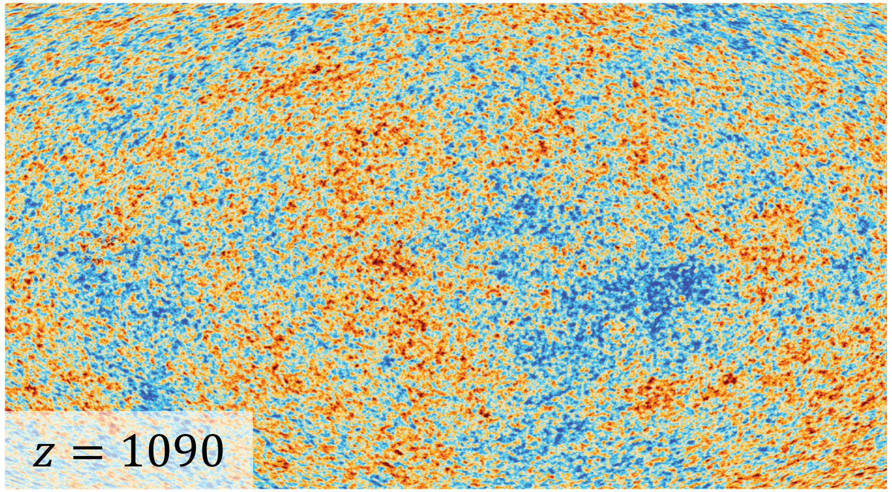
Large Scale Structure Beyond the Two-Point Function

Oliver Philcox (Princeton / IAS)

December 2021

THE EARLY UNIVERSE IS GAUSSIAN



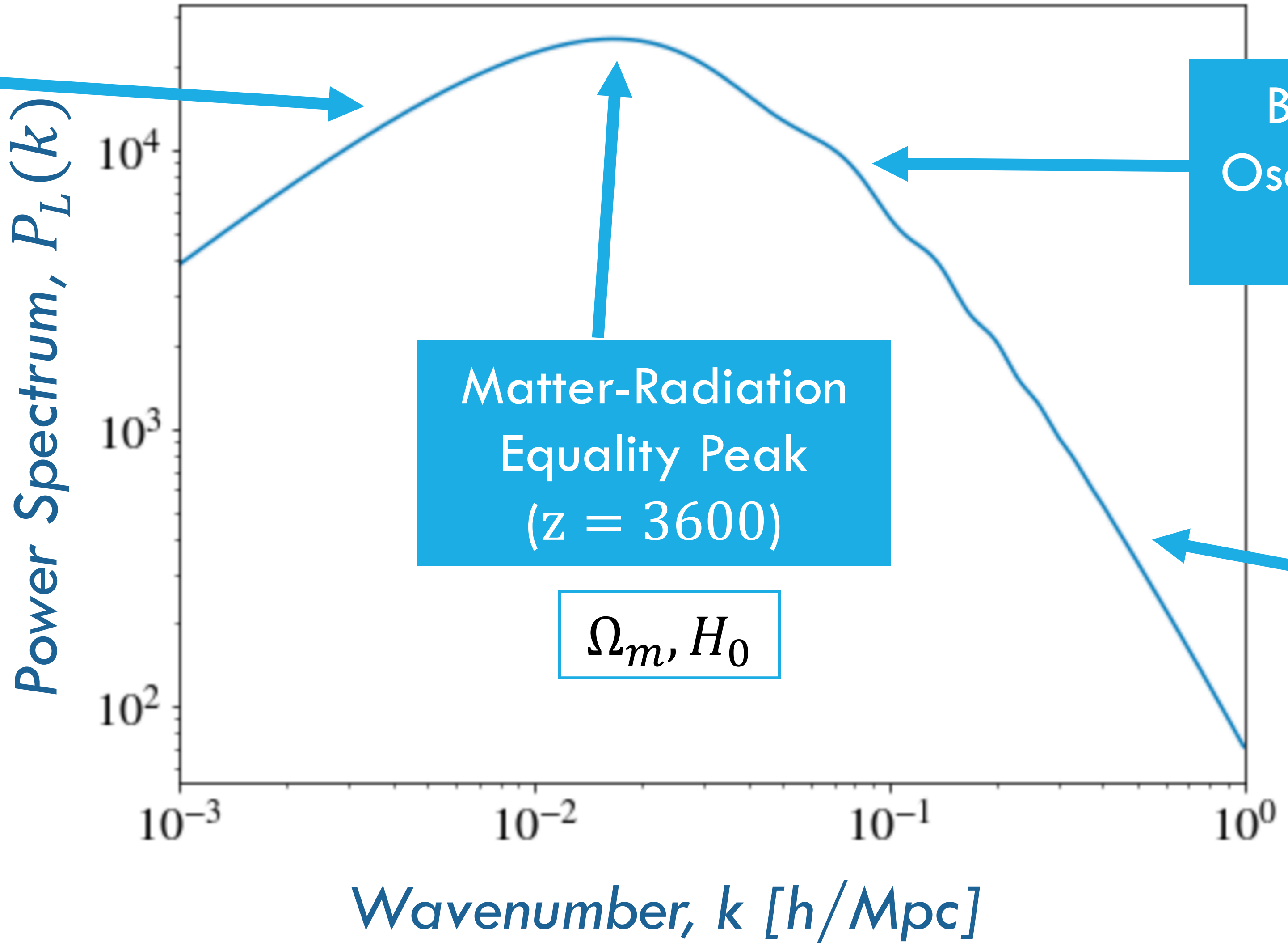
$$\delta(\mathbf{k}) \sim \mathcal{N}\left(0, P_L(\mathbf{k})\right)$$

- All information contained in the power spectrum
- **No** higher order statistics needed!

Planck 2018, IllustrisTNG

LINEAR POWER SPECTRUM

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = (2\pi)^3 \delta_{\mathrm{D}}\left(\mathbf{k}+\mathbf{k}'\right) P_L(\mathbf{k})$$



Slope from Inflation
$(z \to \infty)$

n_s, A_s

Matter-Radiation
Equality Peak
$(z = 3600)$

Ω_m, H_0

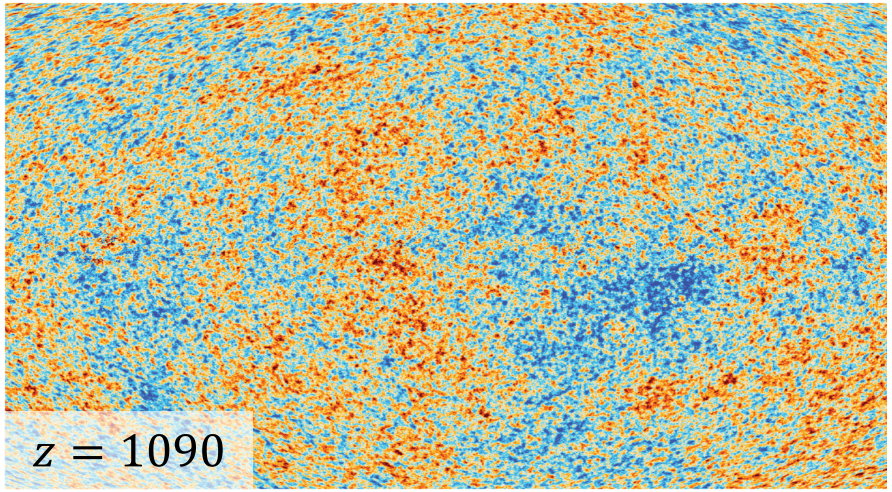
Baryon Acoustic
Oscillation Wiggles
$(z = 1090)$

Ω_b, H_0

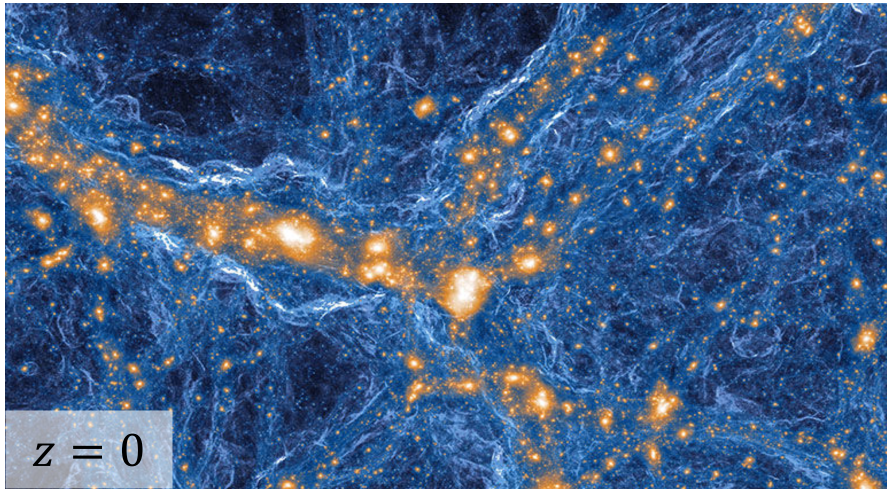
Baryon and
Neutrino
Suppression

$\Omega_b, \sum m_\nu$

THE LATE UNIVERSE IS NOT GAUSSIAN



Gravitational Collapse



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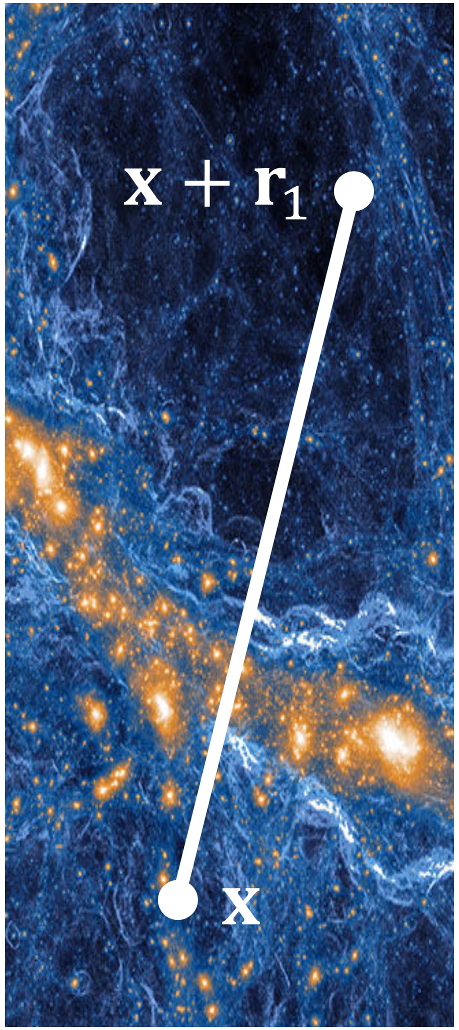
- **Not** all information contained in the power spectrum
- Higher-order statistics needed!

Planck 2018, IllustrisTNG

NON-GAUSSIAN DENSITY ⇒ NON-GAUSSIAN STATISTICS



Gaussian

1. Power Spectrum:

$$P(\mathbf{k}_1) = \langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle'$$

2. 2-Point Correlation Function:

$$\xi(\mathbf{r}_1) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}_1)\rangle$$

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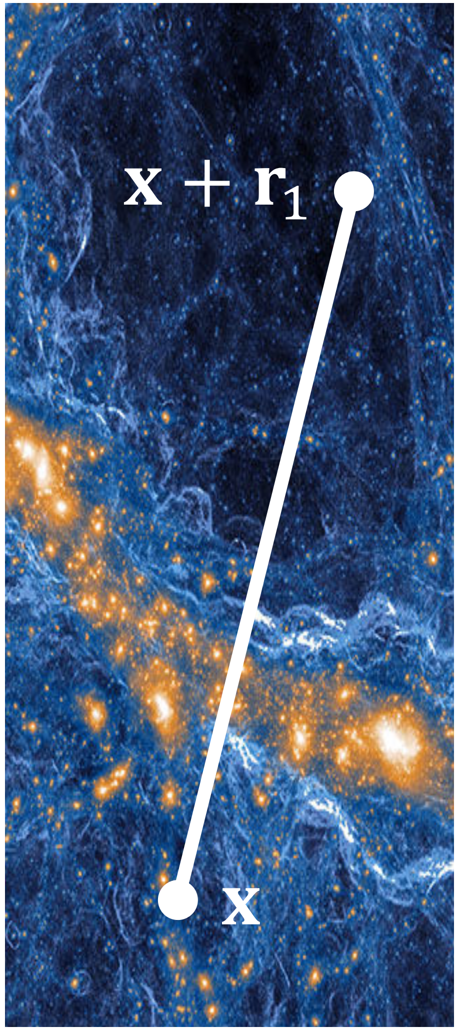

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Non-Gaussian

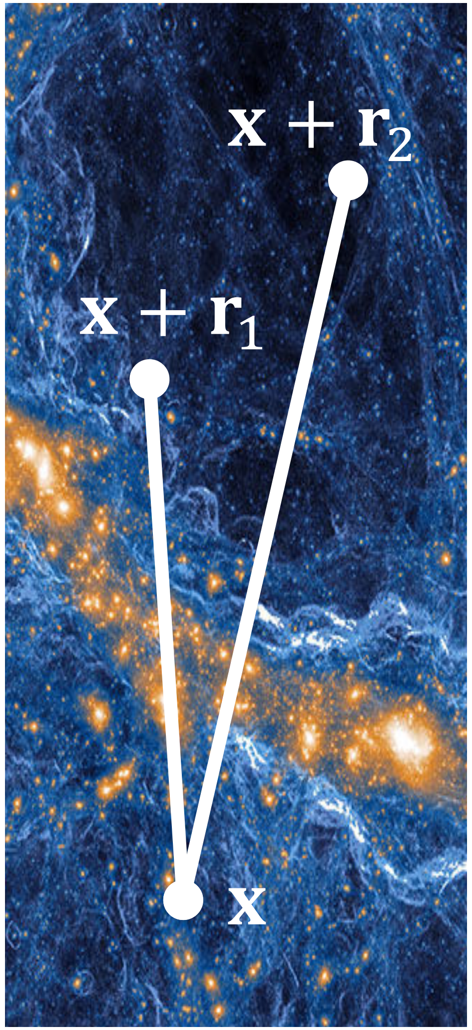
1. Bispectrum:

$$B(\mathbf{k}_1,\mathbf{k}_2) = \langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle'$$

2. 3-Point Correlation Function:

$$\zeta_3(\mathbf{r}_1,\mathbf{r}_2) \equiv \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r}_1)\delta(\mathbf{x}+\mathbf{r}_2)\rangle$$

And beyond…



WHAT MAKES UP THE BISPECTRUM?

$$B_g(\mathbf{k}_1, \mathbf{k}_2) = \left[2b_1^3 F_2(\mathbf{k}_1, \mathbf{k}_2) + b_2 b_1^2 + 2b_{s^2} b_1^2 \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2 - 1/3\right)\right] P_L(k_1) P_L(k_2) + 2 \text{ perms.}$$

The galaxy bispectrum depends on galaxy formation physics, gravity, and early-Universe cosmology.

