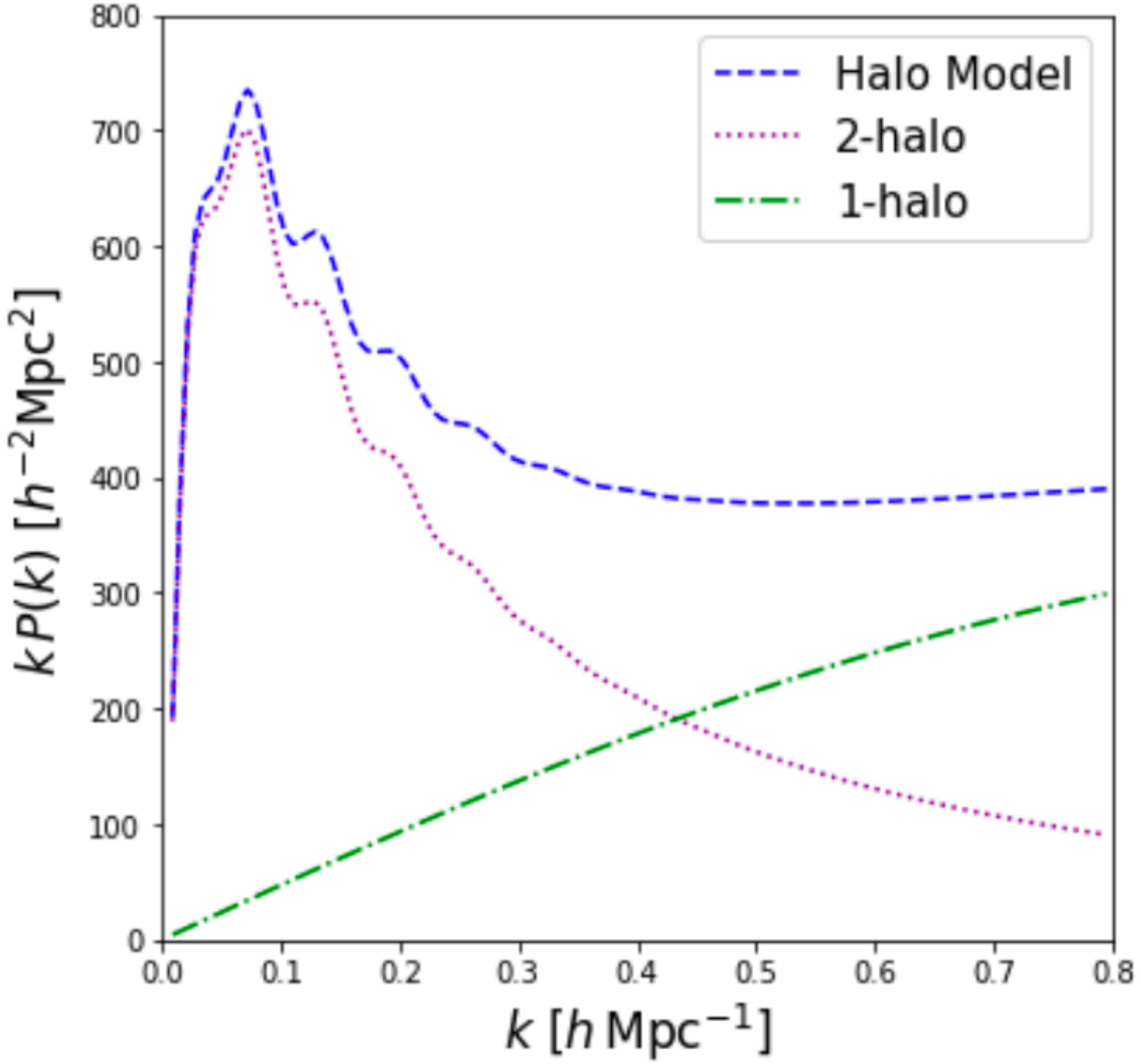
- Leads to a power spectrum;

2-halo 1-halo

$$P(k) = \underbrace{\left[I_1^1(k)\right]^2 P_{\rm L}(k)}_{\text{2-halo}} + \underbrace{I_2^0(k)}_{\text{1-halo}}$$

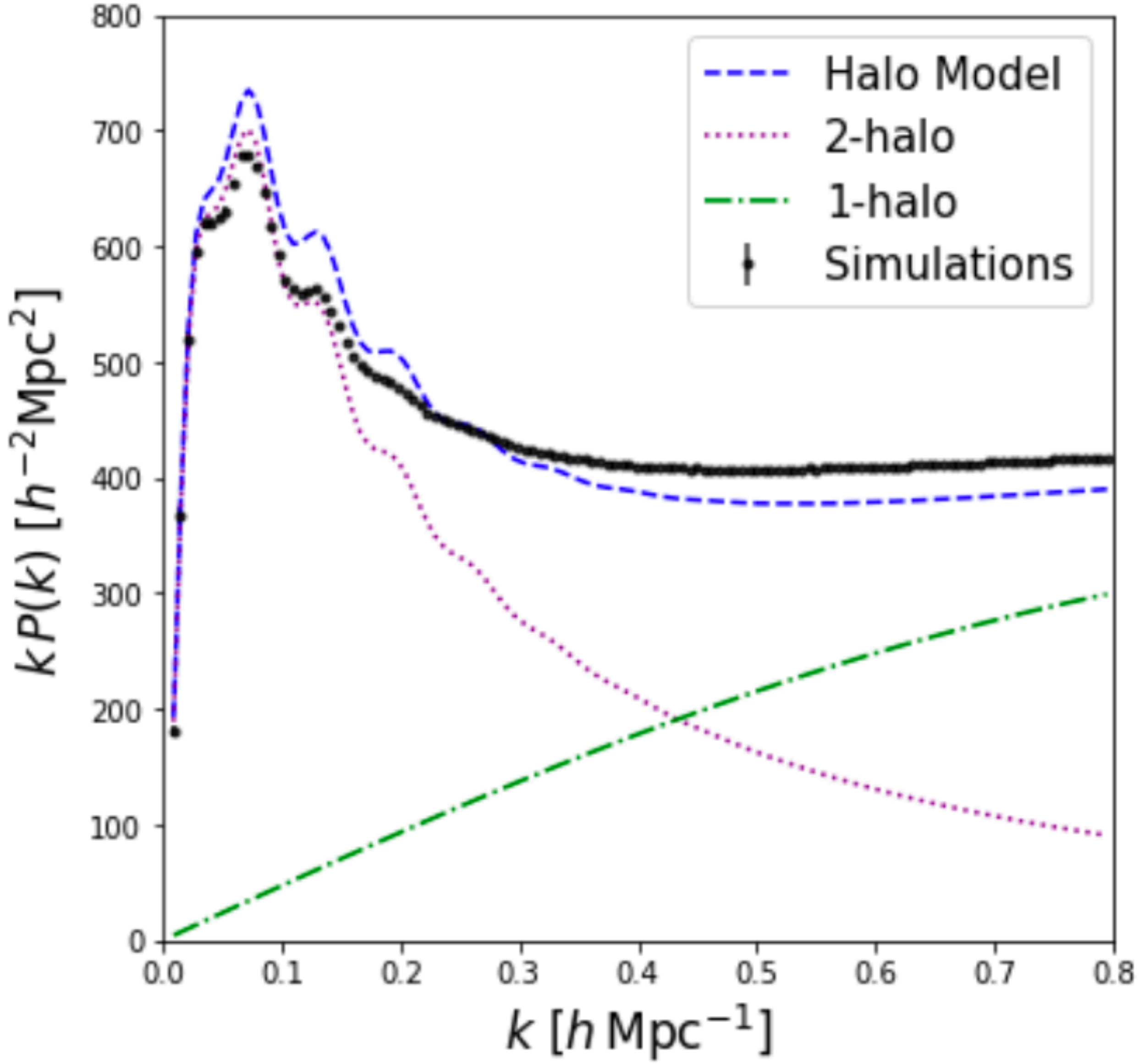
Mass Integrals: $I_1^1(k)$, $I_2^0(k)$

Linear Power: $P_{\rm L}(k)$



Standard Halo Model Prediction

Models computed with ***EffectiveHalos*** *Python code*



N-body Simulations

Standard Halo Model Prediction

Using 100 high-resolution Quijote simulations
(Villaescusa-Navarro+19)

*Models computed with **EffectiveHalos** Python code*

Standard Halo Model Assumptions

- Halos are **Poisson distributed**

$$\hat{n}(m_1, \mathbf{x}_1)\hat{n}(m_2, \mathbf{x}_2) = n(m_1, \mathbf{x}_1)n(m_2, \mathbf{x}_2) + n(m_1, \mathbf{x}_1)\delta_D(\mathbf{x}_1 - \mathbf{x}_2)\delta_D(m_1 - m_2)$$

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Effective Halo Model Assumptions

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- Position is set by the ~~linear density field~~ **smoothed non-linear** density field $\delta_R(\mathbf{x})$

$$n(m,\mathbf{x}) = n(m)\left[1 \cancel{+ b(m)\delta_{\rm L}(\mathbf{x})} + b(m)\delta_{\rm R}(\mathbf{x}) + \ldots\right]$$

Effective Halo Model Assumptions

- Halos are **Poisson distributed**

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- **Justification:**

  - Halos are well-described by **non-linear** Effective Field Theory models
  - Most of halo accretion occurs at **late-times** so needs **Eulerian** description
  - Ensures our model is equal to **perturbation theory** on large scales

Effective Halo Model Power

$$P(k) = \left[I_1^1(k)\right]^2 P_{\rm L}(k) + I_2^0(k)$$

Standard Model

Philcox+20

Effective Halo Model Power

$$P(k) = \left[I_1^1(k)\right]^2 P_{\rm L}(k) + I_2^0(k) \;\Rightarrow\; P(k) = \left[I_1^1(k)\right]^2 P_{\rm NL}(k) W^2(kR) + I_2^0(k)$$

2-halo

1-halo

Smoothing Function

Mass Integrals

Non-Linear Power

Standard Model

Effective Model

$P_{\rm NL}(k)$ is modelled with **Effective Field Theory**

Effective Field Theory

- Evaluate non-linear power with (IR-resummed) 1-loop **Effective Field Theory**

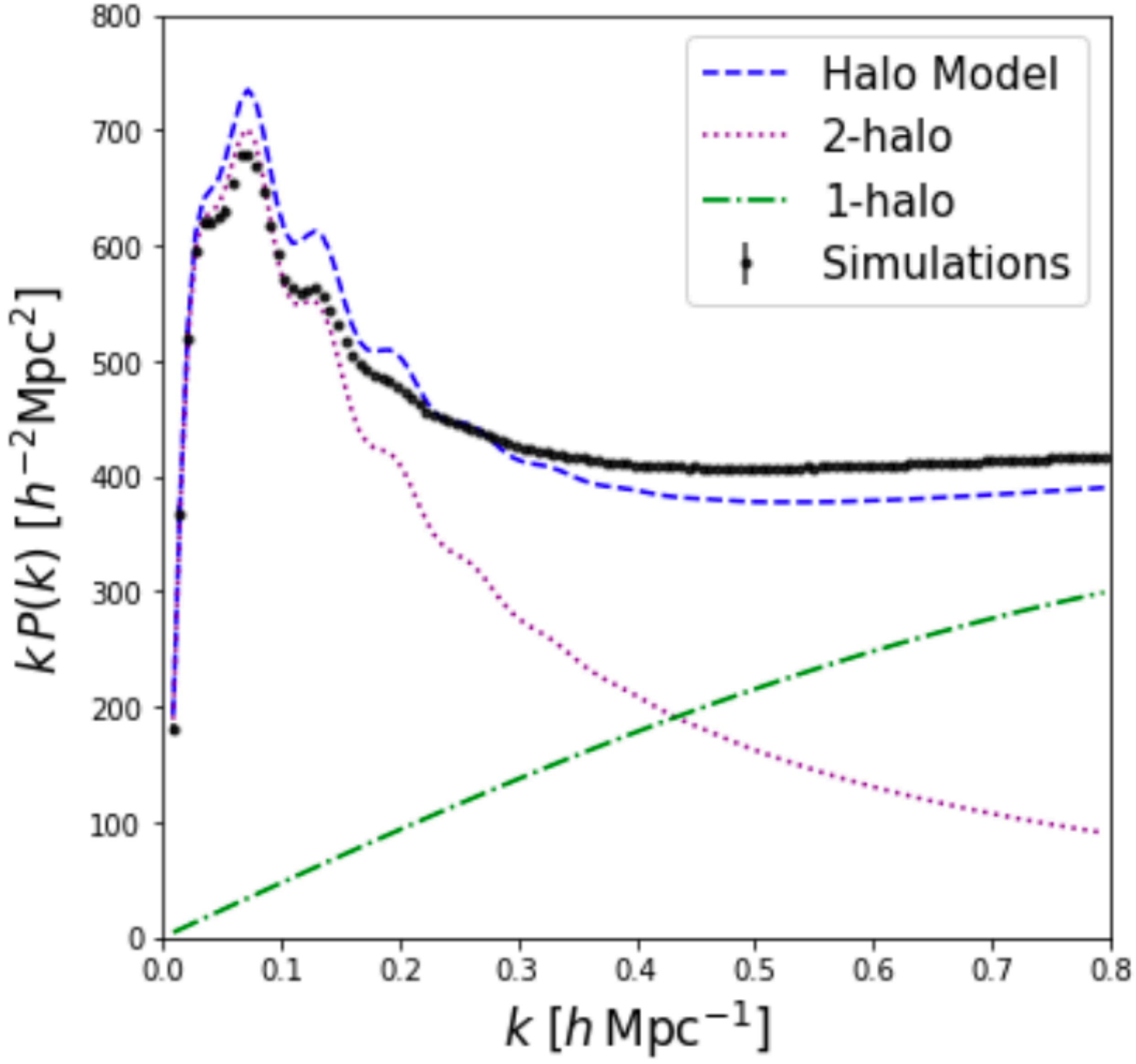
$$P_{\mathrm{NL}}(k) = P_{\mathrm{L}}(k) + P_{1-\mathrm{loop}}(k) - c_s^2 \frac{k^2}{(1+k^2)} P_{\mathrm{L}}(k)$$