▷ To obtain **all** the large-scale information in the initial conditions, we need:*

- Power Spectra / 2-Point Functions $\sim P_L(k)$
- Bispectra / 3-Point Functions $\sim P_L^2(k)$
- Trispectra / 4-Point Functions $\sim P_L^3(k)$
- *etc.*

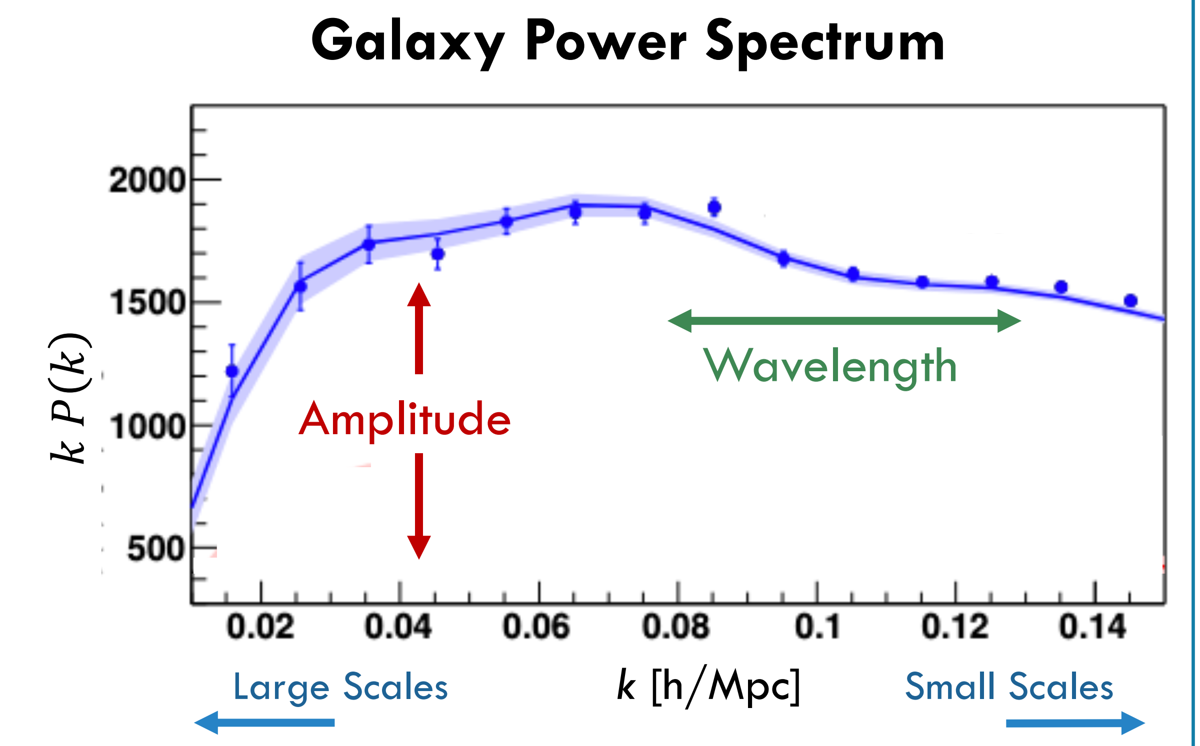
*ignoring higher-order perturbative effects, redshift-space distortions, renormalization, *etc.* e.g. Ivanov+21

THE CURRENT STATE OF PLAY

- Analyze the galaxy **power spectrum** using a **scaling analysis**

- This measures:
  - Overall **amplitude** (= primordial amplitude)
  - **Wiggle** positions (= BAO feature)

- Robust way to constrain **growth rate** and **expansion history** $H(z)$

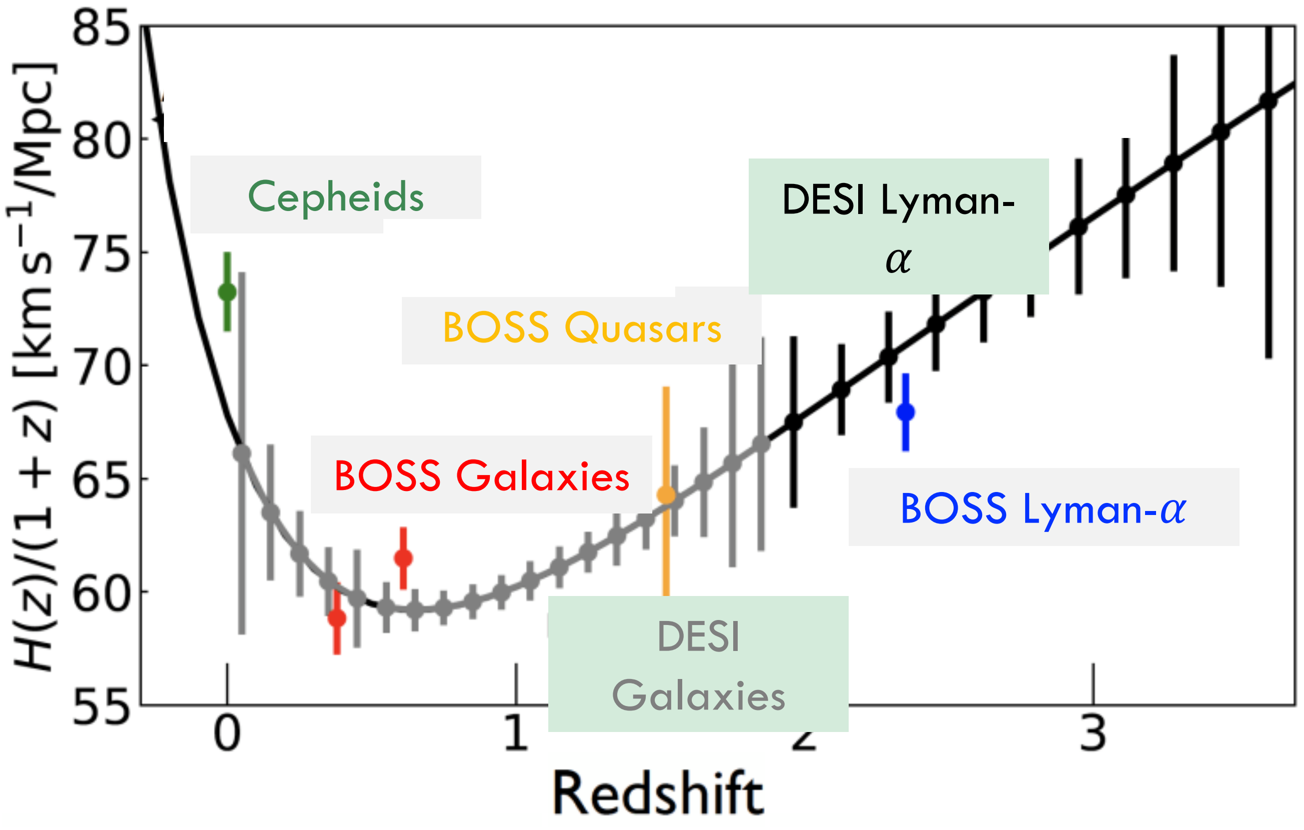


BOSS DR12 Galaxy Survey (monopole only)

e.g. Beutler+17, Gil-Marin+15,17, Alam+20

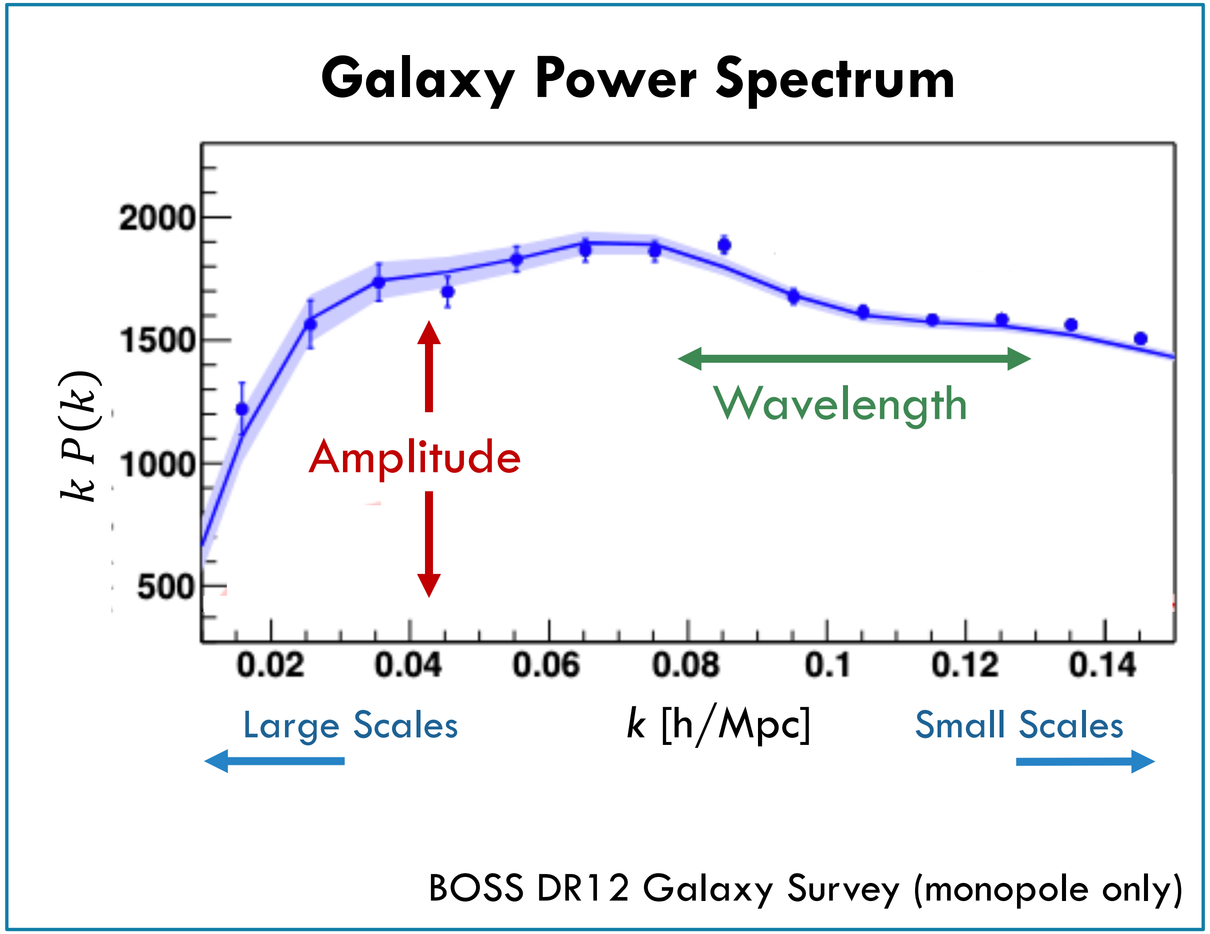
THE CURRENT STATE OF PLAY

- Analyze the galaxy **power spectrum** using a **scaling analysis**
- Measure **wiggle positions** (= BAO feature) and **overall amplitude**
- Robust way to constrain **growth rate** and **expansion history** $H(z)$



THE CURRENT STATE OF PLAY

- This is *not* all the available information!

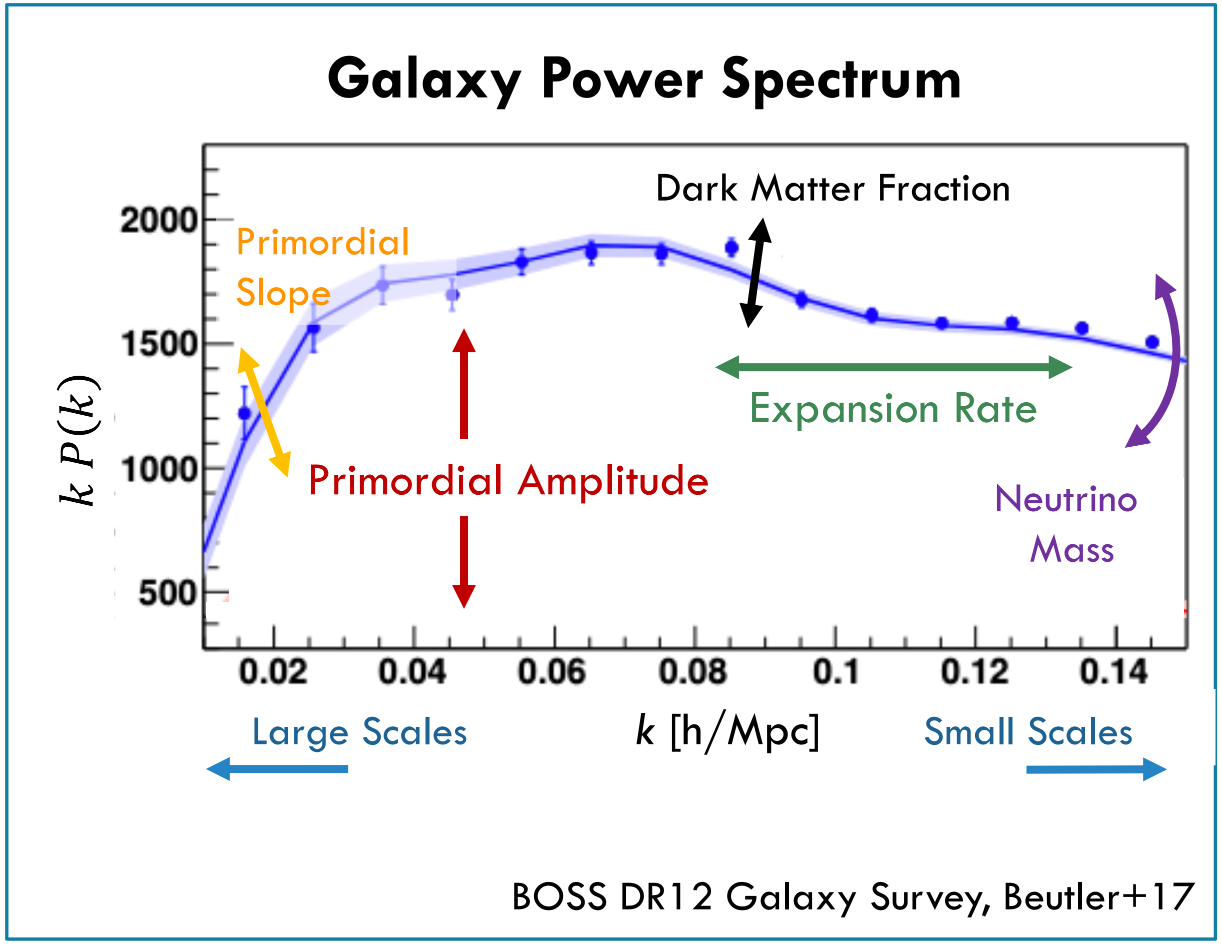


e.g. Beutler+17, Gil-Marin+15,17, Alam+20

THE CURRENT STATE OF PLAY

- This is *not* all the available information!