Linear Power — 1-loop Power — Counterterm

- Counterterm has a **free parameter** c_s^2 and is Pade resummed.

- Percent-accurate up to $k \sim 0.2h\,\mathrm{Mpc}^{-1}$

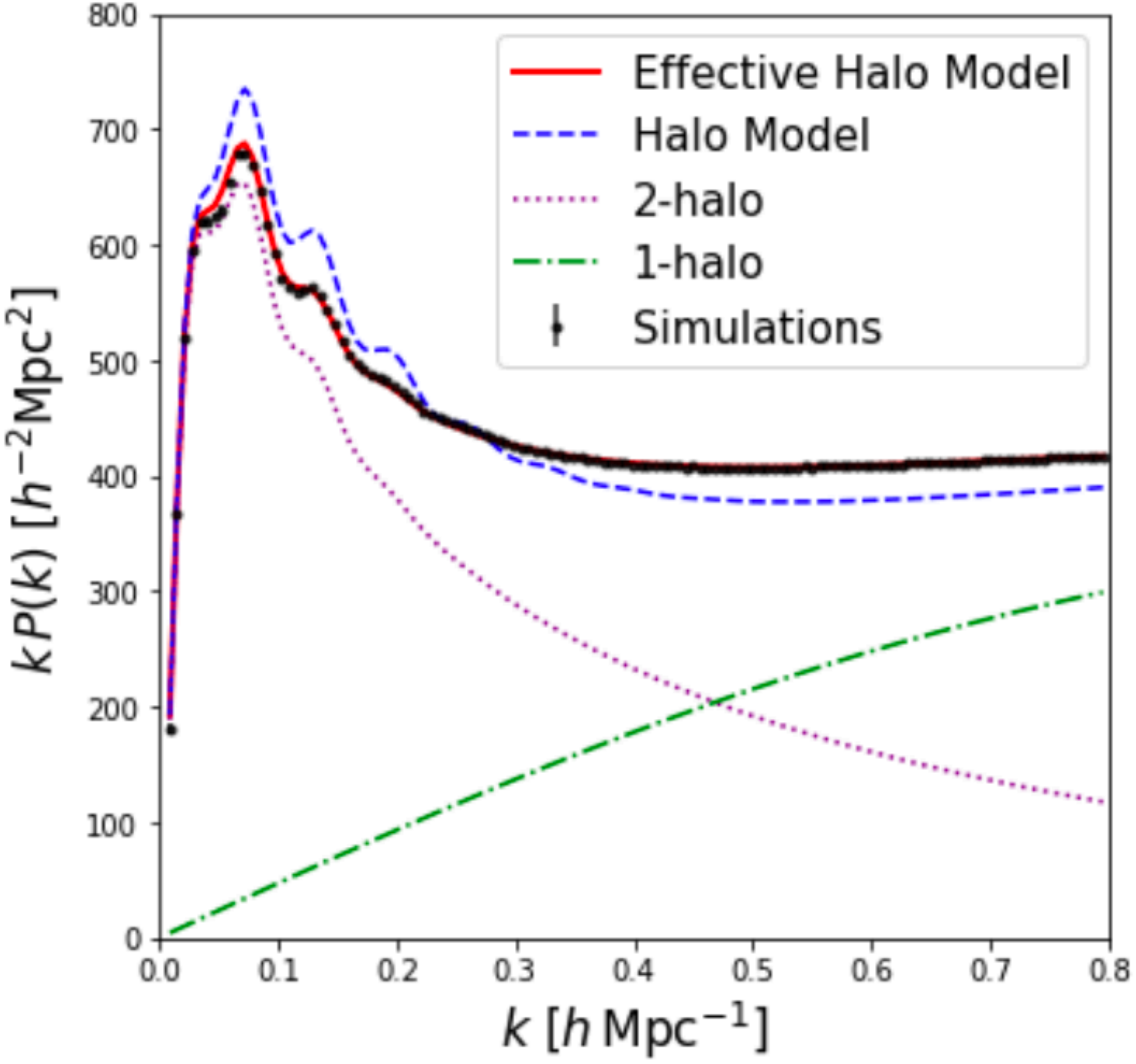
Carrasco+12, Baumann+12



N-body Simulations

Standard Halo Model Prediction

Philcox+20



Effective Halo Model

N-body Simulations

Standard Halo Model Prediction

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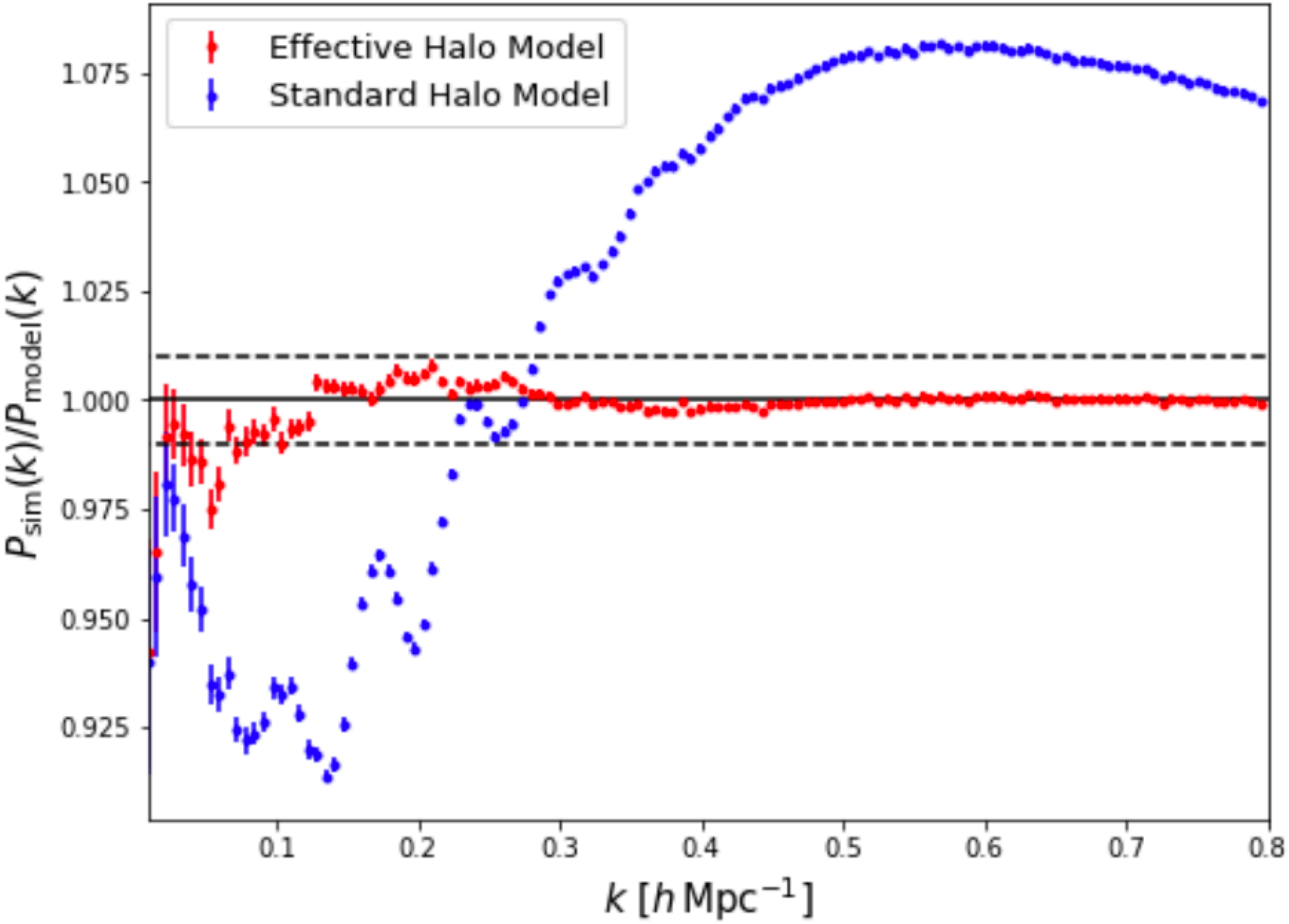
Ratio of Simulation and Model Power



The Effective Halo Model is percent-accurate up to large k!

Philcox+20

Beyond the Power Spectrum

- The Effective Halo Model can be used to predict **other statistics**, e.g.

  - **Power-spectrum covariance** $\mathrm{cov}\left(P(k), P(k')\right)$
  - **Cluster-count covariance** $\mathrm{cov}\left(N(m), N(m')\right)$
  - **Power-spectrum x cluster-count covariance** $\mathrm{cov}\left(N(m), P(k)\right)$