- Measure parameters **directly** from the **full shape** of the galaxy power spectrum

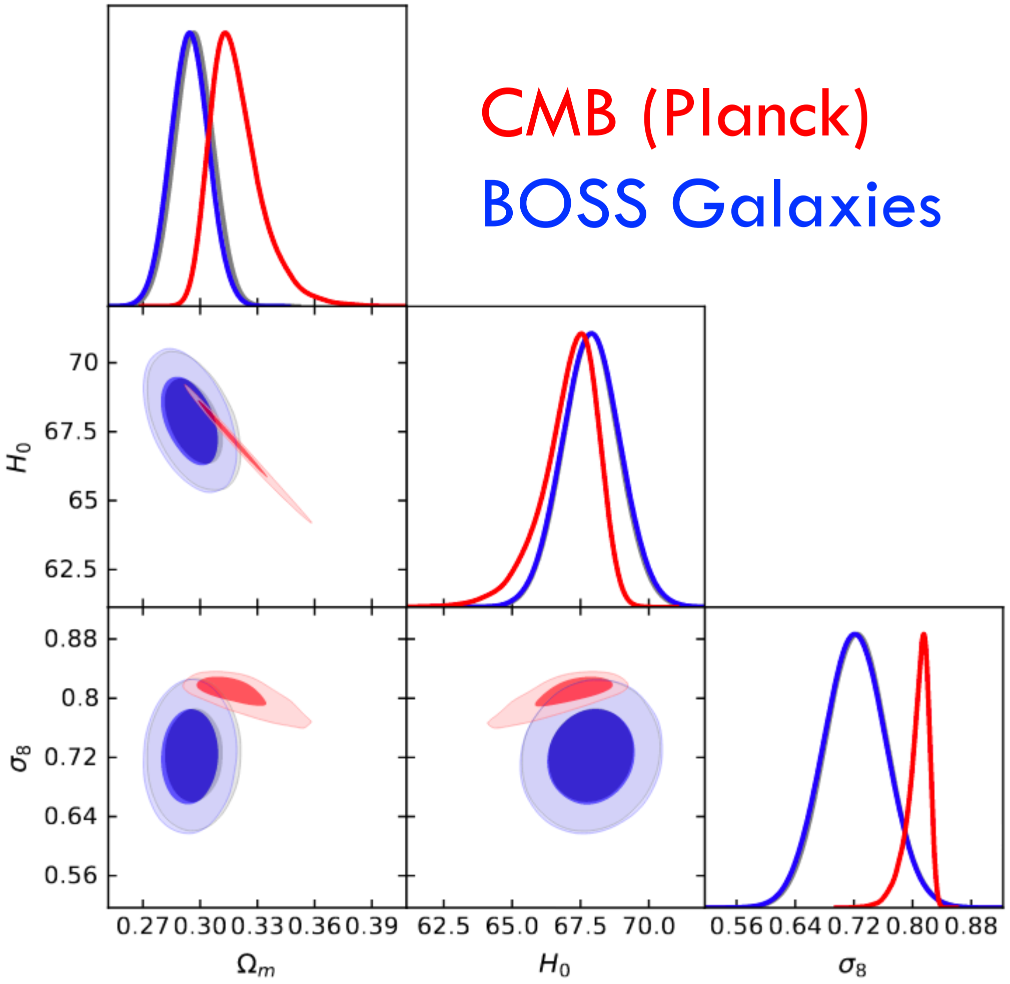


e.g. *Ivanov+19,20, d'Amico+19,20,* ***Philcox****+20ab, Chen+21, Kobayashi+21*

THE CURRENT STATE OF PLAY

- This is *not* all the available information!
- Measure parameters **directly** from the **full shape** of the galaxy power spectrum
- Constrain parameters in **new** ways e.g. expansion rate from **equality** scale.
  [Farren, **Philcox** & Sherwin (in prep.)]

*Can we go **beyond** the power spectrum?*



e.g. *Ivanov+19,20, d'Amico+19,20,* ***Philcox****+20ab, Chen+21, Kobayashi+21*

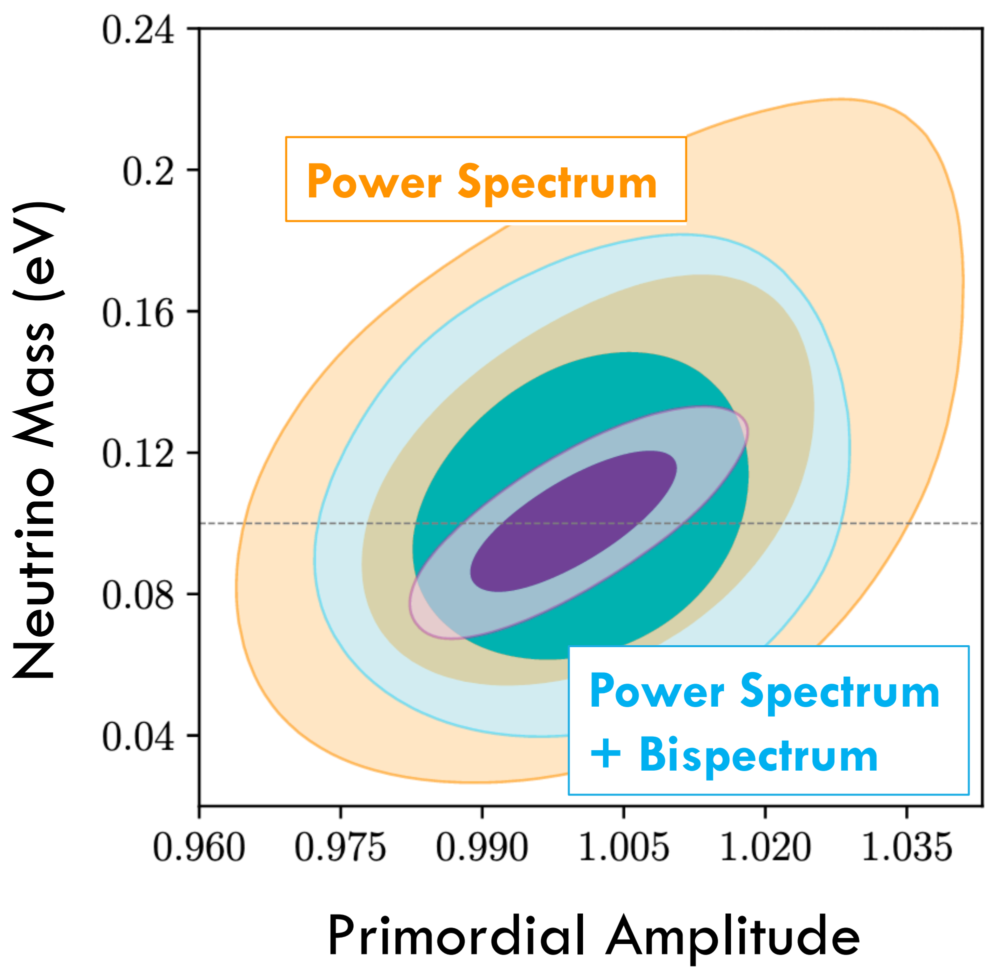
WHY USE HIGHER-ORDER STATISTICS?

- **Sharpen** parameter constraints!
- **Break** parameter **degeneracies**!

  [e.g. $P_g \sim b_1^2 \sigma_8^2,\ B_g \sim b_1^3 \sigma_8^4$]

<u>Euclid Forecast</u>

- Bispectrum improves constraints by $\approx 40\%$
- 1σ constraint of $\sigma_{M_\nu} = 0.013$ eV [including *Planck*]



Euclid Forecast, Chudaykin+19

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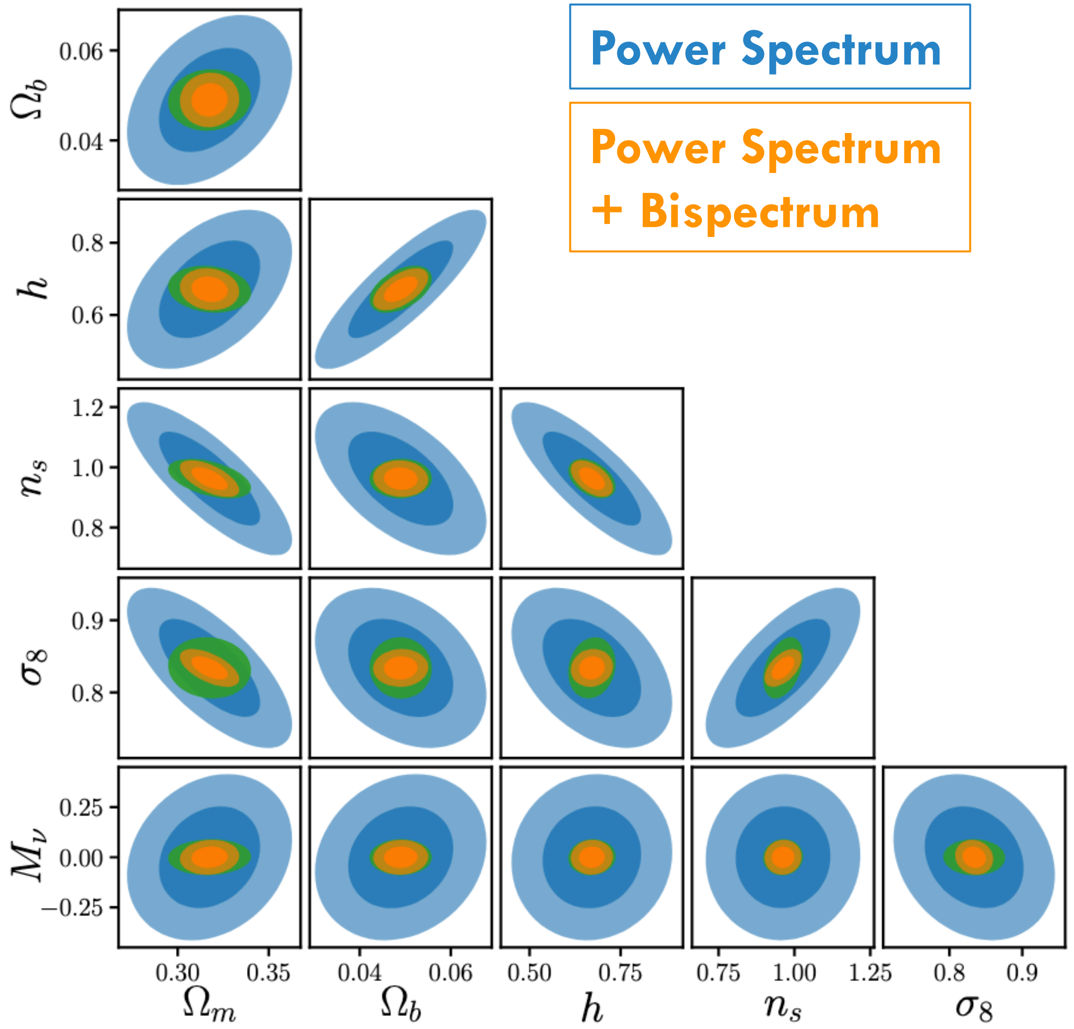
<u>Simulation-Based Forecast</u>

- Galaxy Bispectrum improves constraints by $> 2\times$
- Neutrino constraint improves by $5\times$



$1\ (\mathrm{Gpc}/h)^3$ *volume*, Hahn+20

NON-GAUSSIAN INFLATION

*Are the primordial perturbations **Gaussian** and **adiabatic**?*

Standard Model of Inflation:

- Scalar field ϕ rolling down a potential $V(\phi)$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)\right]$$

Gravity *Kinetic Energy* *Potential*

- Action, *S*, encodes **statistics** of the **primordial** curvature perturbations, ζ

Second Order ⇒ Power Spectrum

$$S^{(2)} \Rightarrow P_\zeta(k) \approx A_s k^{n_s - 4}$$

Third Order ⇒ Bispectrum

$$S^{(3)} \Rightarrow B_\zeta(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\mathrm{NL}} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}$$

Generates **non-Gaussianity** proportional to f_{NL}

NON-GAUSSIAN INFLATION

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\rm NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}$$

The **consistency condition** states that

$$\lim_{k_1 \to 0} B_\zeta(\mathbf{k}_1, \mathbf{k}_2) = (1 - n_s) P_\zeta(k_1) P_\zeta(k_2)$$

$$f_{\rm NL} \sim (1 - n_s) \ll 1$$

Non-Gaussianity is too small to be detected!

Non-standard inflation can beat this, e.g.

- Multifield Inflation [Local Bispectrum]
- New Kinetic Terms [Equilateral Bispectrum]
- New Vacuum States [Folded Bispectrum]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right]$$

Maldacena 03, Creminelli+04

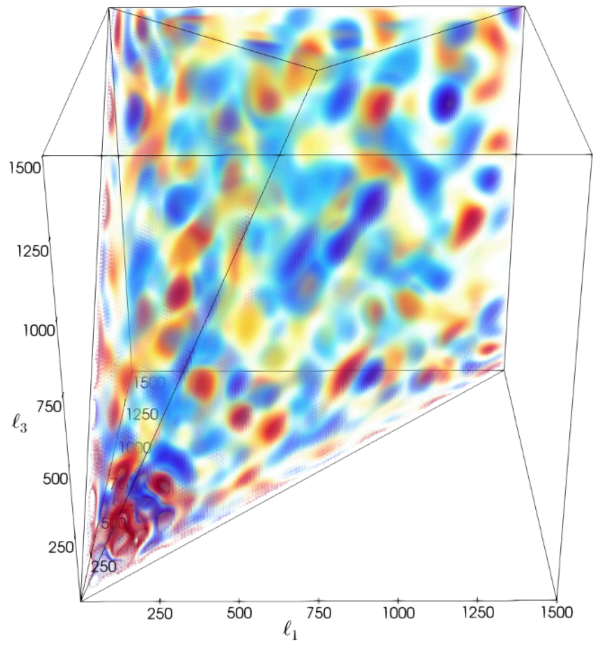
NON-GAUSSIAN INFLATION

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\rm NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}$$

How do we measure this?

1. **CMB Bispectrum**

Planck TTT Bispectrum



$\approx 2\times$ better with CMB-S4!

f_{NL} Constraints

Local	6.7 ± 5.6
Equilateral	6 ± 66
Orthogonal	−38 ± 36

Planck 2018 IX

NON-GAUSSIAN INFLATION

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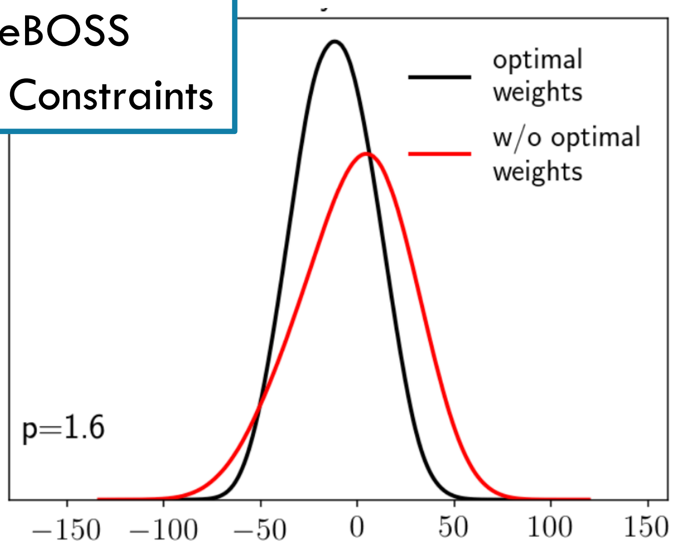
How do we measure this?