- These are useful for **tSZ** and **weak-lensing** analyses

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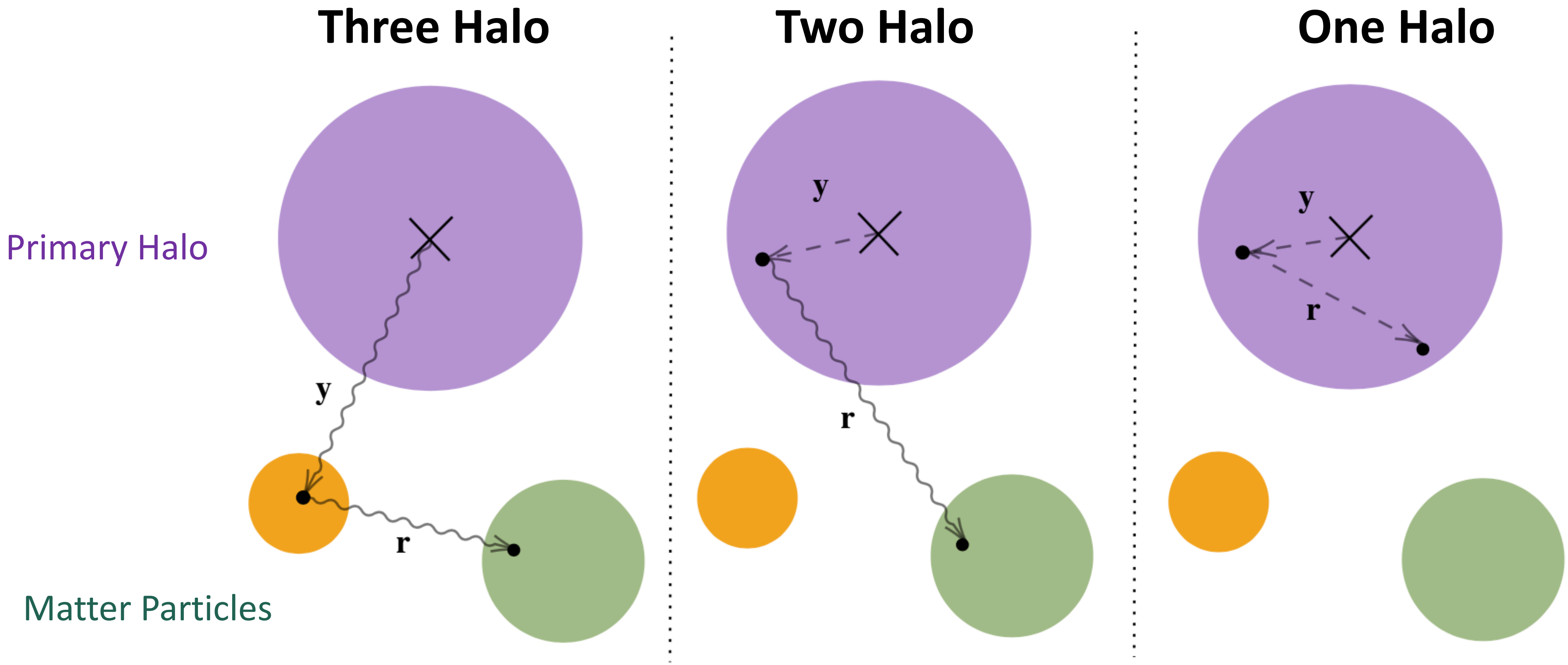
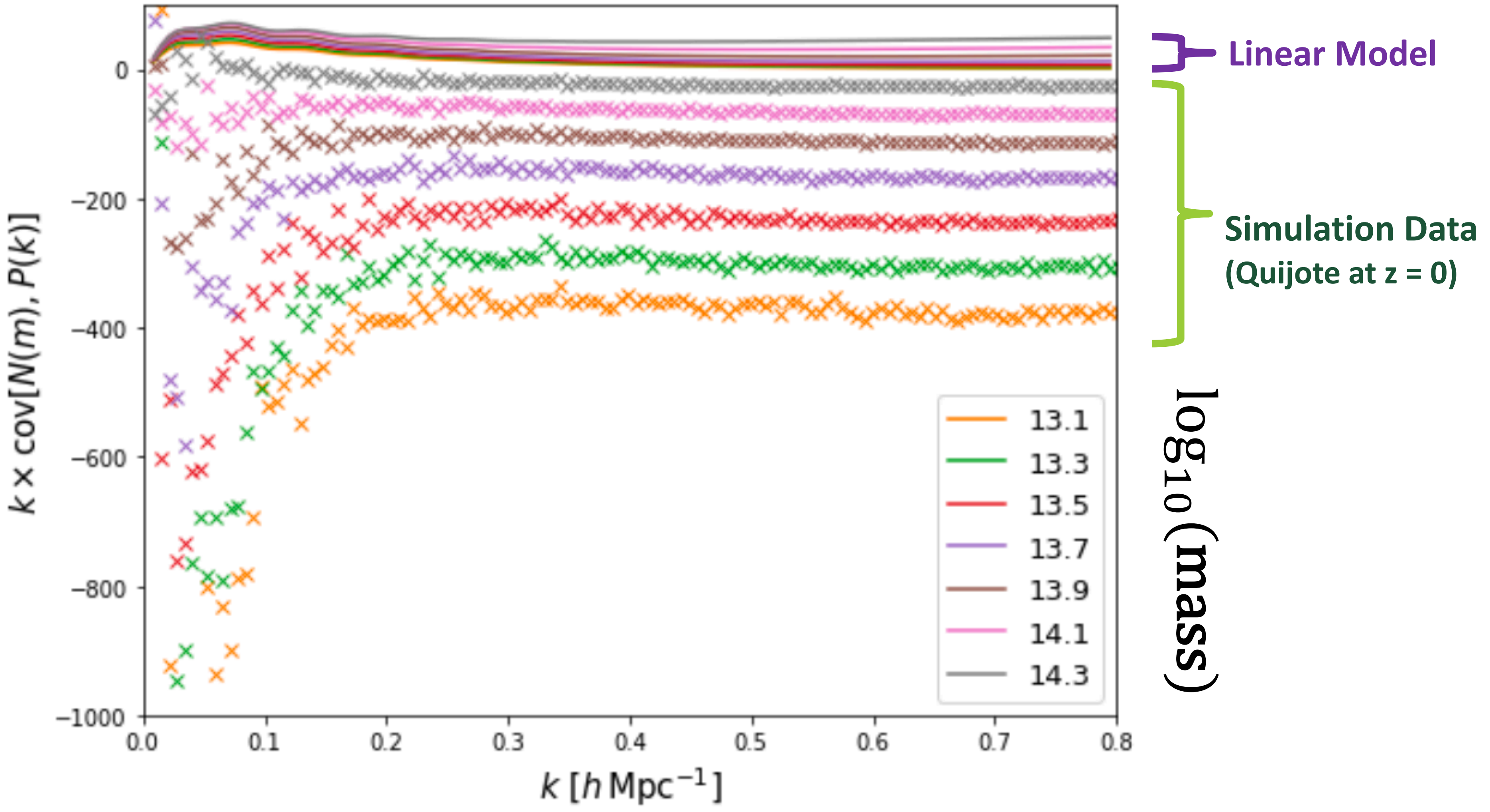
1) Intrinsic Covariance



Old Model



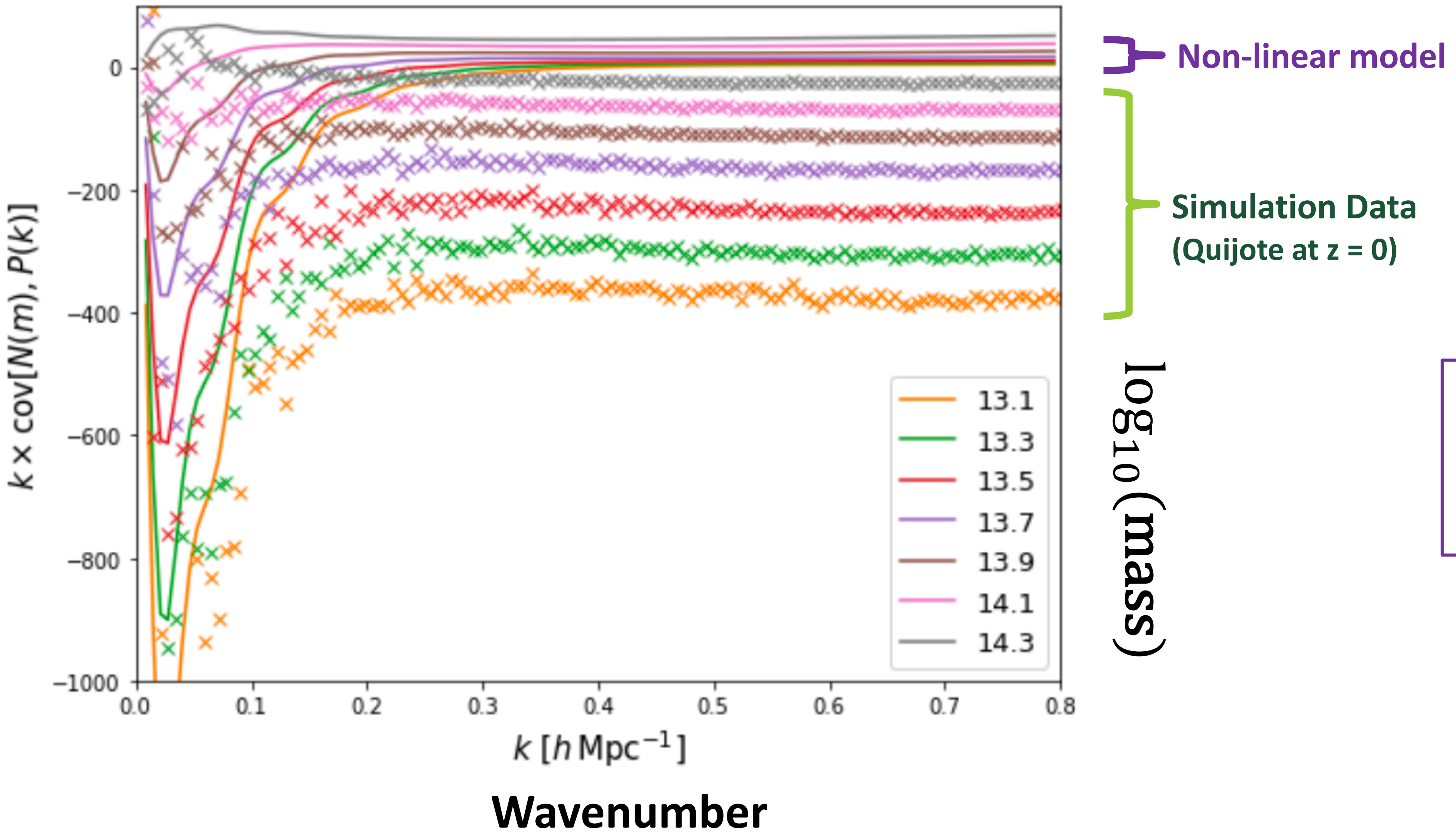
Wavenumber

Model assumes:
- Linear bias
- Linear power

Schaan+14, Takada+07,14

Using 15000 Quijote simulations at standard resolution (no super-sample effects)

New Intrinsic Covariance Model



Model assumes:
- **Quadratic** bias
- **Non-Linear** power

Philcox+20

Using 15000 Quijote simulations at standard resolution (no super-sample effects)

2) Exclusion Covariance

- Halos cannot overlap!
  - For halo mass m, there must be **no halos** with $m' > m$ within some **exclusion radius** $R_{\rm ex}(m, m')$

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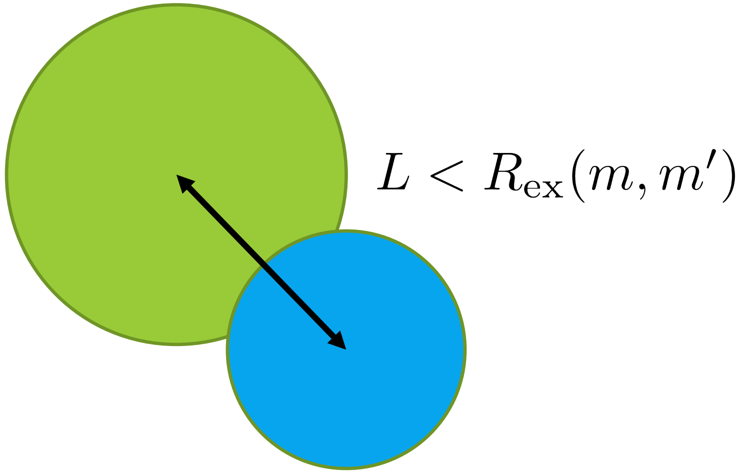
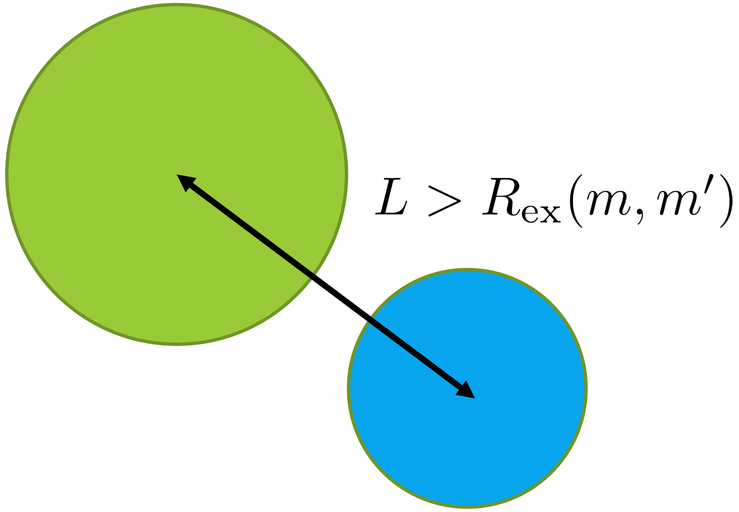