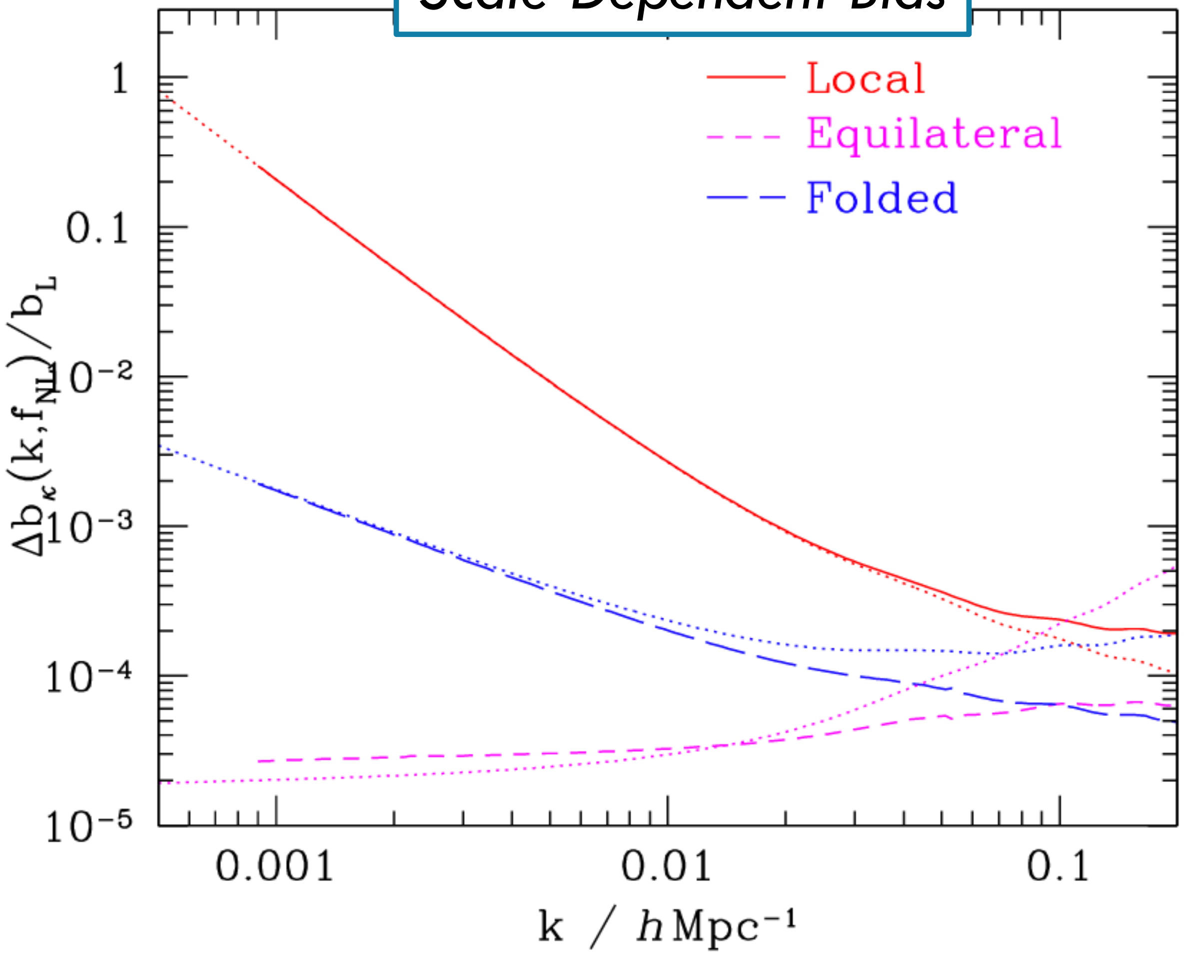
1. CMB Bispectrum

2. **Galaxy Power Spectrum**

eBOSS
$f_{\rm NL}^{\rm loc}$ Constraints



Scale-Dependent Bias



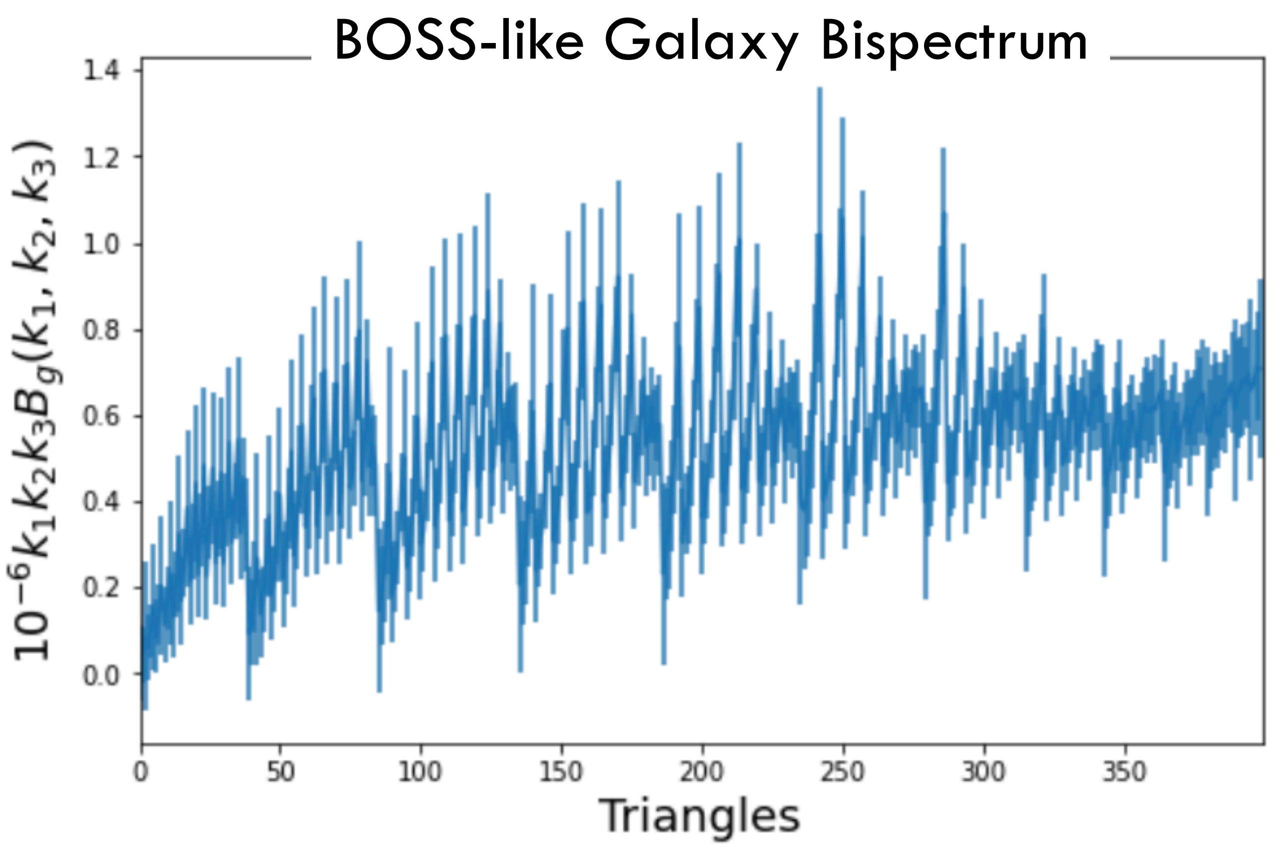
Desjacques & Seljak 10, eBOSS 21

NON-GAUSSIAN INFLATION

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\rm NL} P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}$$

How do we measure this?

1. CMB Bispectrum
2. Galaxy Power Spectrum
3. **Galaxy Bispectrum**



Ivanov, **Philcox**+21, **Philcox** & Ivanov (in prep.)

CHERN-SIMONS INTERACTIONS VIOLATE PARITY

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} + \frac{\gamma}{4}f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

- Add a **gauge field** A_μ to the inflationary action, via $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$
- This can include a **Chern-Simons coupling** to the (pseudo-)scalar ϕ [motivated by baryogenesis]
- $f(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu}$ violates **parity symmetry** ⇒ parity-violating correlators!

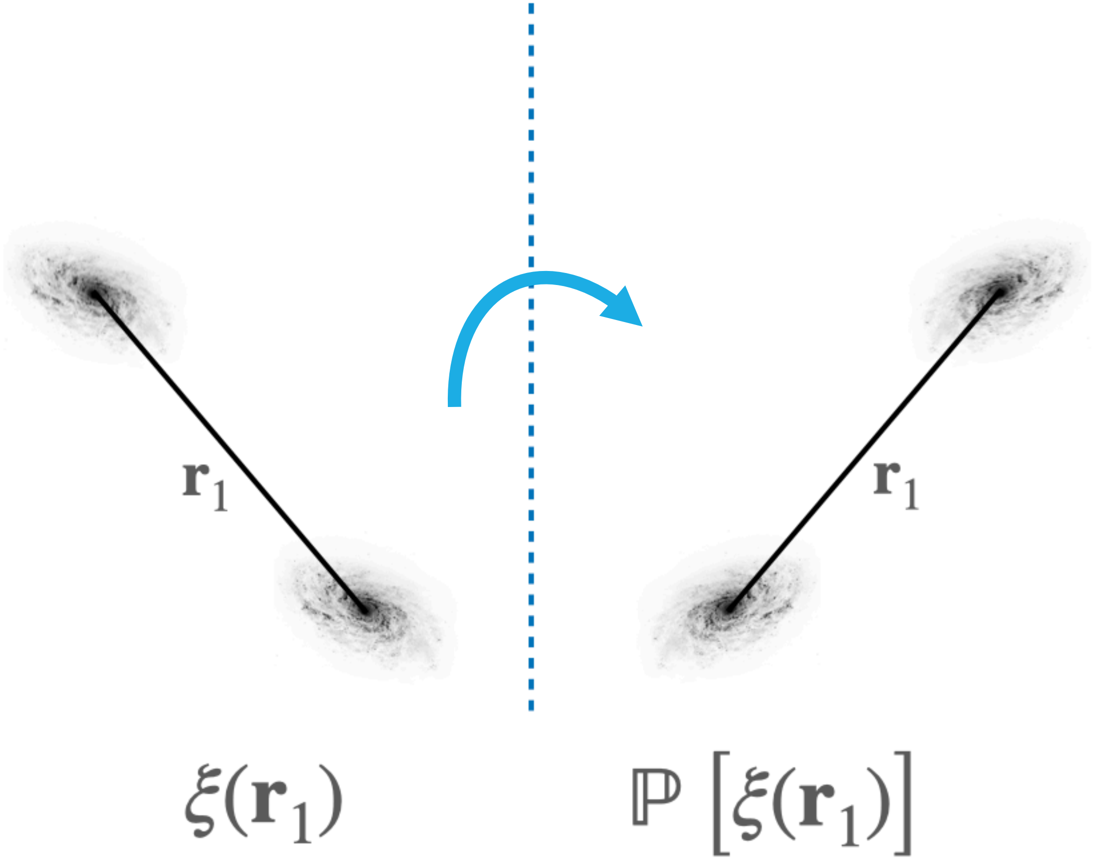
Where should we look for these signatures?

e.g. Lue+98, Jeong+12, Alexander+11, Shiraishi 16

THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY

2-Point Correlation Function (2PCF):

Parity Inversion = Rotation



Lue+98, Jeong+12, Shiraishi 16, Cahn+21, **Philcox** (in prep.)

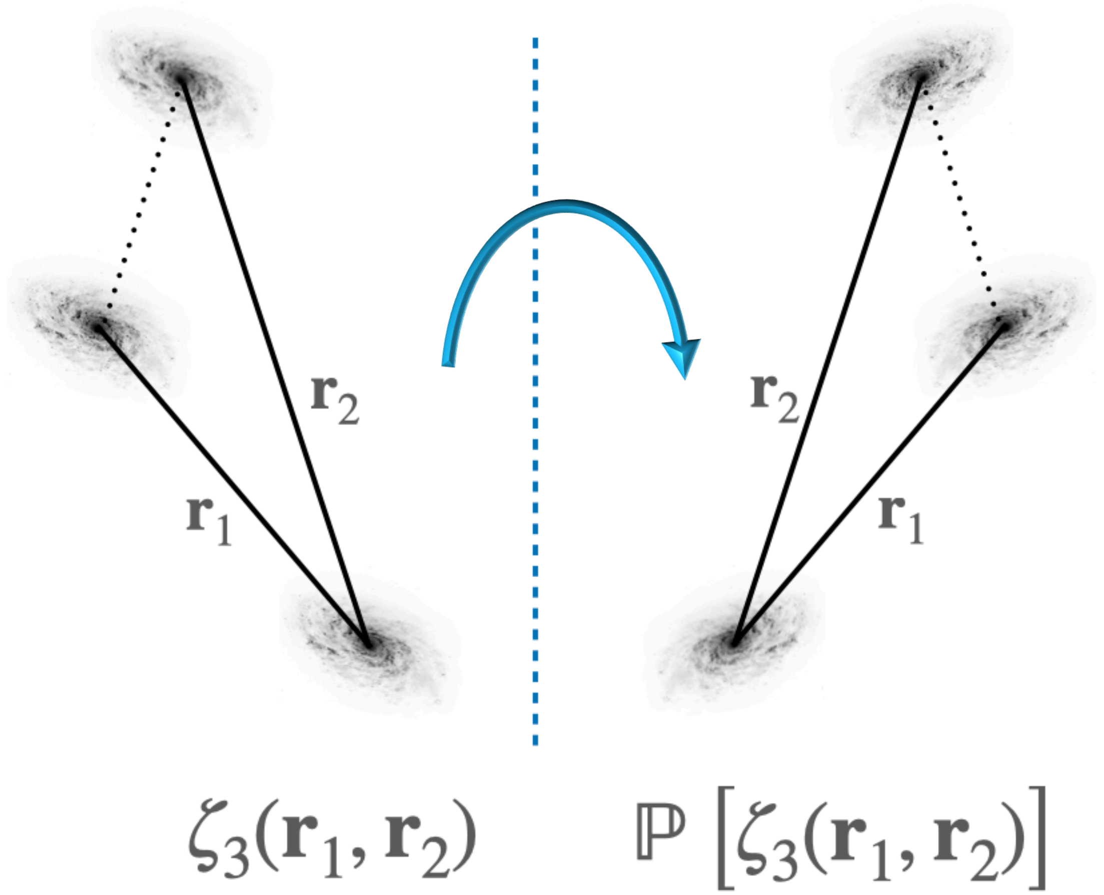
THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY

2-Point Correlation Function (2PCF):

Parity Inversion = Rotation

3-Point Correlation Function (3PCF):

Parity Inversion = Rotation



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THE 2PCF + 3PCF ARE NOT SENSITIVE TO PARITY

2-Point Correlation Function (2PCF):

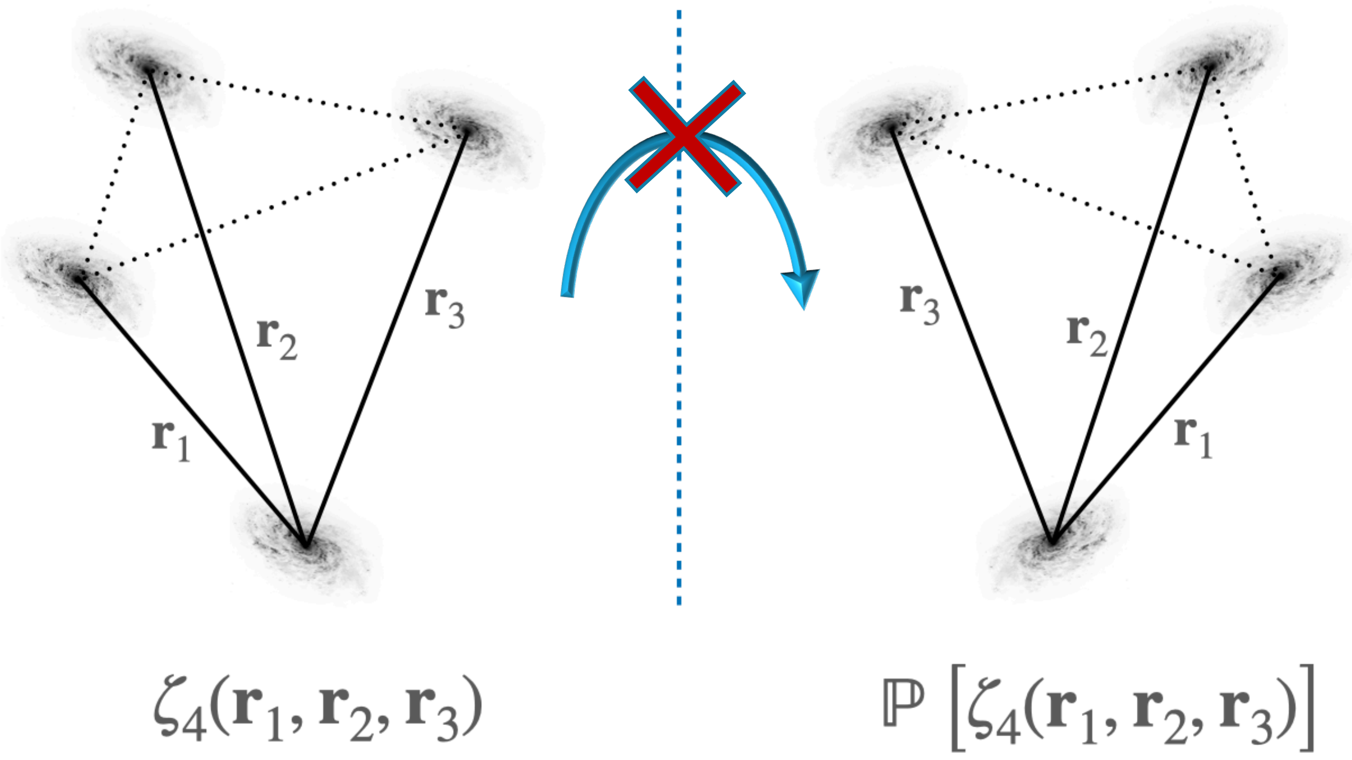
Parity Inversion = Rotation

3-Point Correlation Function (3PCF):