Overlapping Halos: Classified as One Object



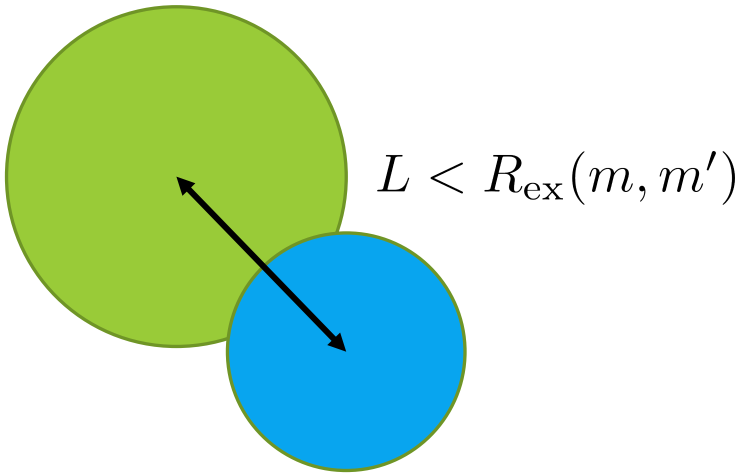
Separate Halos: Classified as Two Objects

2) Exclusion Covariance

- Halos cannot overlap!
  - For halo mass m, there must be **no halos** with $m' > m$ within some **exclusion radius** $R_{\rm ex}(m,m')$

$$R_{\rm ex}(m,m') = \alpha\left[R_{\rm Lag}(m) + R_{\rm Lag}(m')\right]$$

$\alpha < 1$: Halo Exclusion Scale



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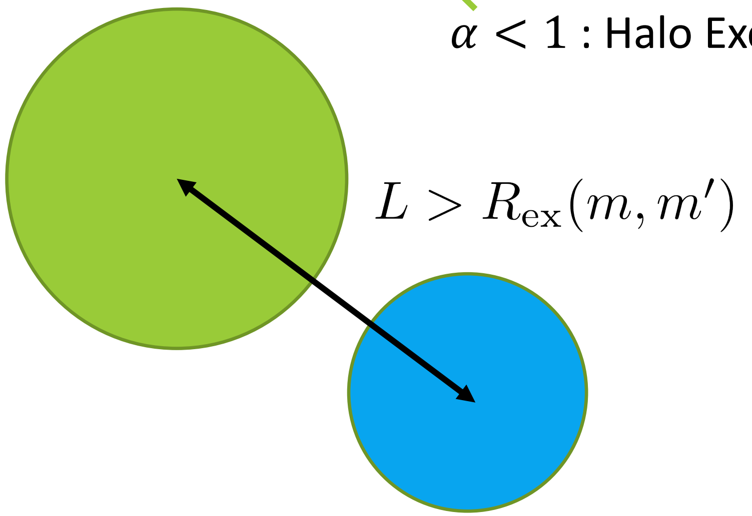
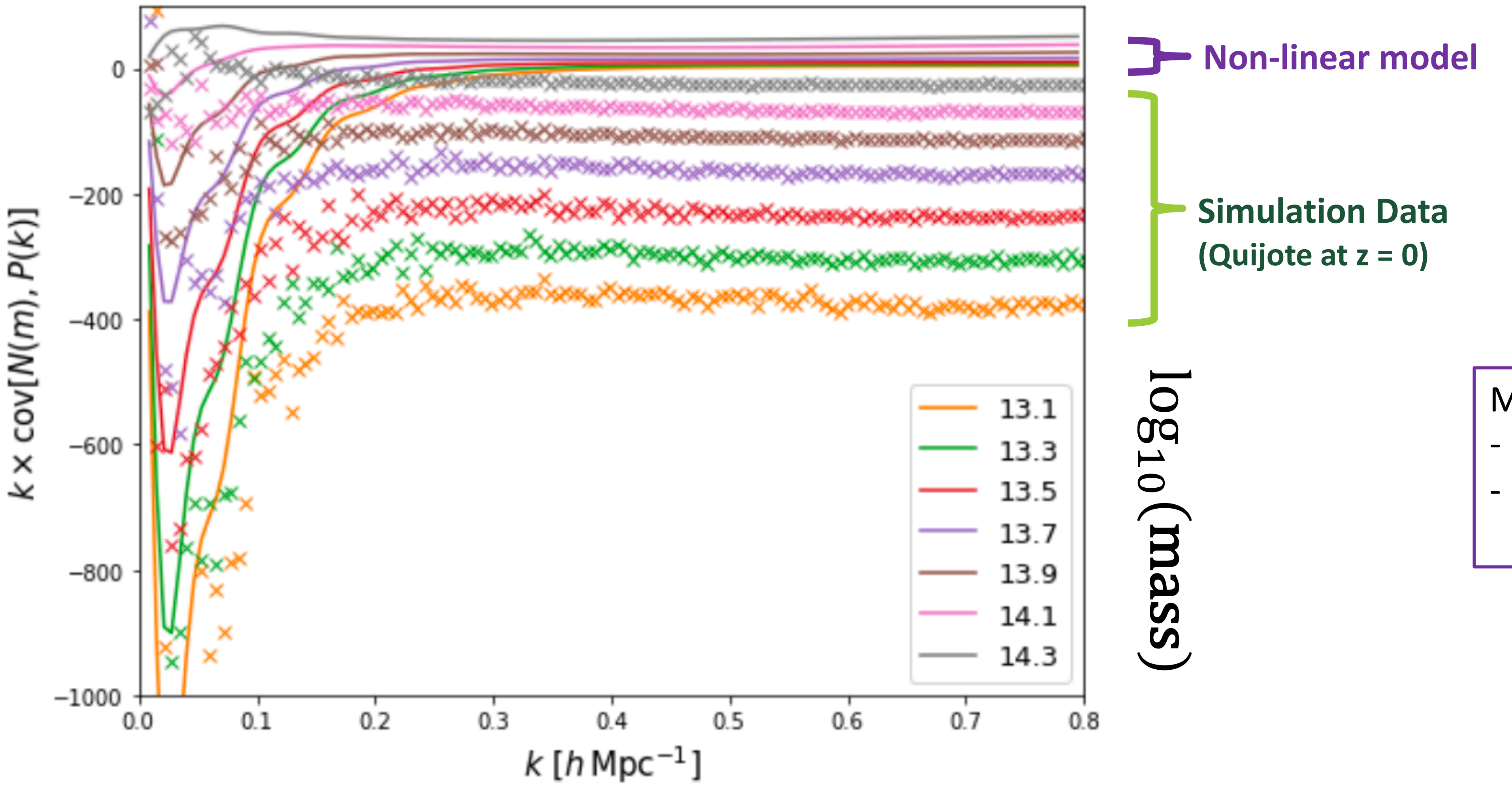


Separate Halos: Classified as Two Objects

New Intrinsic Covariance Model



Wavenumber

Model assumes:

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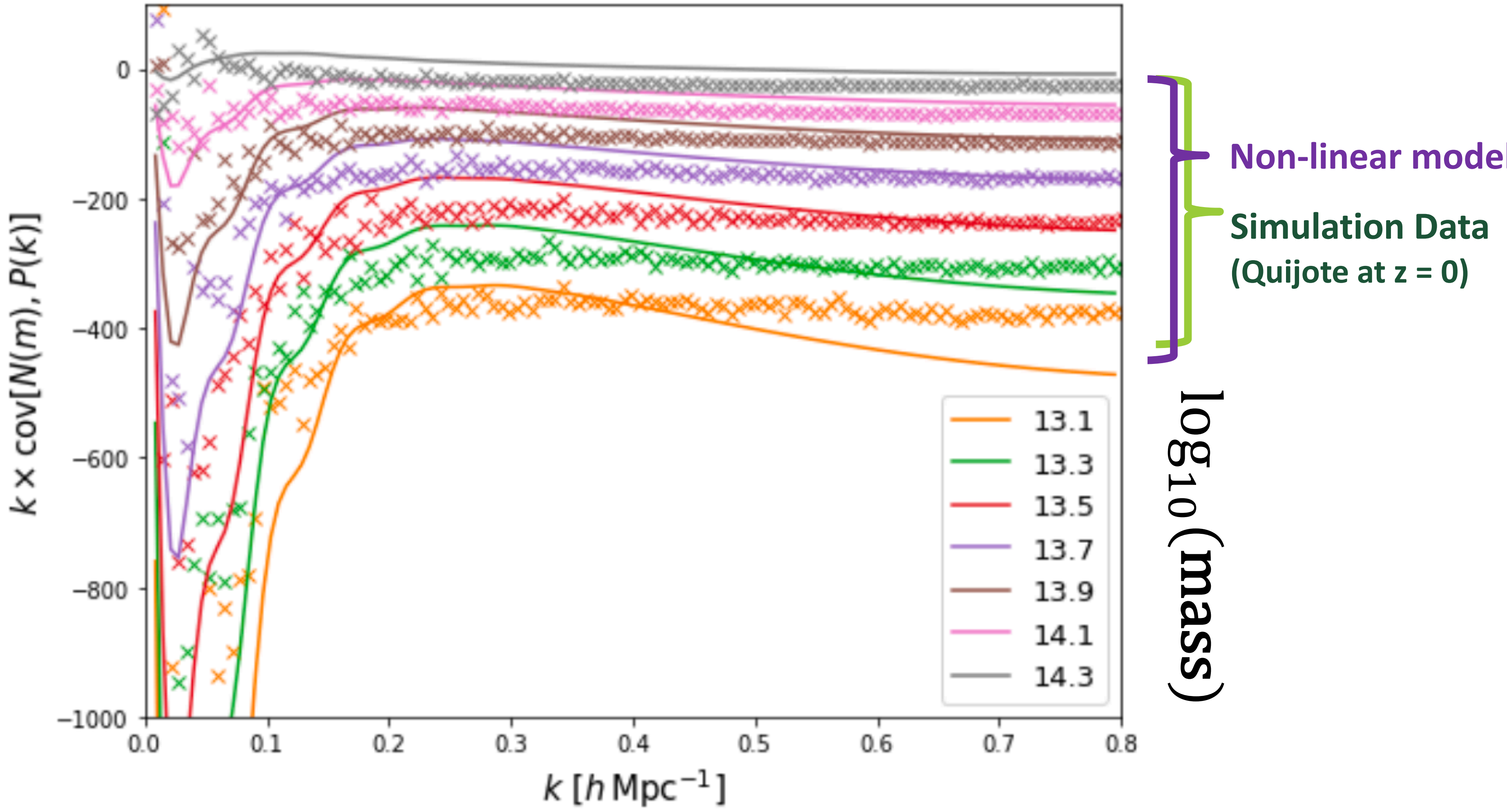
- **Non-Linear** power

Philcox+20

Using 15000 Quijote simulations at standard resolution (no super-sample effects)

New Intrinsic + Exclusion Covariance Model



Wavenumber

Model assumes:
- **Quadratic** bias
- **Non-Linear** power
- **Halo exclusion**

Philcox+20

Using 15000 Quijote simulations at standard resolution (no super-sample effects)

3) Super-Sample Covariance

- Density modes δ_b on scales comparable to the survey width modulate the background density

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- This causes an **extra covariance**

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Number count response

Power spectrum response

Variance of δ_b

Takada+13, Schaan+14, Chiang+14

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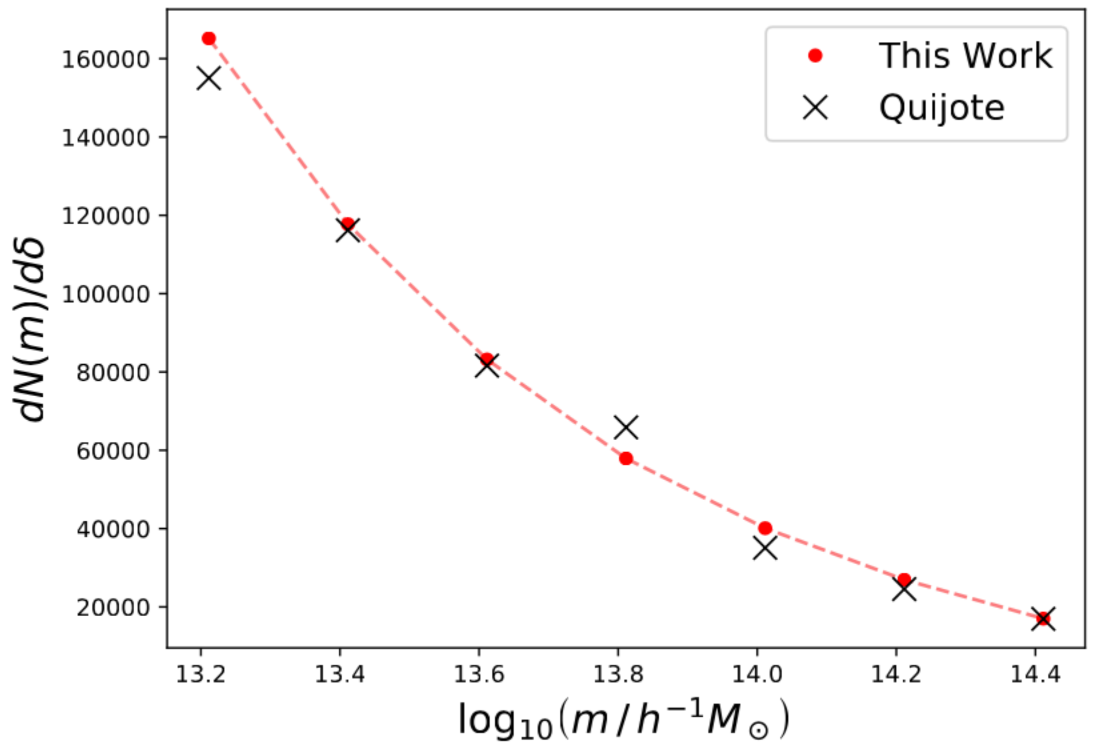
- Number count response
- Power spectrum response
- Variance of δ_b

- But not normally found in N-body simulations
  - Use **separate universe** simulations [Li+14], and **sub-box** simulations

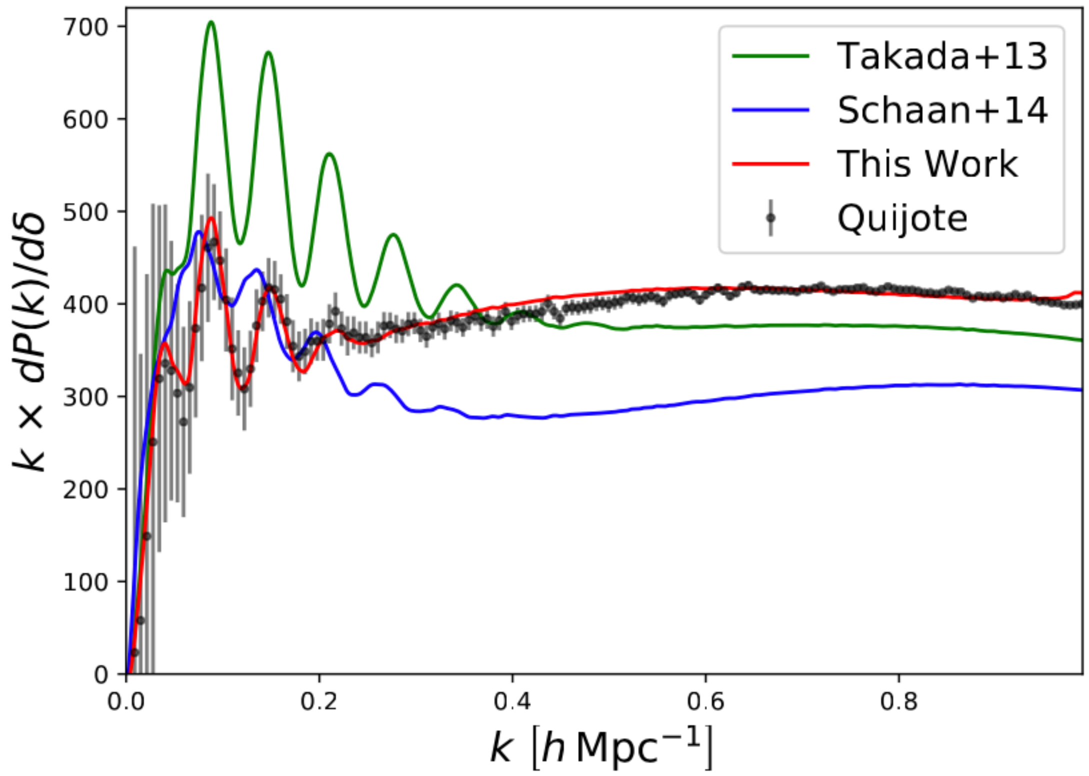
Takada+13, Schaan+14, Chiang+14

Response Terms

Number count response



Power spectrum response



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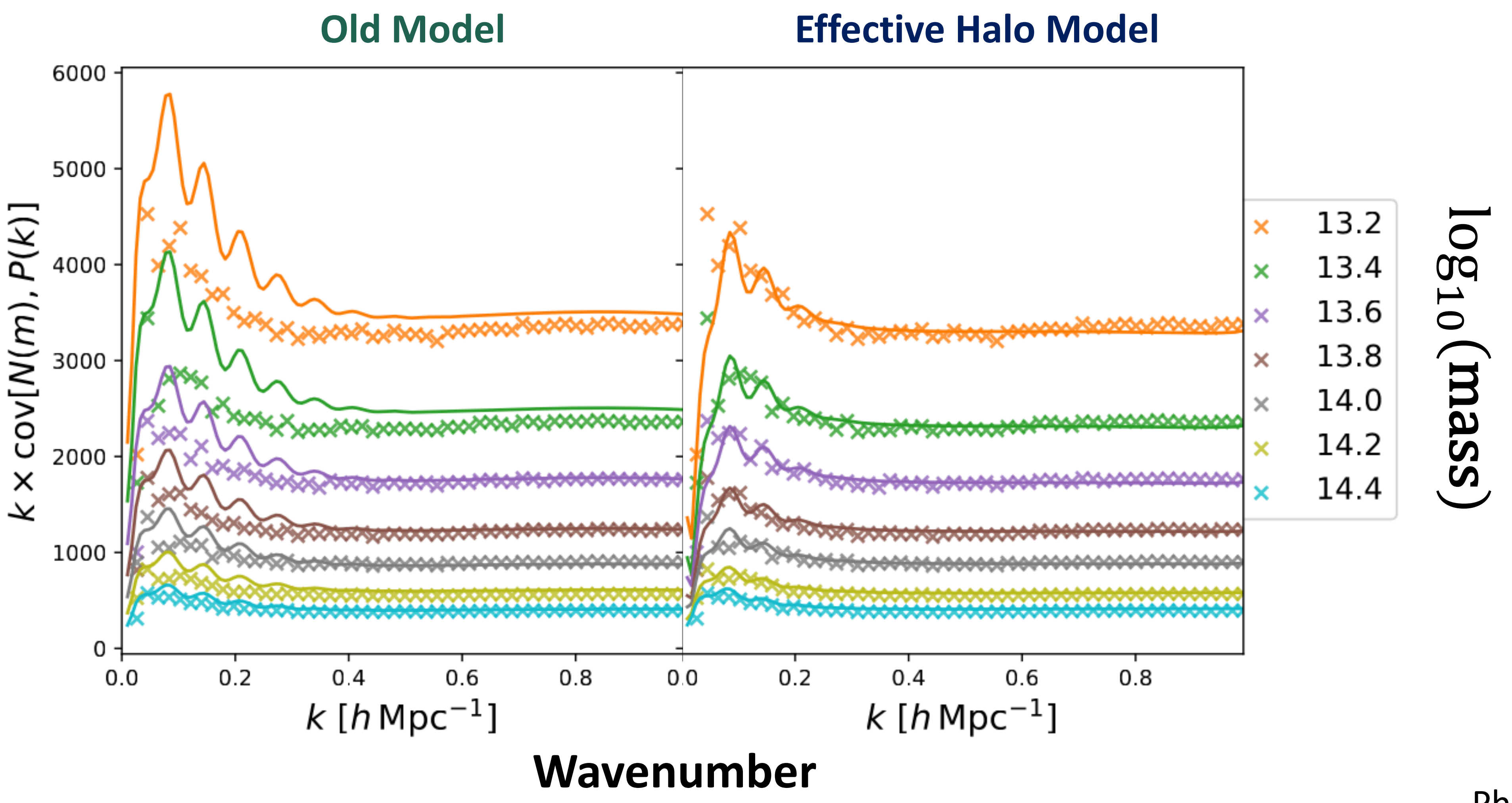
Data from separate universe Quijote simulations at z = 0

Full Covariance using sub-box Simulations



Philcox+20

Conclusions

(arXiv: 2004.09515)

- Using the **Halo Model** and **Effective Field Theory** we get 1% accurate matter power spectra to $k \sim 1h\,\mathrm{Mpc}^{-1}$

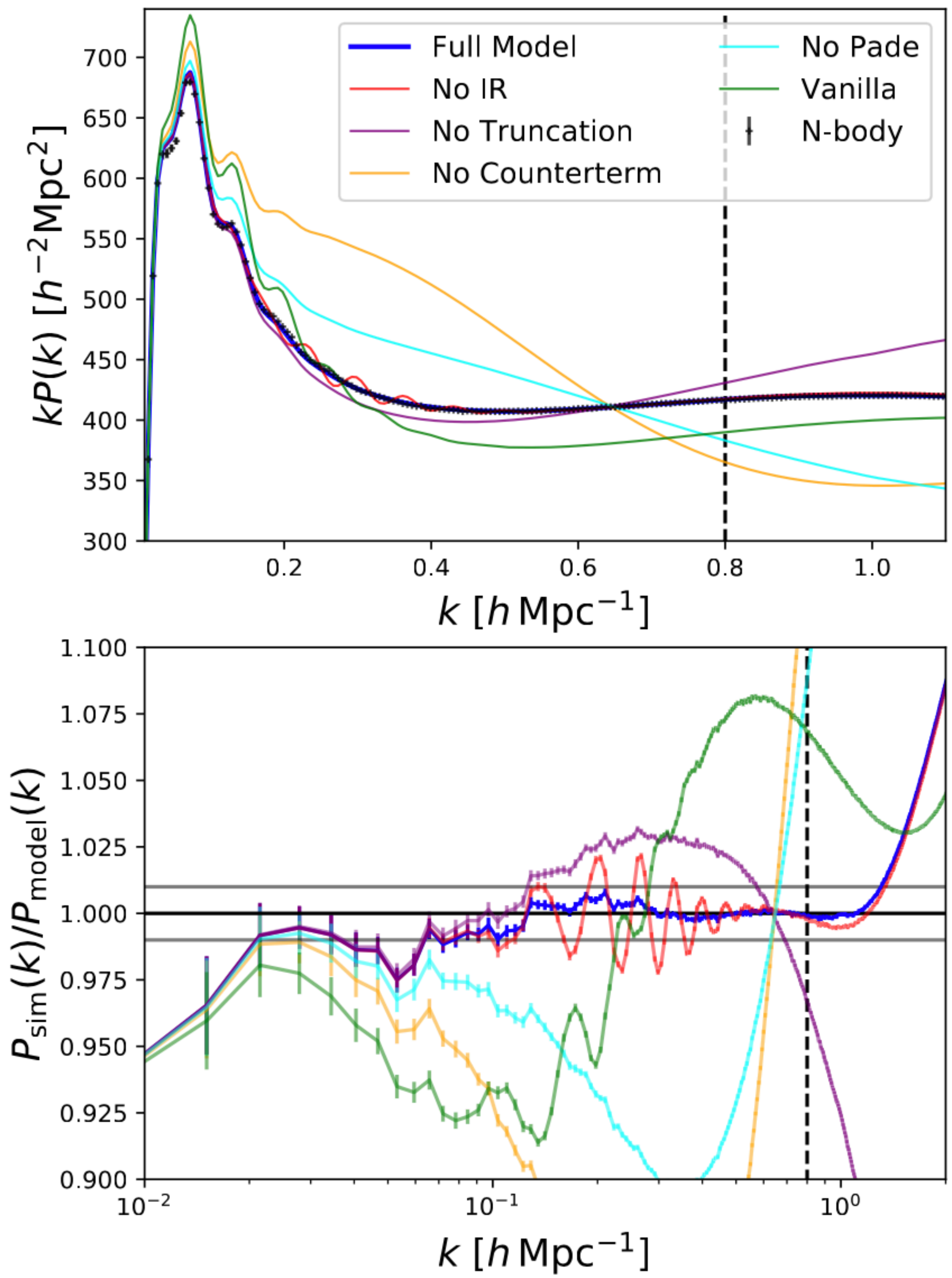
- Can extend this to other statistics, e.g. **cluster count covariances**, but we must consider **halo exclusion**