Parity Inversion = Rotation

4-Point Correlation Function (4PCF):

Parity Inversion ≠ Rotation



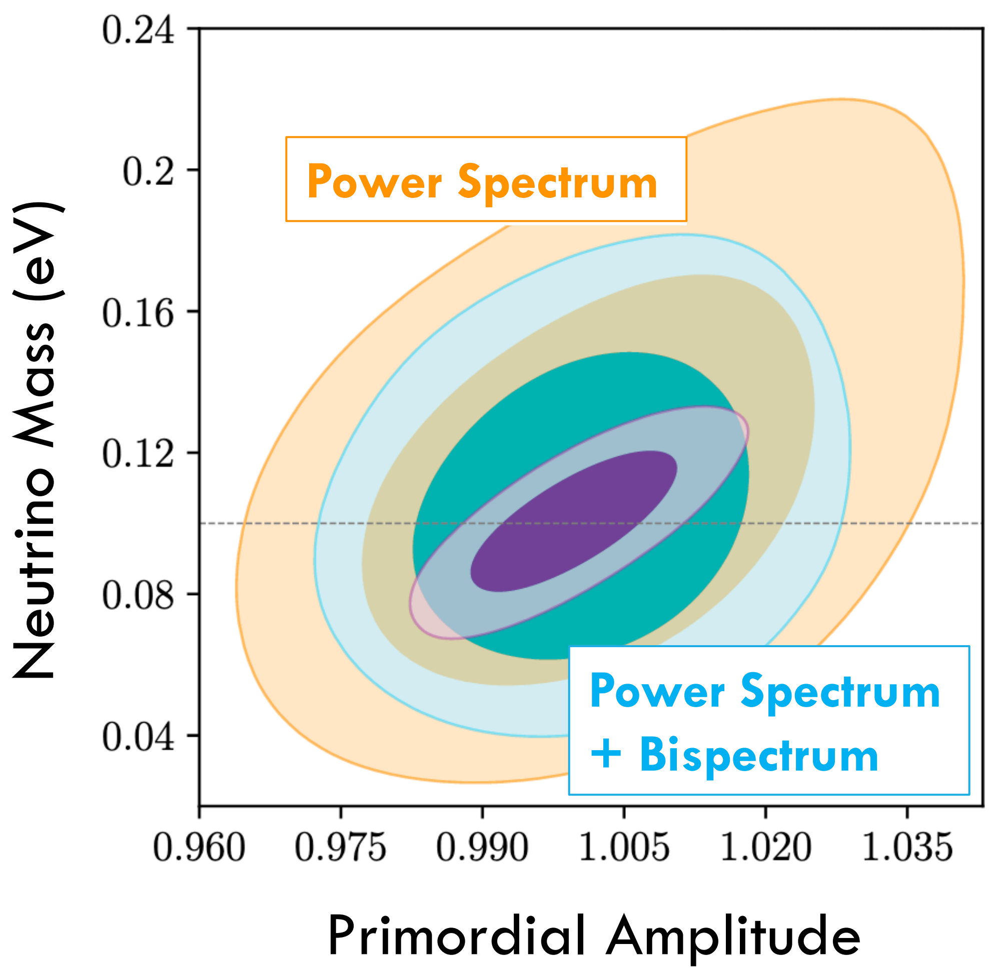
Lue+98, Jeong+12, Shiraishi 16, Cahn+21, **Philcox** (in prep.)

WHY USE HIGHER-ORDER STATISTICS?

- **Sharpen** parameter constraints!
- **Break** parameter **degeneracies**!
- Test **non-standard** physics models!

Why Use Large Scale Structure?

- Signal-to-Noise is **cubic** in number of modes unlike CMB
- New physics constraints **don't** dilute with redshift



Chudaykin+19

HOW TO MEASURE A BISPECTRUM

$$\hat{B}_g(k_1, k_2, k_3) = \int_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 \in \mathrm{bins}} \delta_g(\mathbf{k}_1)\delta_g(\mathbf{k}_2)\delta_g(\mathbf{k}_3)(2\pi)^3\delta_{\mathrm{D}}\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3\right)$$

Problem: We don't measure the density field directly.

$$\delta_g(\mathbf{r}) \to W(\mathbf{r})\delta_g(\mathbf{r}) \qquad \delta_g(\mathbf{k}) \to \int \frac{d\mathbf{p}}{(2\pi)^3} W(\mathbf{k} - \mathbf{p})\delta_g(\mathbf{p})$$

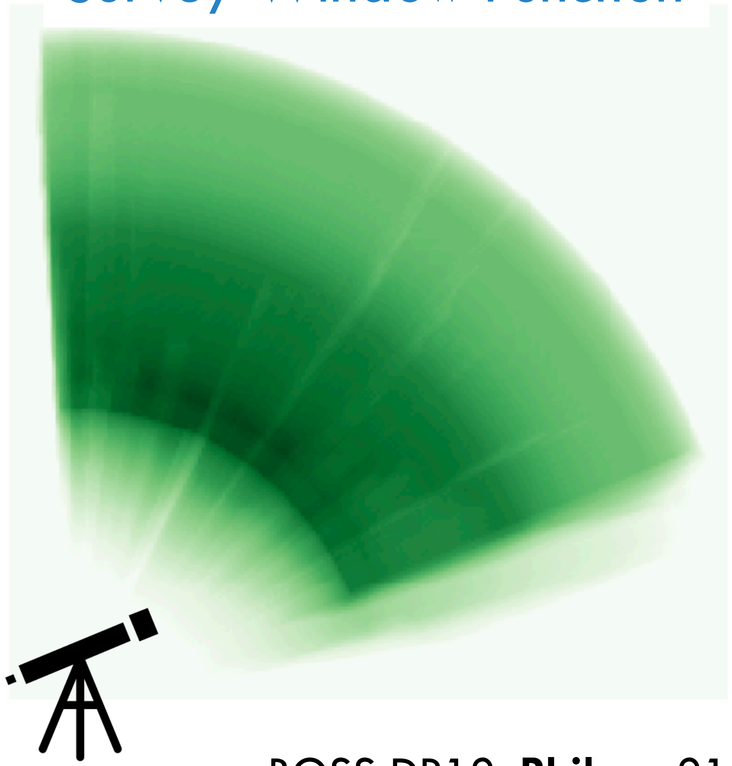
Window Function

The measured bispectrum is a triple **convolution**

$$B_g(\mathbf{k}_1, \mathbf{k}_2) \to \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1)W(\mathbf{k}_2 - \mathbf{p}_2)W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2)B_g(\mathbf{p}_1, \mathbf{p}_2)$$

Solution: Convolve the **theory model** too

Survey Window Function



BOSS DR12, **Philcox** 21

CONVOLUTION IS EXPENSIVE

$$B_g^{\mathrm{win}}(\mathbf{k}_1,\mathbf{k}_2) = \int_{\mathbf{p}_1\mathbf{p}_2} W(\mathbf{k}_1-\mathbf{p}_1)W(\mathbf{k}_2-\mathbf{p}_2)W(\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}_1-\mathbf{k}_2)B_g(\mathbf{p}_1,\mathbf{p}_2)$$

- Window convolution is too costly to do repeatedly!

- Common approximation: apply the window **only** to the power spectrum

$$B_g(\mathbf{k}_1,\mathbf{k}_2) \supset P_L(k_1)P_L(k_2)$$

But:

- This gives **systematic errors** on large scales
- Spectra cannot be used to search for new physics!

Gil-Marin 15,17, Sugiyama+18, **Philcox** 21

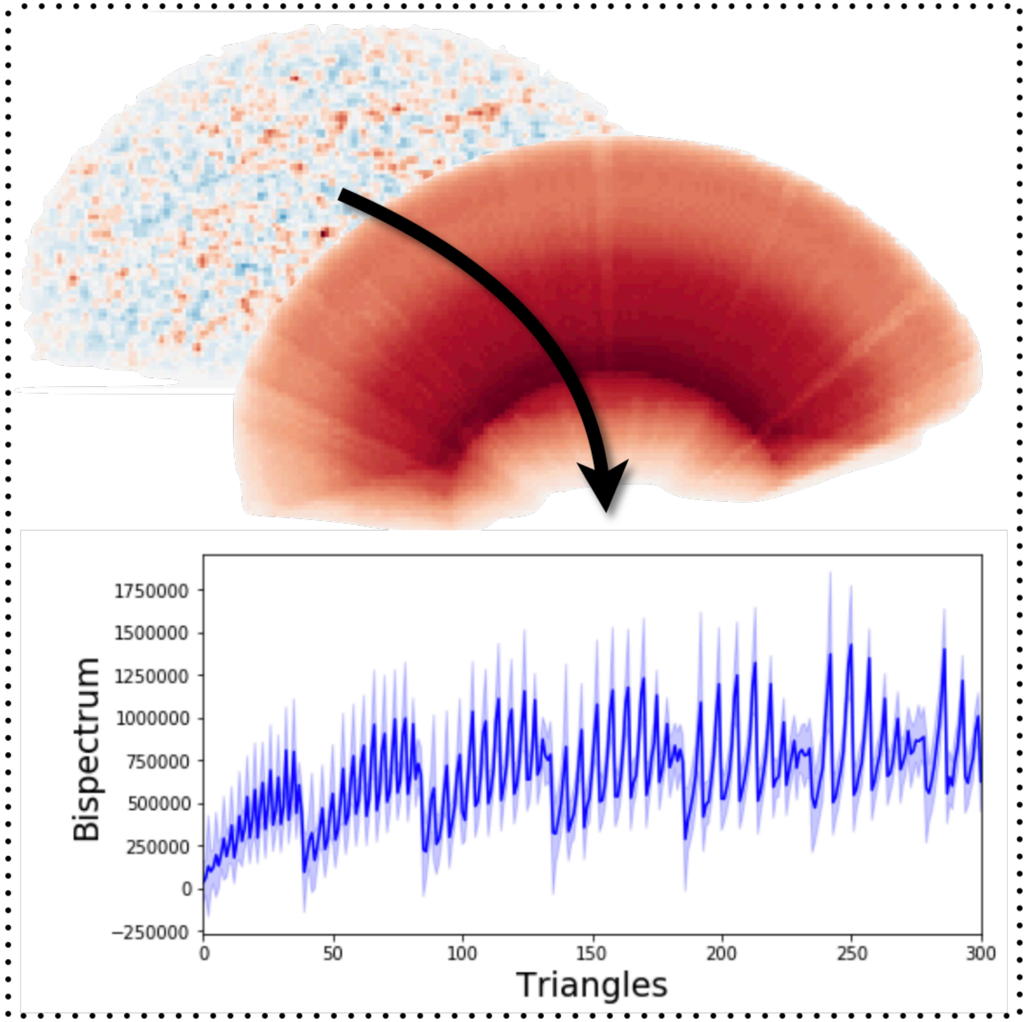
BISPECTRA WITHOUT WINDOWS

Alternatively: estimate the **unwindowed** bispectrum directly

$$B_g^{\rm win}(\mathbf{k}_1,\mathbf{k}_2) = \int_{\mathbf{p}_1\mathbf{p}_2} W(\mathbf{k}_1-\mathbf{p}_1)W(\mathbf{k}_2-\mathbf{p}_2)W(\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}_1-\mathbf{k}_2)B_g(\mathbf{p}_1,\mathbf{p}_2)$$

- Derive a **maximum-likelihood** estimator for the **true** bispectrum
- Effectively **deconvolves** the window

$$\nabla_{B_g} L[\text{data}|B_g] = 0 \quad \Rightarrow \quad \widehat{B}_g = \cdots$$



See GitHub.com/oliverphilcox/BOSS-Without-Windows

Philcox 20, 21

BISPECTRA WITHOUT WINDOWS

New Approach

- Start from the **likelihood** for data **d,** using an Edgeworth expansion

$$L[\mathbf{d}](\mathbf{b}) = L_G[\mathbf{d}](\mathbf{b}) \left[1 + \frac{1}{3!}\mathsf{B}^{ijk}\left\{[\mathsf{C}^{-1}\mathbf{d}]_i [\mathsf{C}^{-1}\mathbf{d}]_j [\mathsf{C}^{-1}\mathbf{d}]_k - (\mathsf{C}^{-1}_{ij} d_k + 2 \text{ perms.})\right\} + \cdots\right]$$

Gaussian Piece   Three-Point Function, $\mathsf{B}^{ijk} \equiv \langle d^i d^j d^k \rangle$ Covariance, $\mathsf{C}^{ij} = \langle d^i d^j \rangle$

- This depends on **survey geometry** through C^{ij} and **bispectrum** through B^{ijk}

$$\nabla_{\mathbf{b}} \log L[\mathbf{d}](\mathbf{b}) = \mathbf{0}$$

- Optimize for true bispectrum, **b:**

$$\hat{b}^{\mathrm{ML}}_\alpha = \sum_\beta F^{-1,\mathrm{ML}}_{\alpha\beta} \hat{q}^{\mathrm{ML}}_\beta,$$

$$\hat{q}^{\mathrm{ML}}_\alpha = \frac{1}{6}\mathsf{B}^{ijk}_{,\alpha}[\mathsf{C}^{-1}\mathbf{d}]_i \left([\mathsf{C}^{-1}\mathbf{d}]_j [\mathsf{C}^{-1}\mathbf{d}]_k - 3\mathsf{C}^{-1}_{jk}\right)$$

Cubic Estimator

$$F^{\mathrm{ML}}_{\alpha\beta} = \frac{1}{6}\mathsf{B}^{ijk}_{,\alpha}\mathsf{B}^{lmn}_{,\beta}\mathsf{C}^{-1}_{il}\mathsf{C}^{-1}_{jm}\mathsf{C}^{-1}_{kn},$$