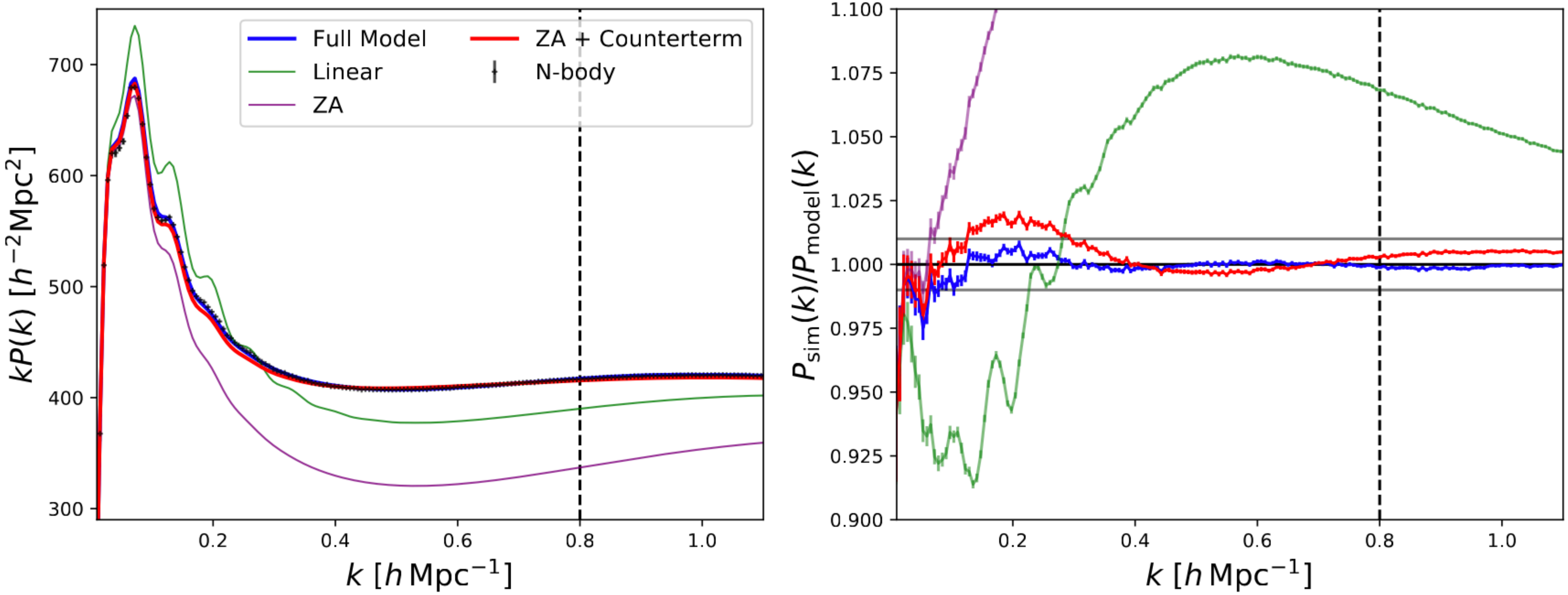
- Includes a **fast** and **easy-to-use** Python package: EffectiveHalos.readthedocs.io

Extensions

- Baryonic Physics
- Joint likelihoods of **tSZ cluster counts** and **weak lensing**

Power Spectrum Model Components



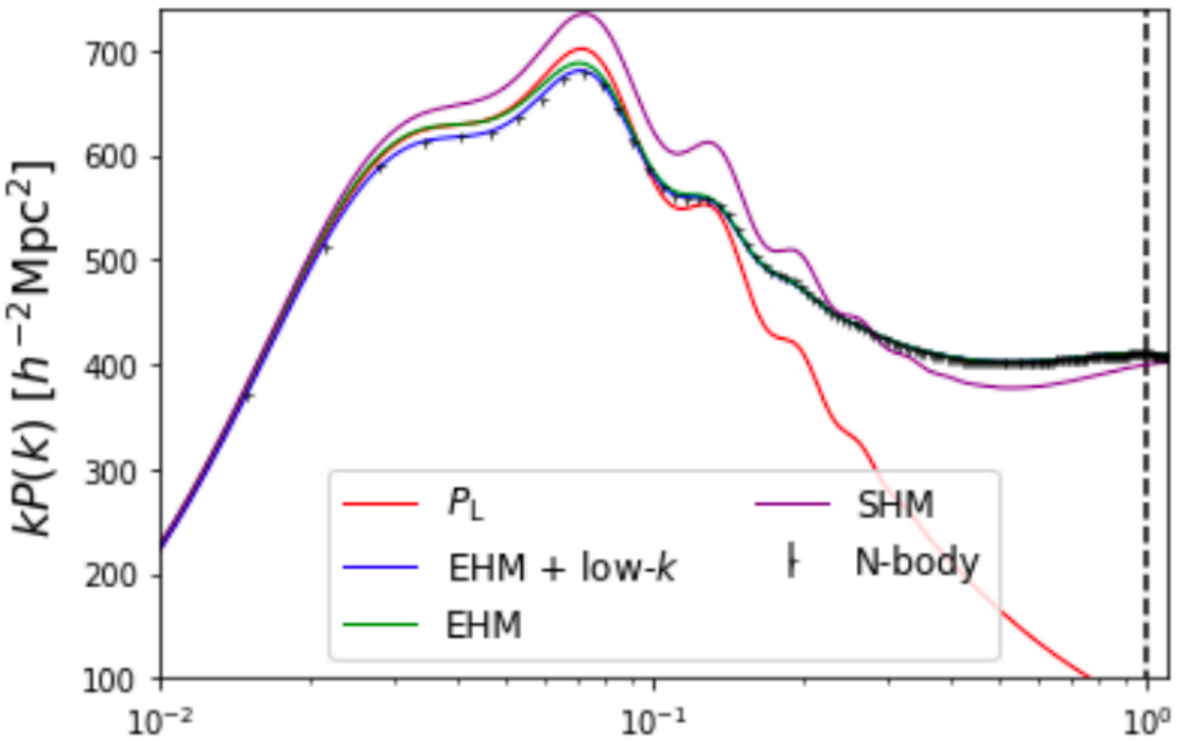


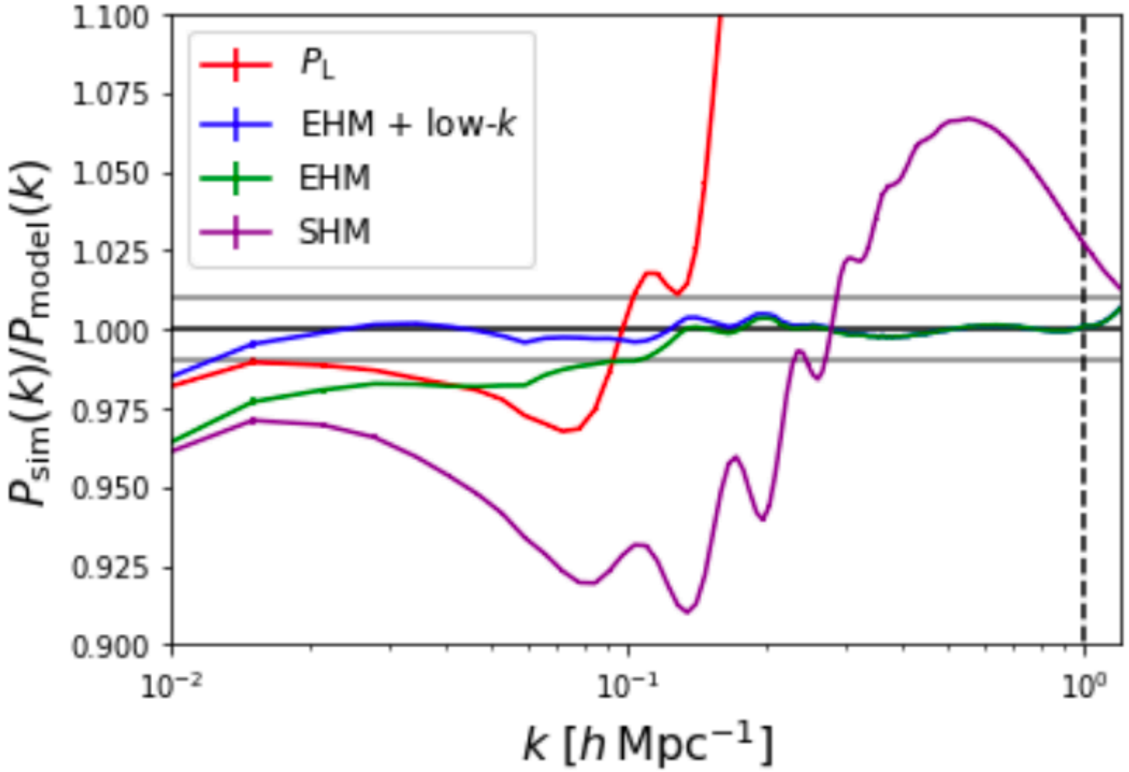
Zel`dovich Approximation vs EFT

Schmidt (2016) Correction

$$P^{1h}(k) \to P^{1h}(k) - \frac{1}{\bar{\rho}\langle m\rangle}\int dm_1 dm_2\, \frac{m_1^2}{\bar{\rho}}\frac{m_2^2}{\bar{\rho}} n(m_1)n(m_2)u(k|m_1)u(k|m_2)\Theta(k|m_1,m_2)$$

$$\Theta(k|m_1,m_2) = \frac{1}{1+[k\,(R_L(m_1)+R_L(m_2))]^4}$$



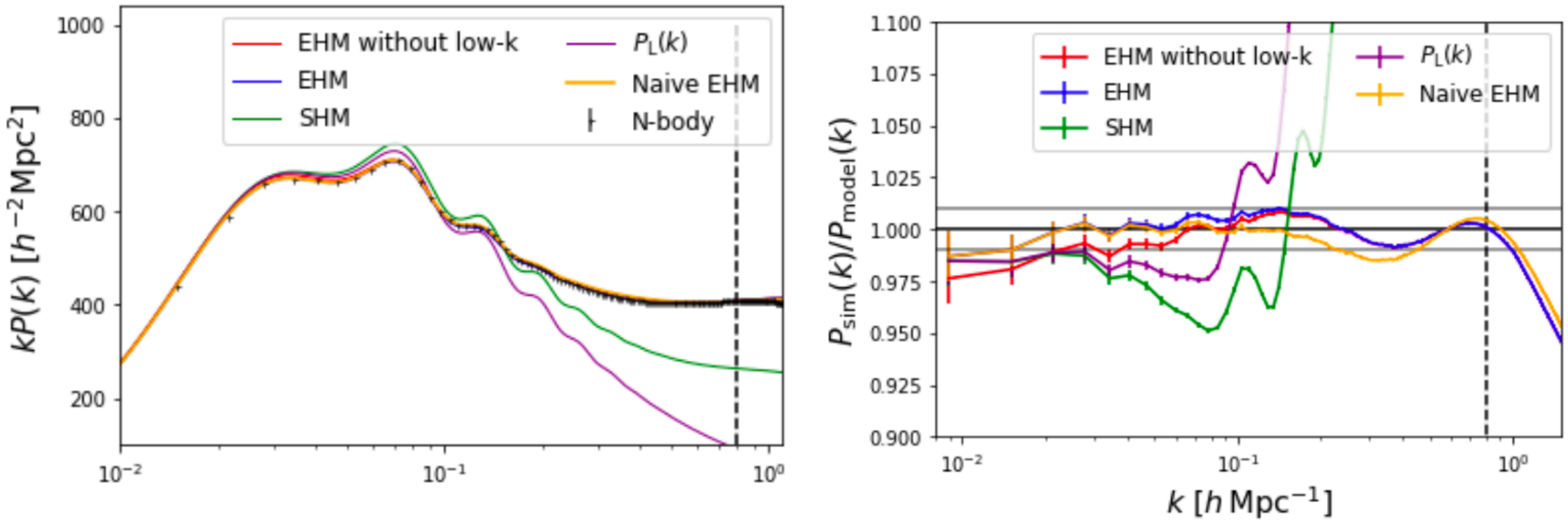


Low-k Effects

From 15,000 Quijote Simulations

Philcox+ (in prep)

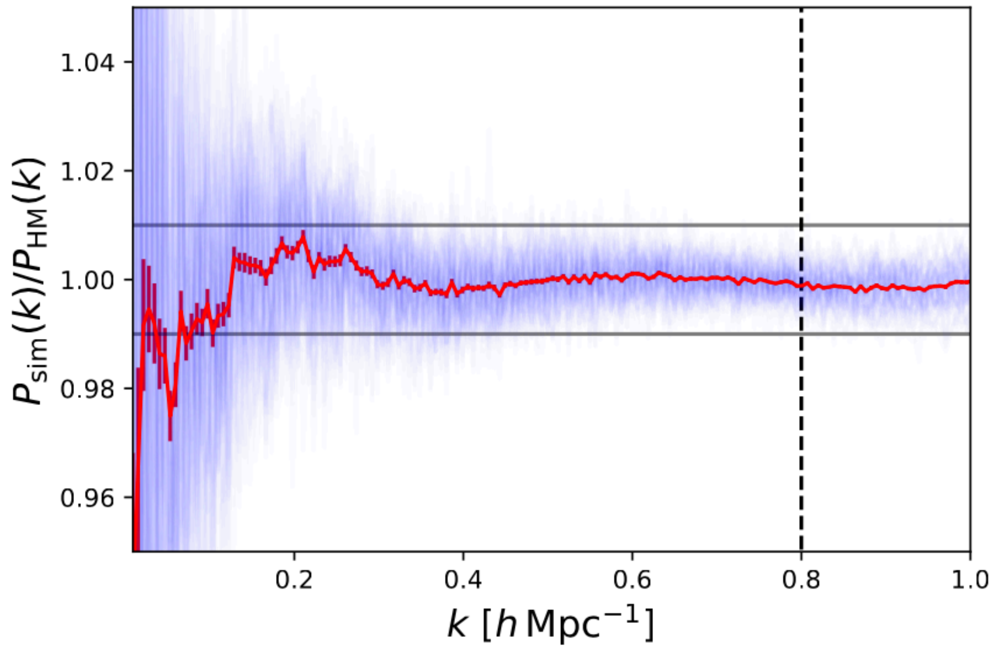
Following Massara et al. (2014), treat CDM+baryons with the halo model and assume **linear theory** for the neutrino correlators:



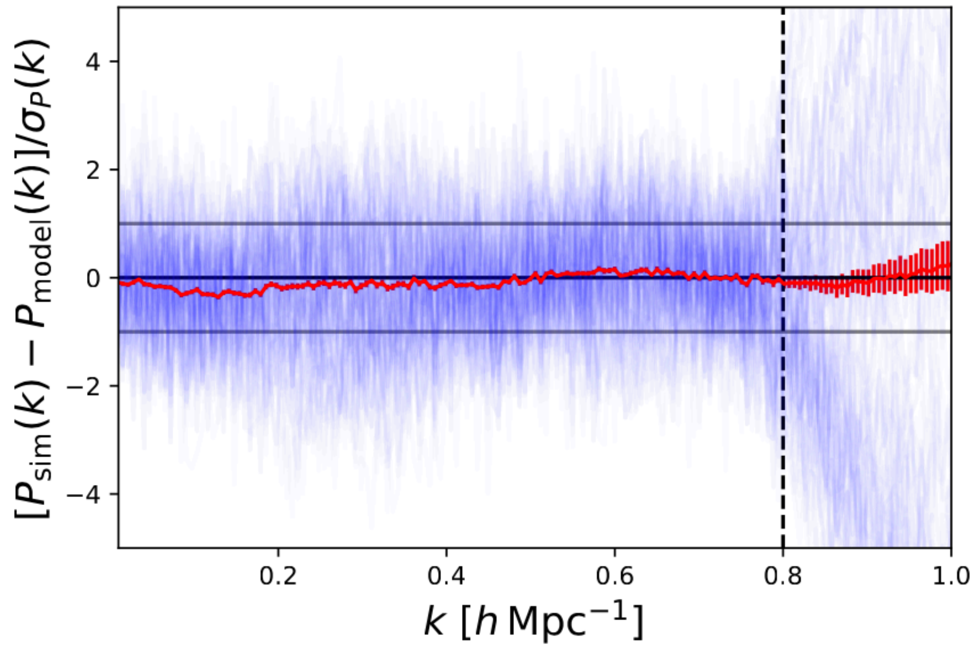
From 500 Quijote Simulations at $\sum m_\nu = 0.4$ eV

Massive Neutrinos

Philcox+ (in prep)



Fitting Individual Simulations



Fitting for non-standard cosmologies

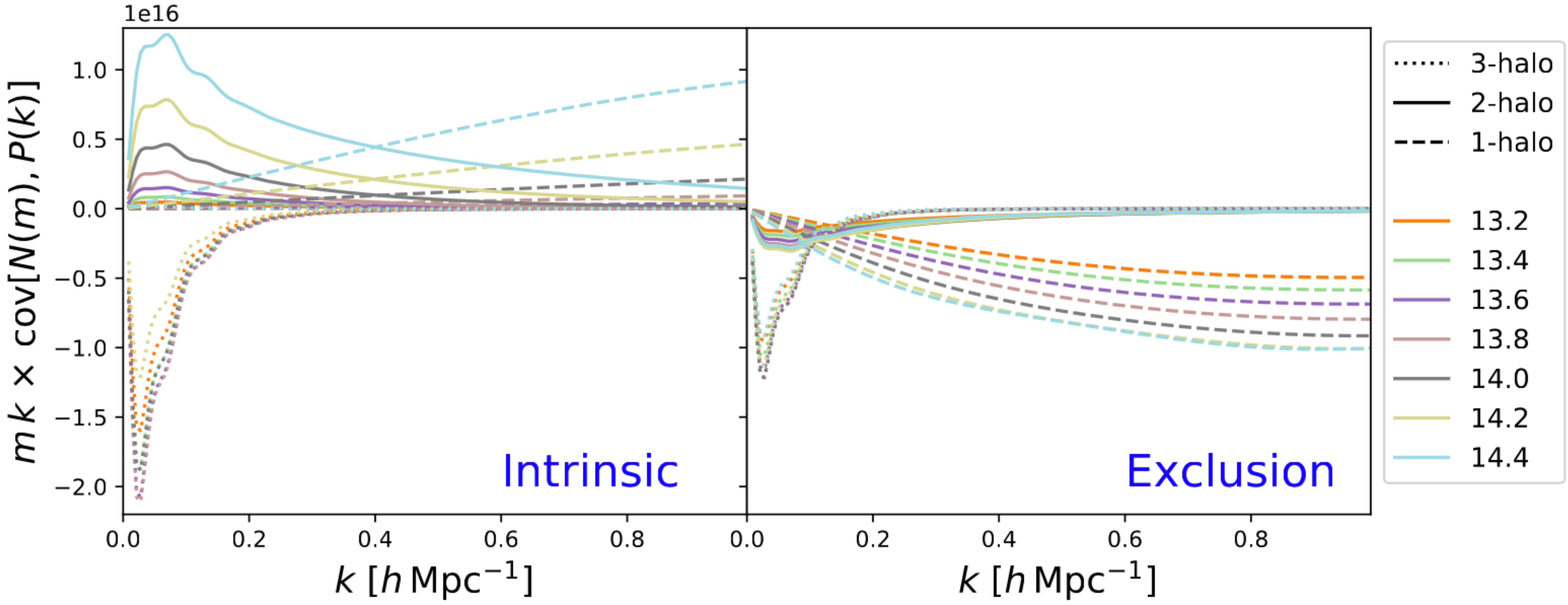
$$
\begin{aligned}
\frac{dP_{\mathrm{HM}}(k)}{d\delta_b} &= 2I_1^2(k)I_1^1(k)W^2(kR)P_{\mathrm{NL}}(k) + I_2^1(k,k) \\
&+ \left[I_1^1(k)\right]^2 W^2(kR)P_{\mathrm{NL}}(k)\left(\frac{68}{21} + \frac{26}{21}\frac{P_{\mathrm{SPT}}(k) + P_{\mathrm{ct}}(k)}{P_{\mathrm{NL}}(k)}\right) \\
&- \frac{1}{3}\frac{d\log k^3 P_{\mathrm{NL}}(k)}{d\log k}P_{\mathrm{HM}}(k),
\end{aligned}
$$

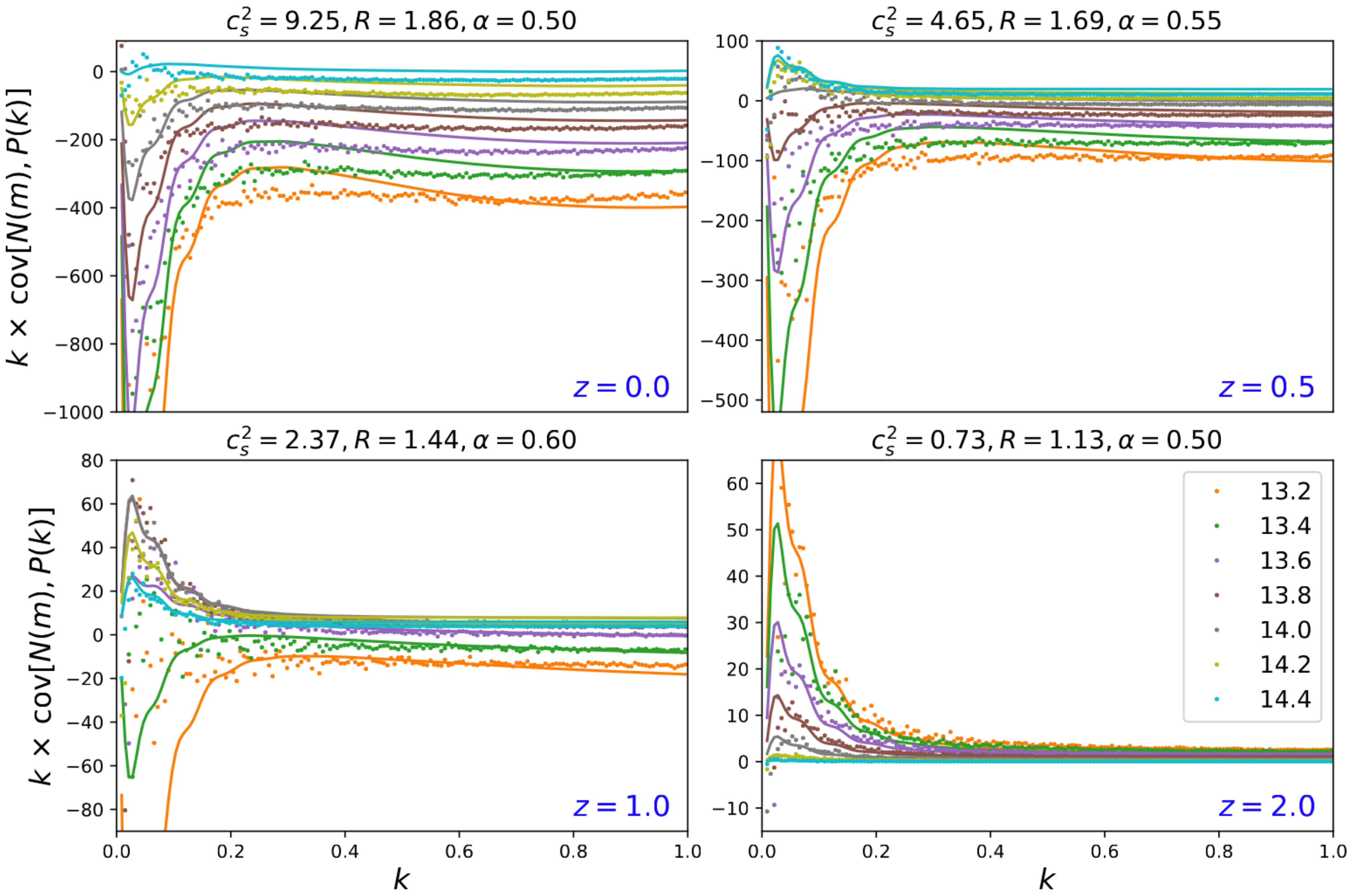
Power Spectrum Response

$$
\frac{\partial N(m)}{\partial \delta_b} \equiv \frac{\partial}{\partial \delta_b}\int d\mathbf{x}\, n(m) = Vn(m)b^{(1)}(m),
$$