Fisher Matrix

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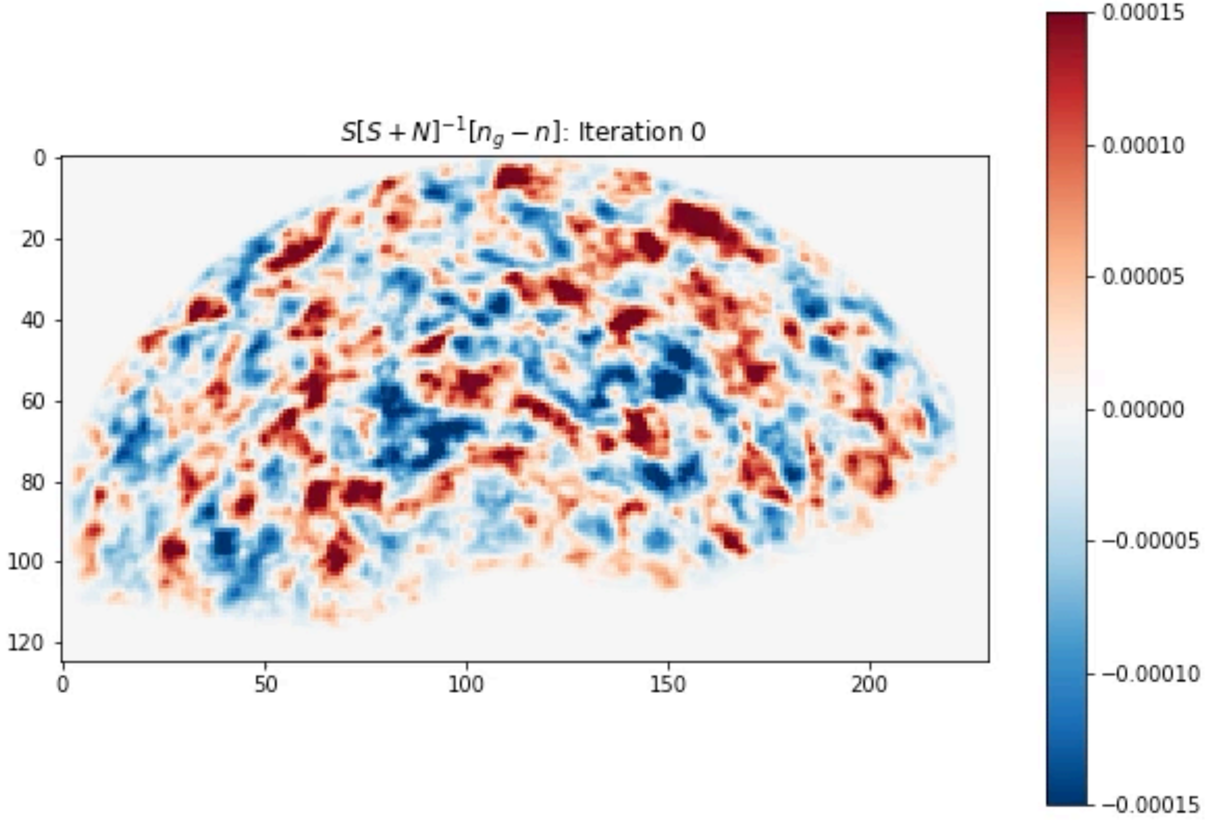
INVERSE VARIANCE WEIGHTING

Compute $\mathrm{C}^{-1}\boldsymbol{d}$ iteratively via **conjugate gradient descent**



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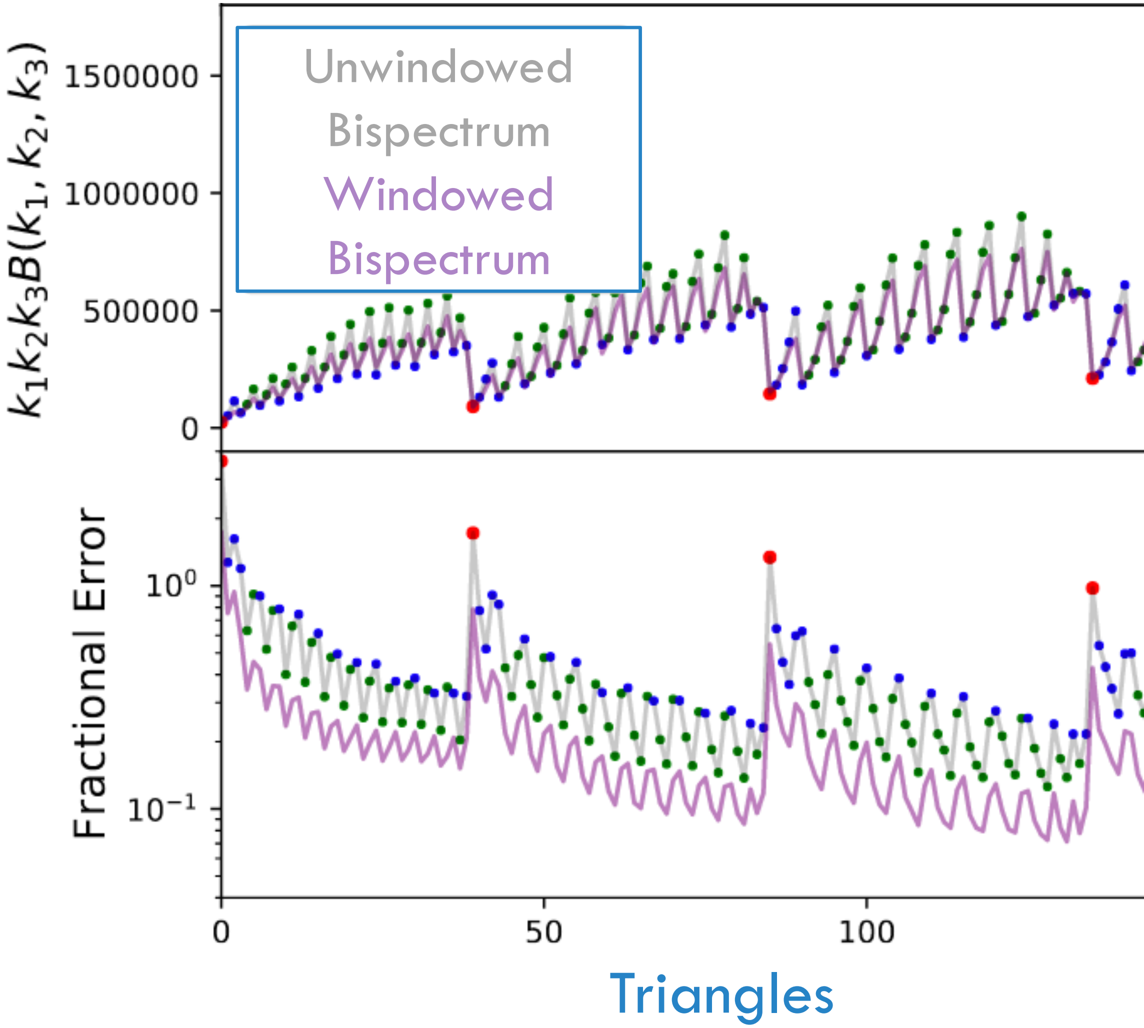
BISPECTRA WITHOUT WINDOWS

Properties of the **cubic estimator:**

1. Unbiased
2. Minimum variance [as $B(k_1, k_2, k_3) \to 0$]
3. Window-free [effectively a deconvolution]

▷ Requires various tricks for dealing with high-dimensional data [e.g. conjugate gradient descent, Monte Carlo estimation etc.]

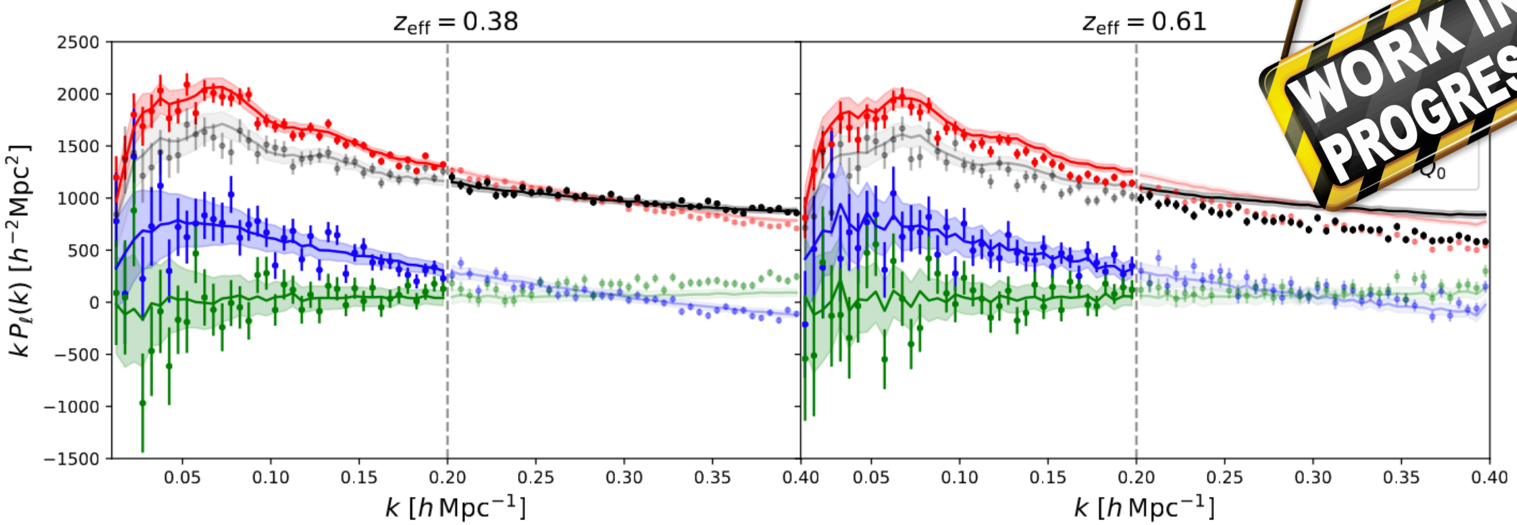


See GitHub.com/oliverphilcox/BOSS-Without-Windows

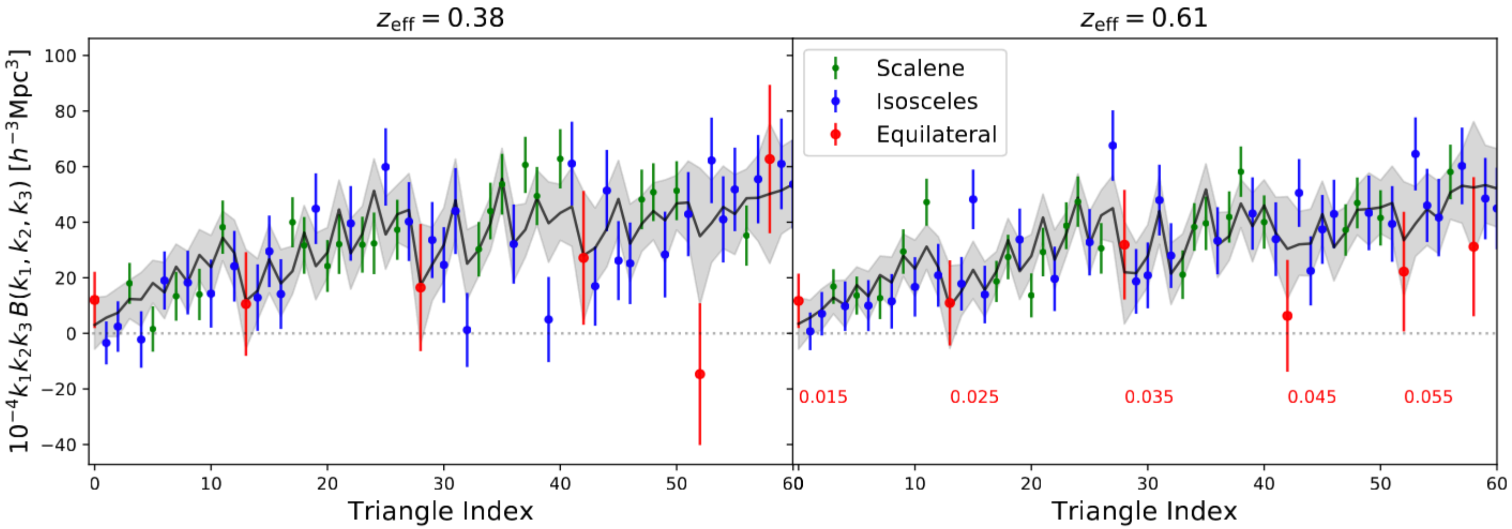
Philcox 20, 21

BOSS WITHOUT WINDOWS

Power Spectra



Bispectra



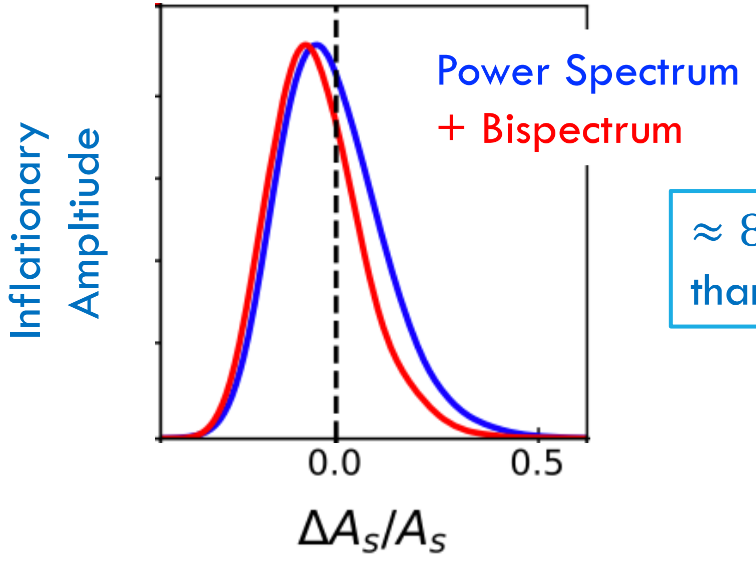
Theory Model

Cosmological Parameters

Ivanov, **Philcox**+21
Philcox & Ivanov (in prep.)

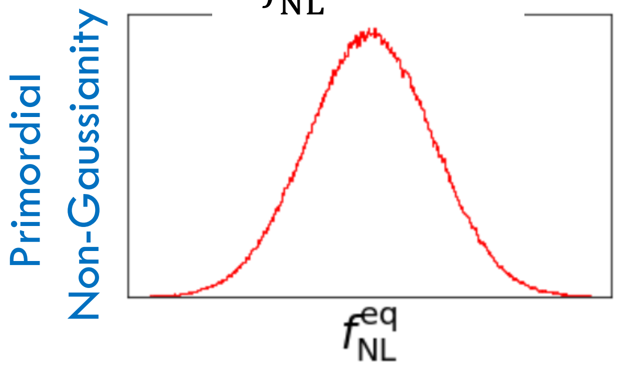
WHAT WILL WE MEASURE?