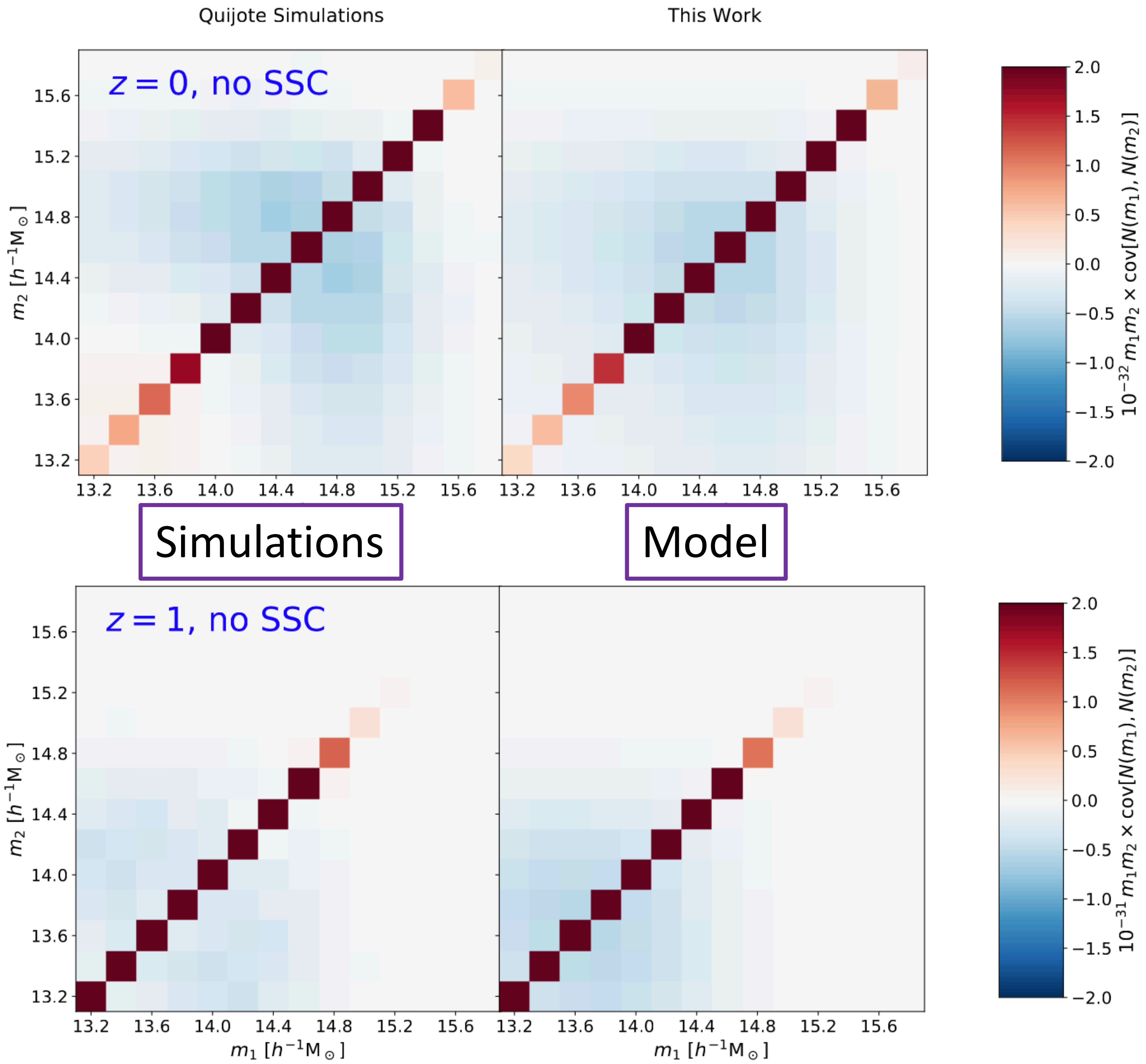
Number Count Response

Covariance Components

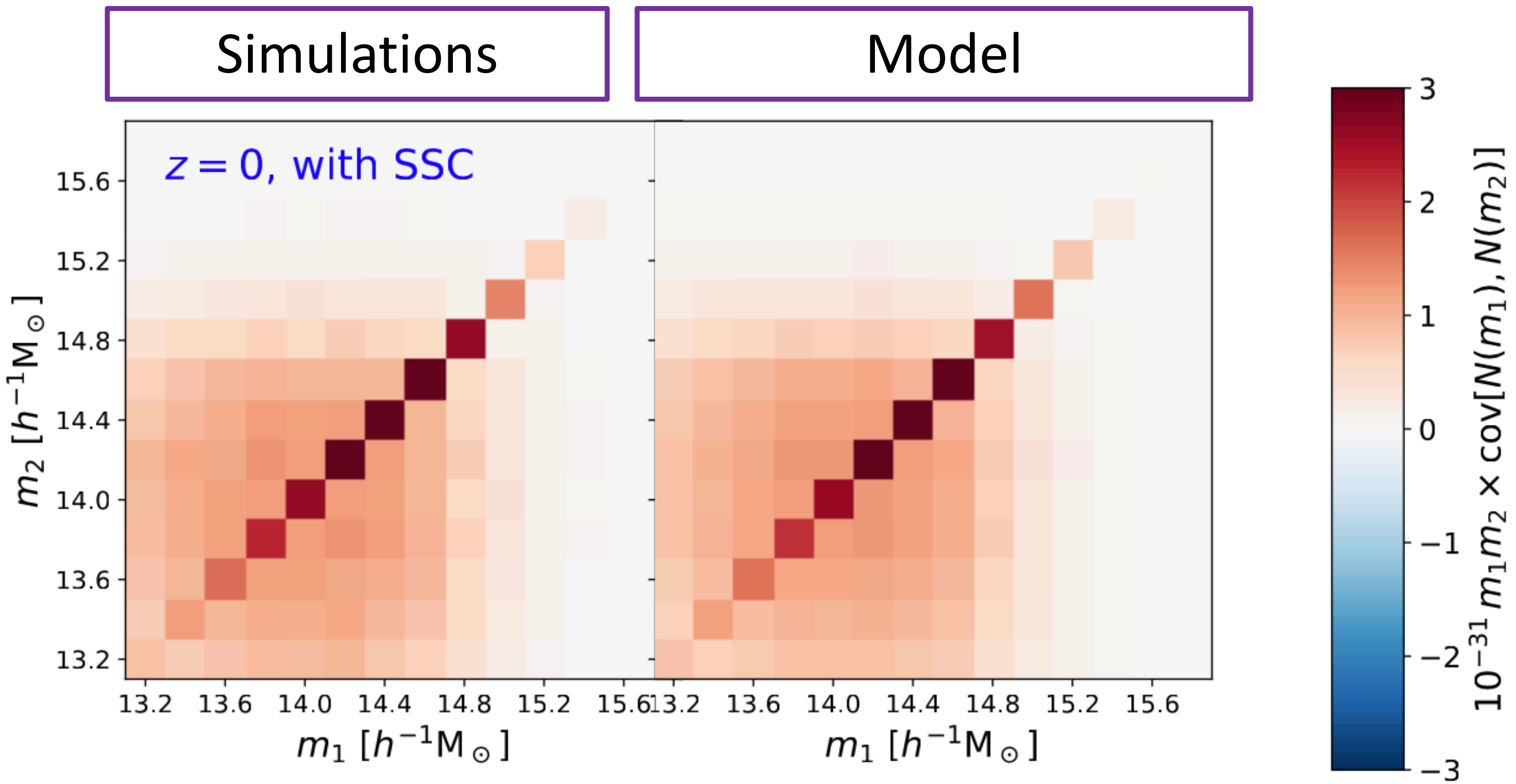




$c_s^2 = 9.25, R = 1.86, \alpha = 0.50$
$c_s^2 = 4.65, R = 1.69, \alpha = 0.55$
$c_s^2 = 2.37, R = 1.44, \alpha = 0.60$
$c_s^2 = 0.73, R = 1.13, \alpha = 0.50$
$k \times \mathrm{cov}[N(m), P(k)]$
k
$z = 0.0$
$z = 0.5$
$z = 1.0$
$z = 2.0$
13.2
13.4
13.6
13.8
14.0
14.2
14.4

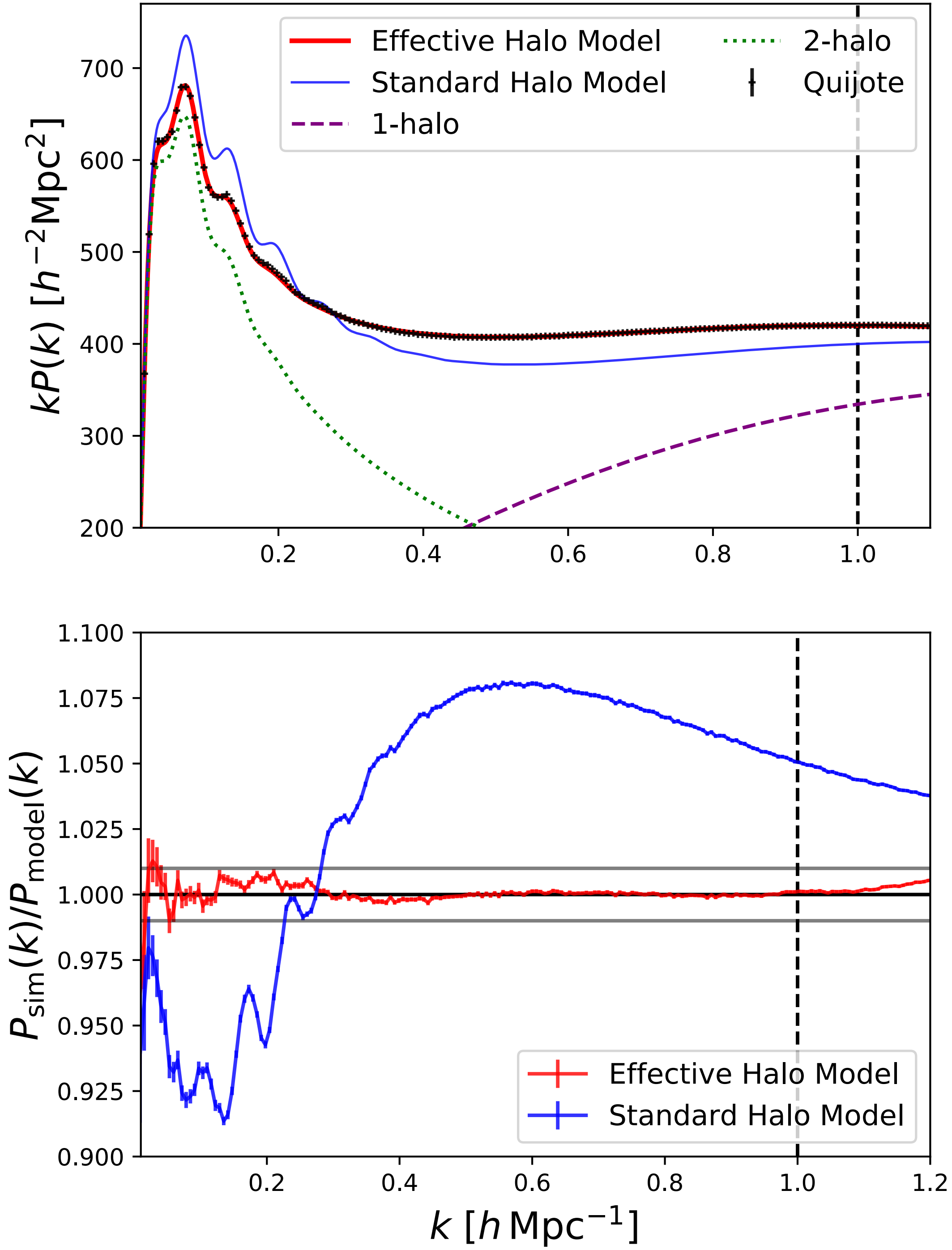


Halo Covariance



Simulations
Model
$z = 0$, with SSC
15.6
15.2
14.8
14.4
14.0
13.6
13.2
$m_2\ [h^{-1}\mathrm{M}_\odot]$
13.2 13.6 14.0 14.4 14.8 15.2 15.6
$m_1\ [h^{-1}\mathrm{M}_\odot]$
13.2 13.6 14.0 14.4 14.8 15.2 15.6
$m_1\ [h^{-1}\mathrm{M}_\odot]$
3
2
1
0
−1
−2
−3
$10^{-31}\, m_1 m_2 \times \mathrm{cov}[N(m_1), N(m_2)]$

The Effective Halo Model



1% Accuracy to k = 1 h/Mpc

- Combine the Halo Model and Perturbation Theory
- One Assumption:
  - **Halos are distributed according to the smoothed non-linear density field**
- Gives **accurate** predictions for matter power spectra and covariances with halo counts
- Test with **Quijote** simulations

Philcox, Spergel & Villaescucsa-Navarro 2020