- Tighter constraints on **cosmological** and **galaxy formation** parameters
  - σ_8 improves by 10%
  - Tidal bias improves by 50%
- Bounds on **all** flavors of **Primordial Non-Gaussianity**
  - First equilateral-type measurement from LSS



≈ 8× weaker than *Planck*

$\sigma_{f_{\rm NL}^{\rm eq}} \approx 500$

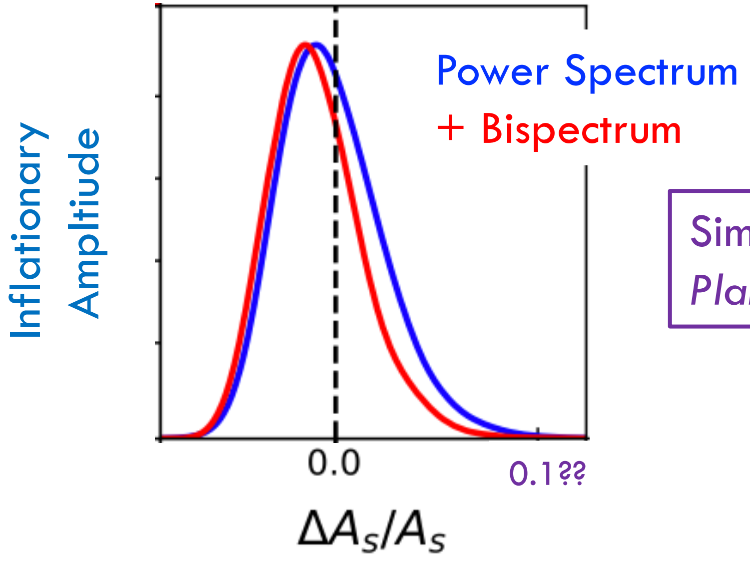


≈ 7× weaker than *Planck*

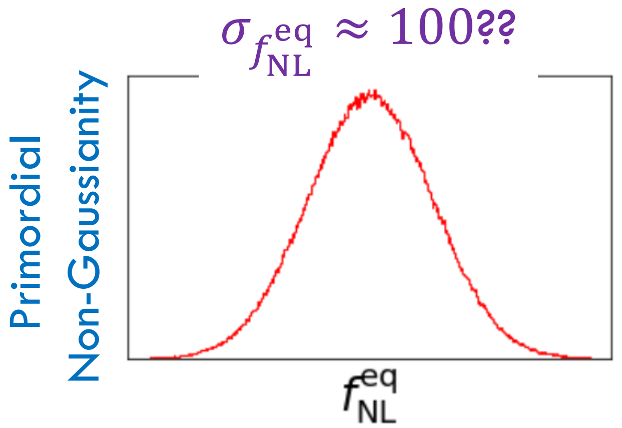
Ivanov, **Philcox**+21
Philcox & Ivanov (in prep.)

WHAT'S NEXT FOR BISPECTRA?

- Improve bispectrum **modeling**
  - Higher-order perturbation theory
  - Add **redshift-space** information
  - Better treatment of **fingers-of-God**
- Apply to **DESI** data
  - **Pipelines** already available and **tested**
  - Expect $\mathcal{O}(5)\times$ **stronger** constraints



Similar to *Planck*??



Similar to *Planck*??

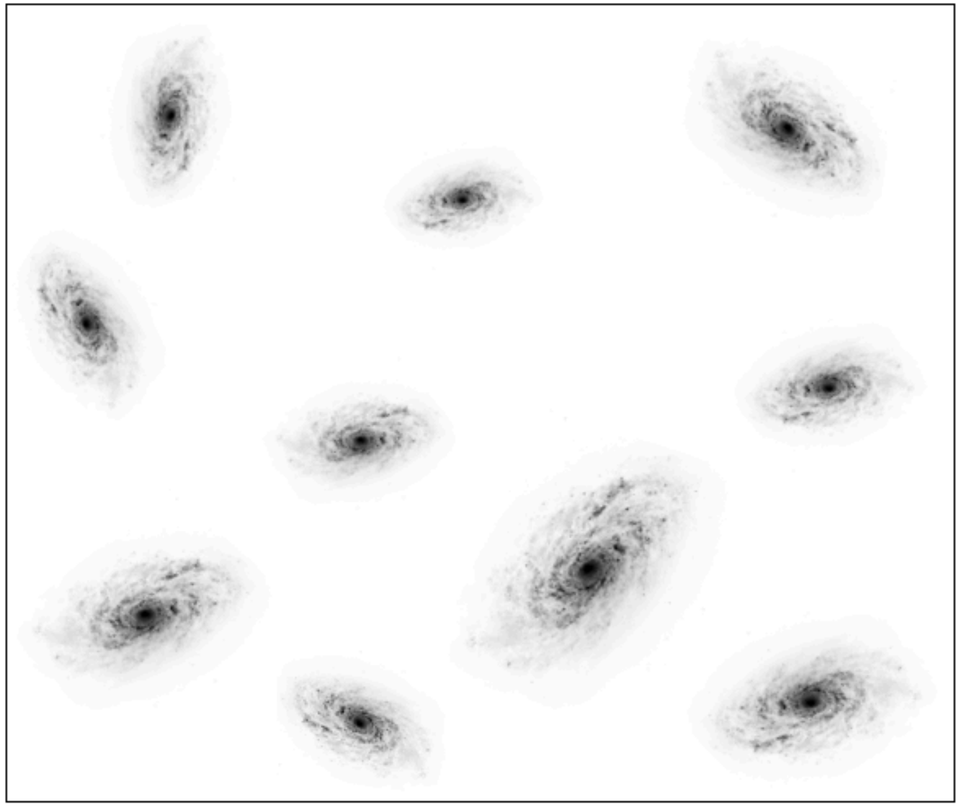
https://github.com/oliverphilcox/BOSS-Without-Windows

Ivanov, **Philcox**+21
Philcox & Ivanov (in prep.)

HOW TO MEASURE A CORRELATION FUNCTION

$$\hat{\zeta}_{g,4}(r_1, r_2, r_3, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_3, \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_3) = \int \frac{d\mathbf{x}}{V} \int_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \in \text{bins}} \delta_g(\mathbf{x})\delta_g(\mathbf{x}+\mathbf{r}_1)\delta_g(\mathbf{x}+\mathbf{r}_2)\delta_g(\mathbf{x}+\mathbf{r}_3)$$

Measuring the 4PCF involves counting **quadruplets** of galaxies



Philcox+21

HOW TO MEASURE A CORRELATION FUNCTION

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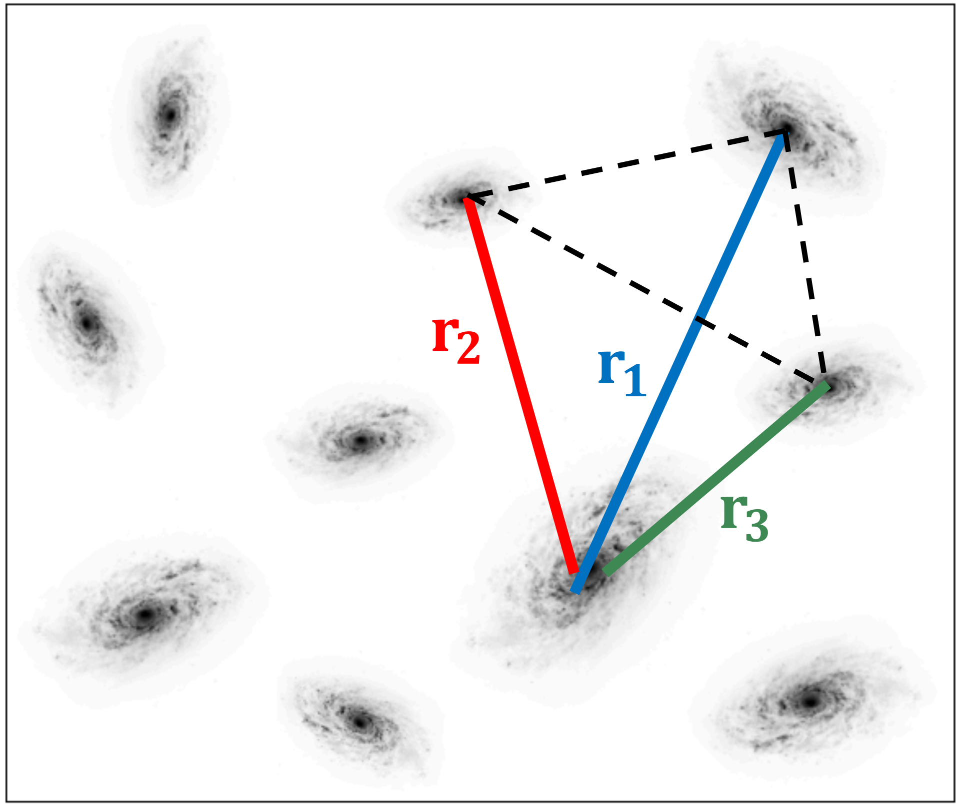
Measuring the 4PCF involves counting **quadruplets** of galaxies

Total number of quadruplets:

$$\mathcal{O}(N_{\mathbf{gal}}^4)$$

This is too many to count…



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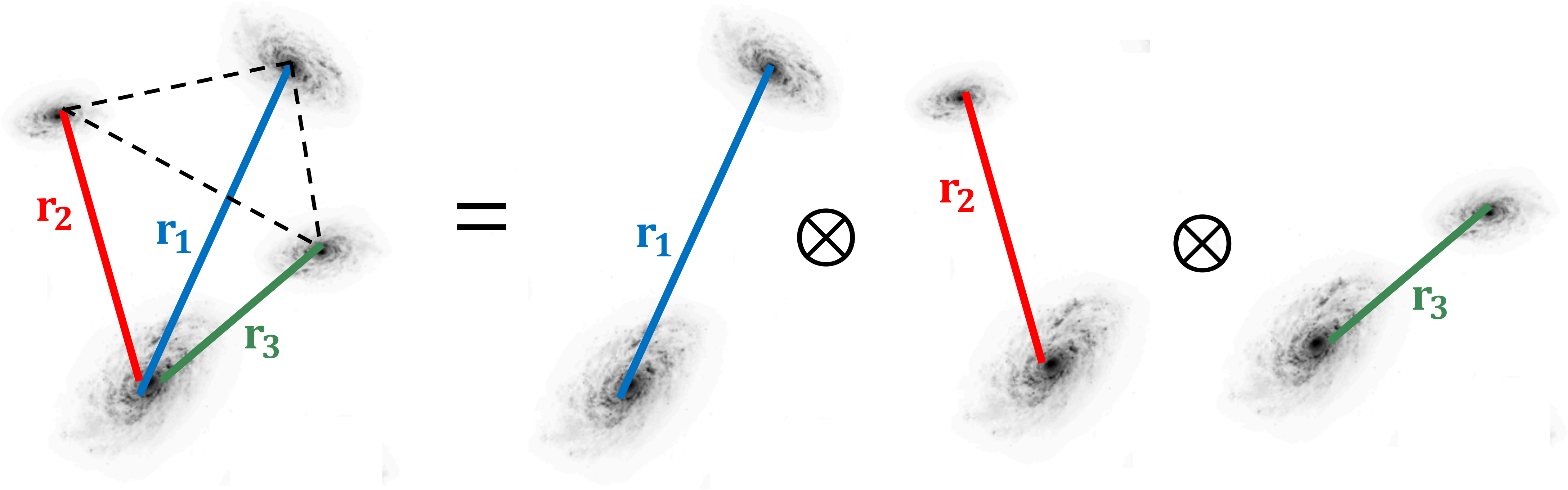
ONE TETRAHEDRON $=$ THREE VECTORS



3 lengths + 3 angles

$(r_1, r_2, r_3, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}}_3, \hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_3)$

1 length + 1 direction

$(r_1, \hat{\mathbf{r}}_1)$

1 length + 1 direction

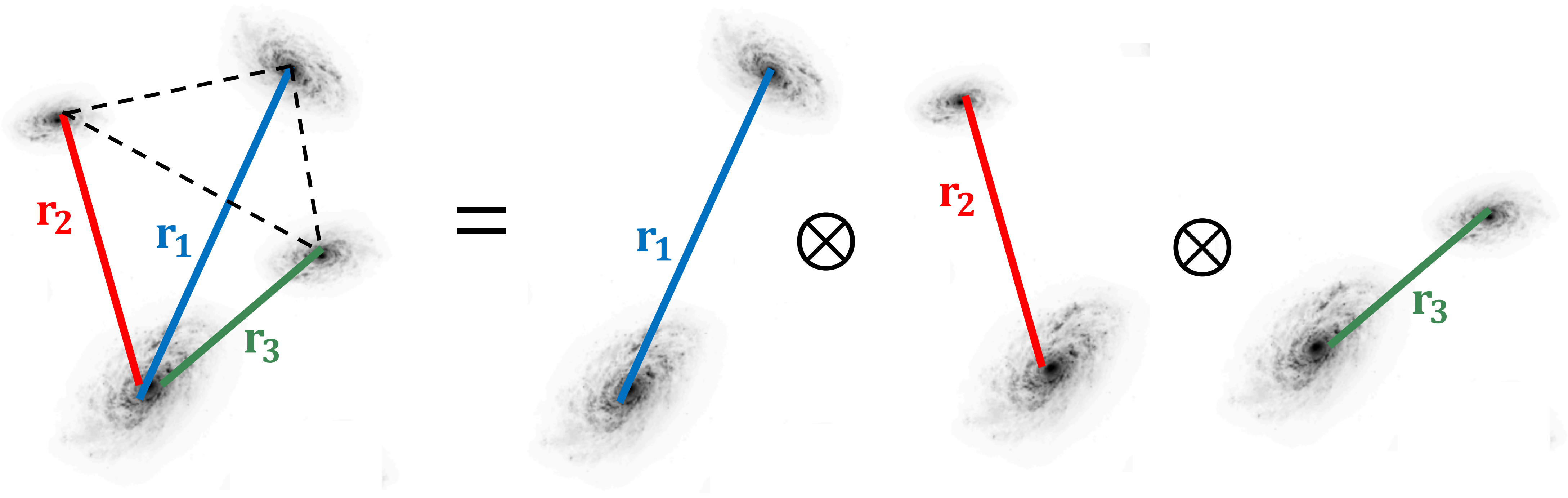
$(r_2, \hat{\mathbf{r}}_2)$

1 length + 1 direction

$(r_3, \hat{\mathbf{r}}_3)$

Philcox+21

ONE TETRAHEDRON = THREE VECTORS



3 lengths + 3 multipoles

$(r_1, r_2, r_3, \ell_1, \ell_2, \ell_3)$

1 length + 2 multipoles

(r_1, ℓ_1, m_1)

1 length + 2 multipoles

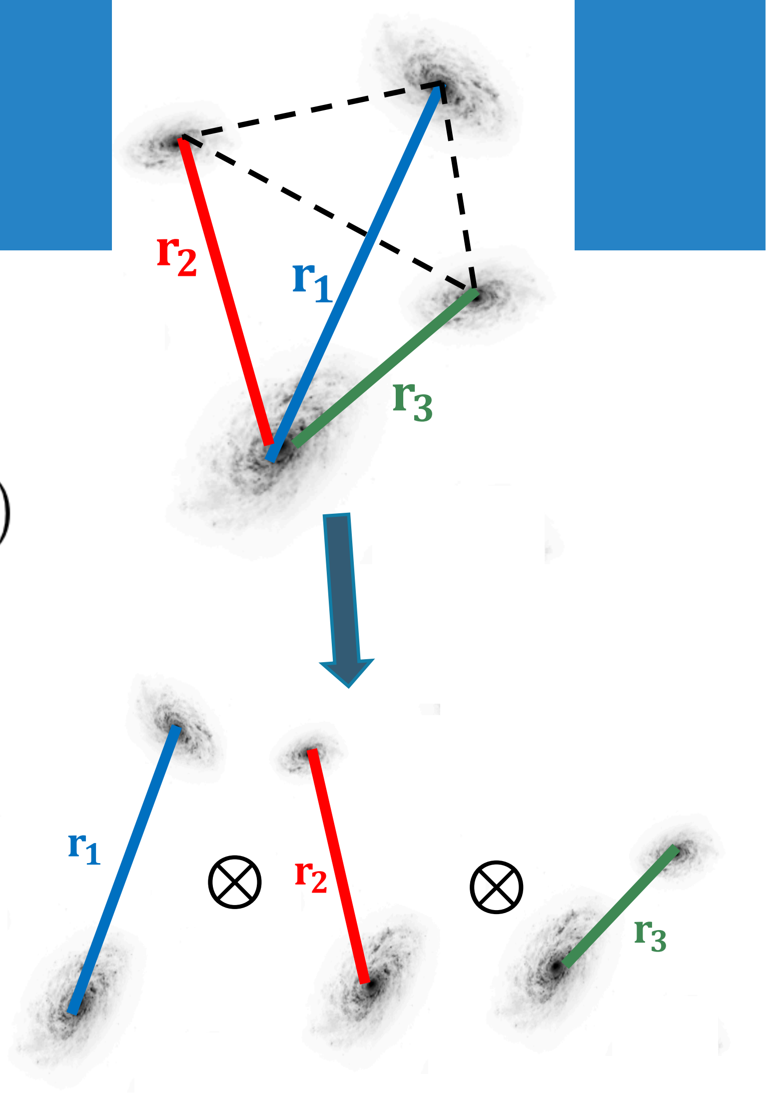
(r_2, ℓ_2, m_2)

1 length + 2 multipoles

(r_3, ℓ_3, m_3)

Philcox+21

ANGULAR MOMENTUM BASIS



Expand 4PCF in basis of **isotropic functions**

$$\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sum_{\ell_1 \ell_2 \ell_3} \zeta_{\ell_1 \ell_2 \ell_3}(r_1, r_2, r_3) \mathcal{P}_{\ell_1 \ell_2 \ell_3}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3)$$

Coefficients — Basis Functions

Basis formed from **angular momentum addition** in 3D

$$\mathcal{P}_{\ell_1 \ell_2 \ell_3}(\hat{\mathbf{r}}_1, \hat{\mathbf{r}}_2, \hat{\mathbf{r}}_3) = \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} Y^*_{\ell_1 m_1}(\hat{\mathbf{r}}_1) Y^*_{\ell_2 m_2}(\hat{\mathbf{r}}_2) Y^*_{\ell_3 m_3}(\hat{\mathbf{r}}_3)$$

This is **separable** in $\hat{\mathbf{r}}_1$, $\hat{\mathbf{r}}_2$, $\hat{\mathbf{r}}_3$

Cahn+20, **Philcox**+21

A SEPARABLE BASIS ⇒ A QUADRATIC ESTIMATOR

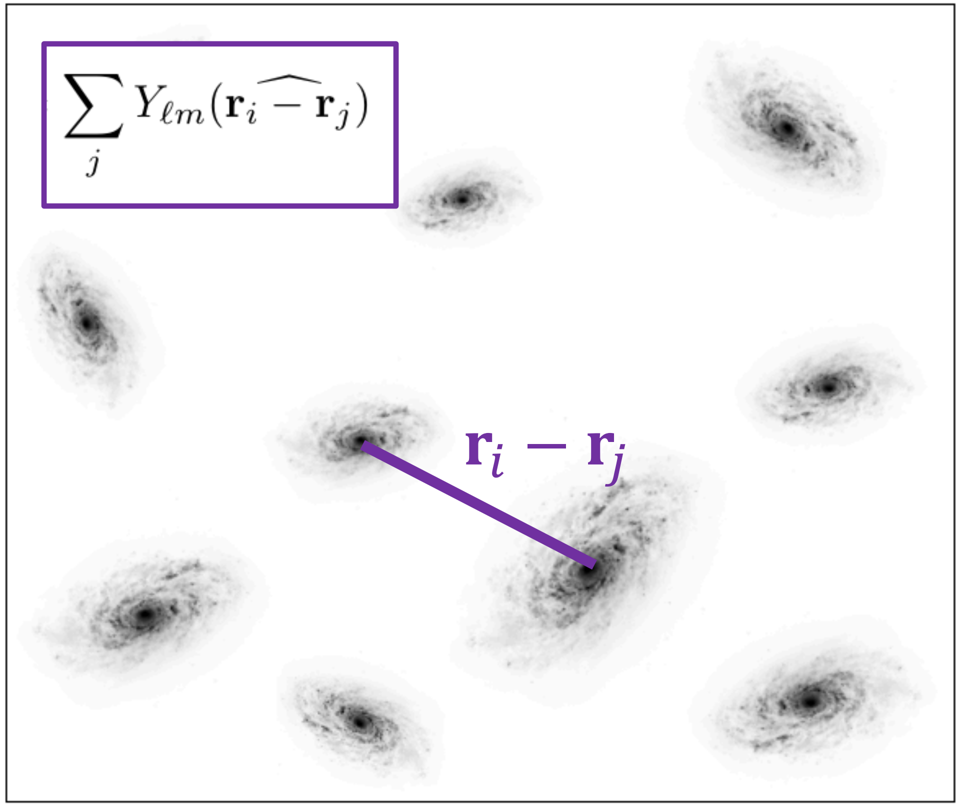
$$\hat{\zeta}_{\ell_1\ell_2\ell_3}(r_1,r_2,r_3) = \sum_{m_1m_2m_3}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\int d\mathbf{x}\,\delta_g(\mathbf{x})\left[\int_{\mathbf{r}_1}\delta_g(\mathbf{x}+\mathbf{r}_1)Y_{\ell_1m_1}(\hat{\mathbf{r}}_1)\right]\left[\int_{\mathbf{r}_2}\delta_g(\mathbf{x}+\mathbf{r}_2)Y_{\ell_2m_2}(\hat{\mathbf{r}}_2)\right]\left[\int_{\mathbf{r}_3}\delta_g(\mathbf{x}+\mathbf{r}_3)Y_{\ell_3m_3}(\hat{\mathbf{r}}_3)\right]$$

The estimator **factorizes** into **independent** pieces

To compute the 4PCF: count *pairs* of galaxies

Total number of pairs: $\mathcal{O}(N_{\mathbf{g}}^2)$

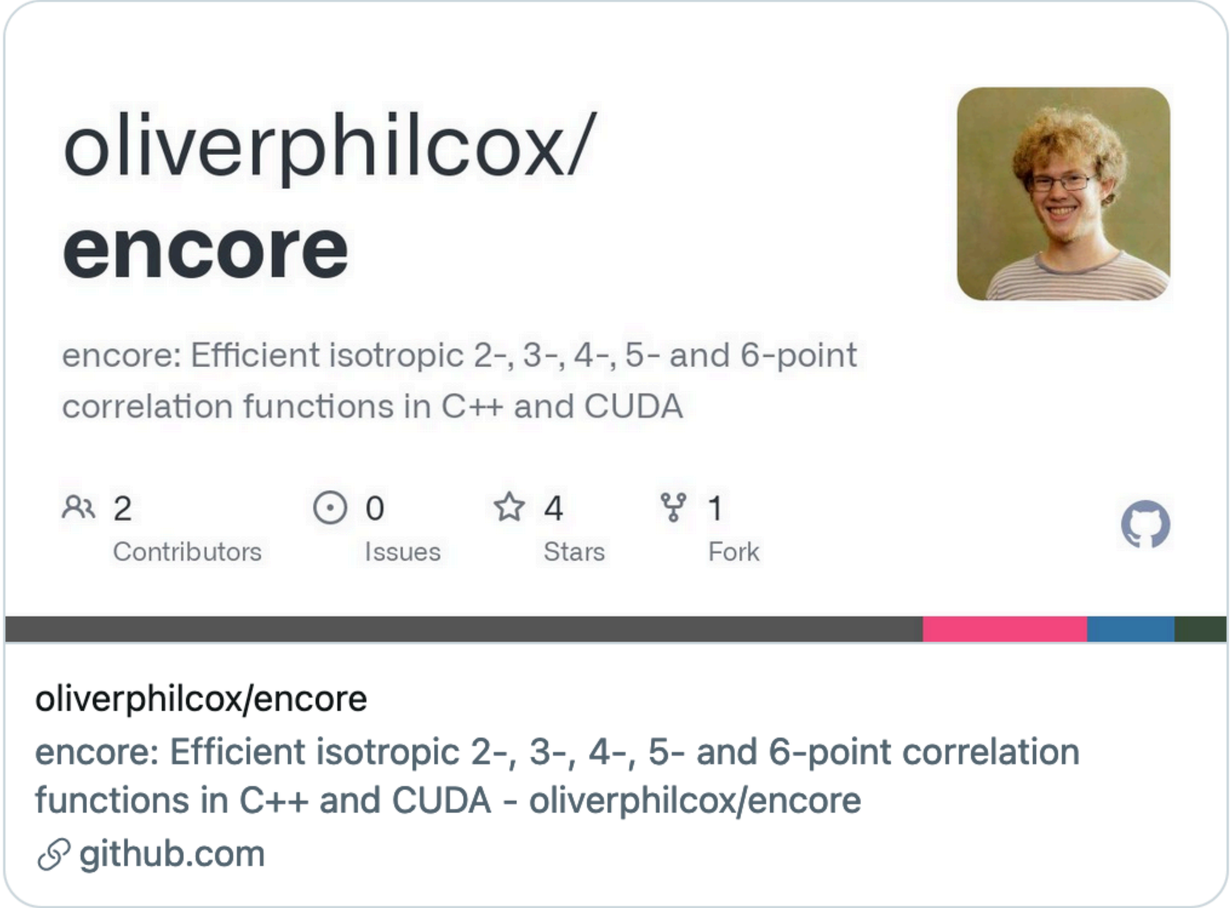
This *can* be computed!



Philcox+21

ENCORE: ULTRA-FAST N-POINT FUNCTIONS

- Public C++/CUDA code
- Computes isotropic 2-, 3-, 4-, 5- and 6-point correlation functions
- Corrects for **survey geometry**
- Requires ~ 10 CPU-hours to compute 4PCF of current data



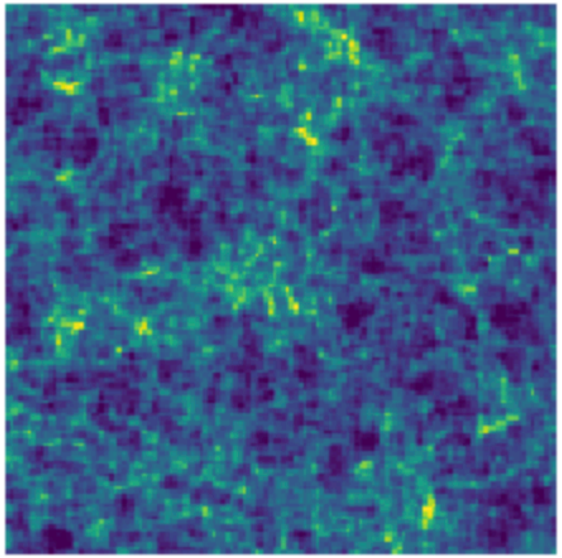
See GitHub.com/oliverphilcox/encore

Philcox+21

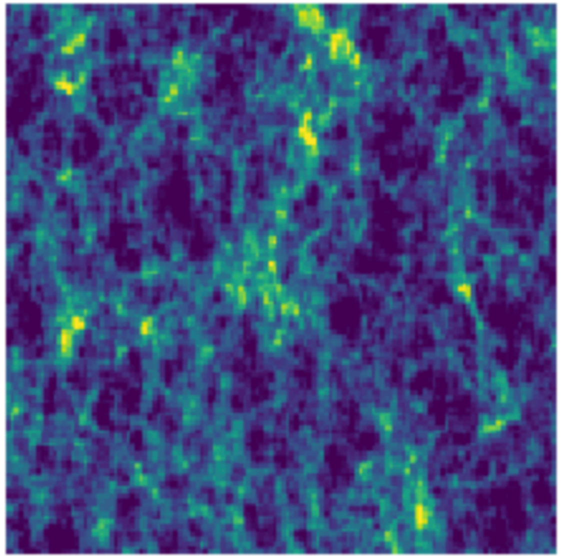
BEYOND THE 4-POINT FUNCTION

This generalizes **beyond** the 4PCF

- 5PCF, 6PCF, …
- **Anisotropic** correlation functions
- Non-Flat Universes
- Two, Three, Four, … Dimensions



Real Space



Redshift Space

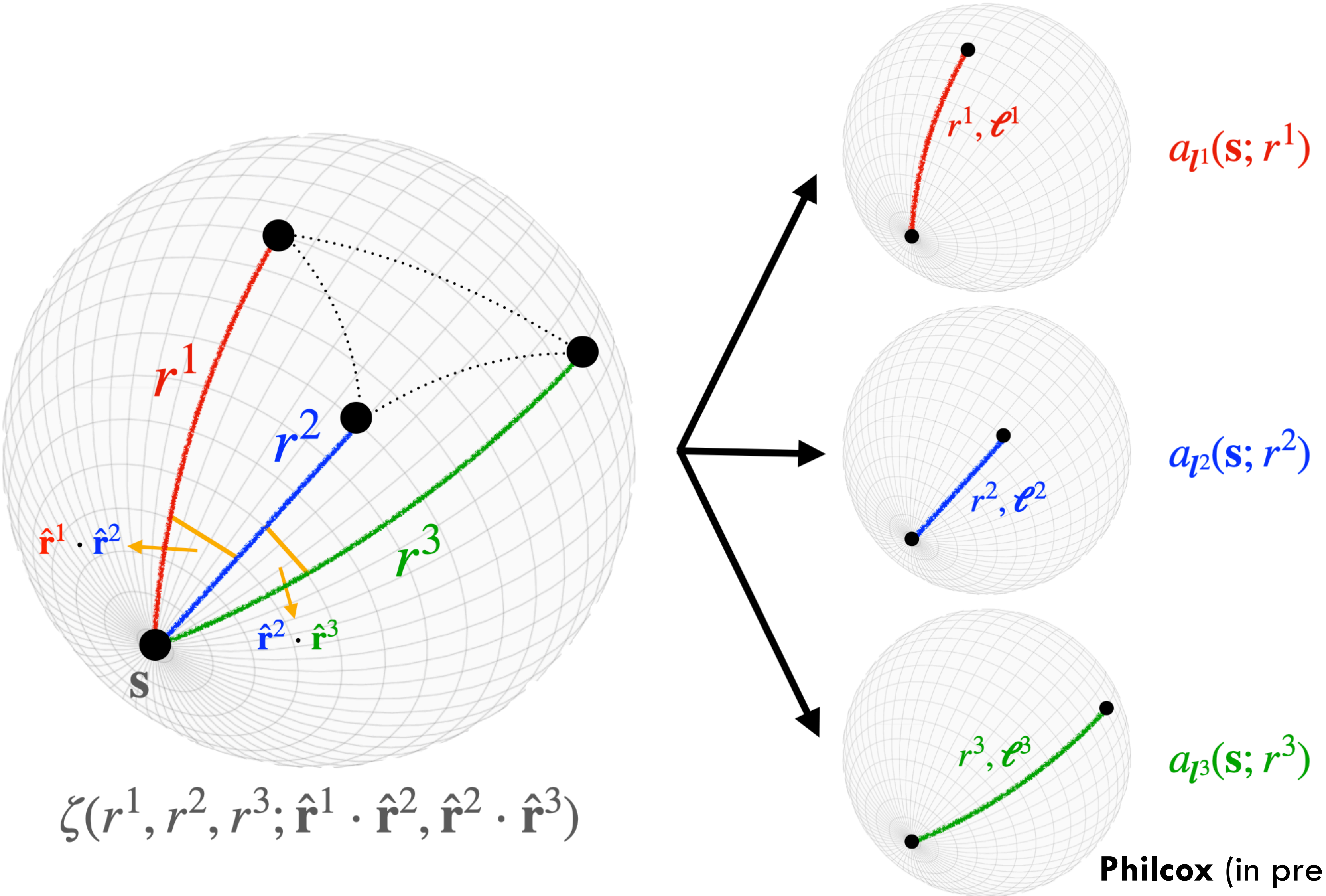
Requires the addition of N angular momenta in D dimensions *[i.e. $\mathfrak{so}(D)$ Lie algebra]*

See GitHub.com/oliverphilcox/NPCFs.jl

Philcox+21

CORRELATION FUNCTIONS ON THE 2-SPHERE

- Create as basis **on** the 2-sphere
- Basis functions are $e^{i\ell(\phi_1-\phi_2)}$
- Also computable in $O(N_g^2)$ time



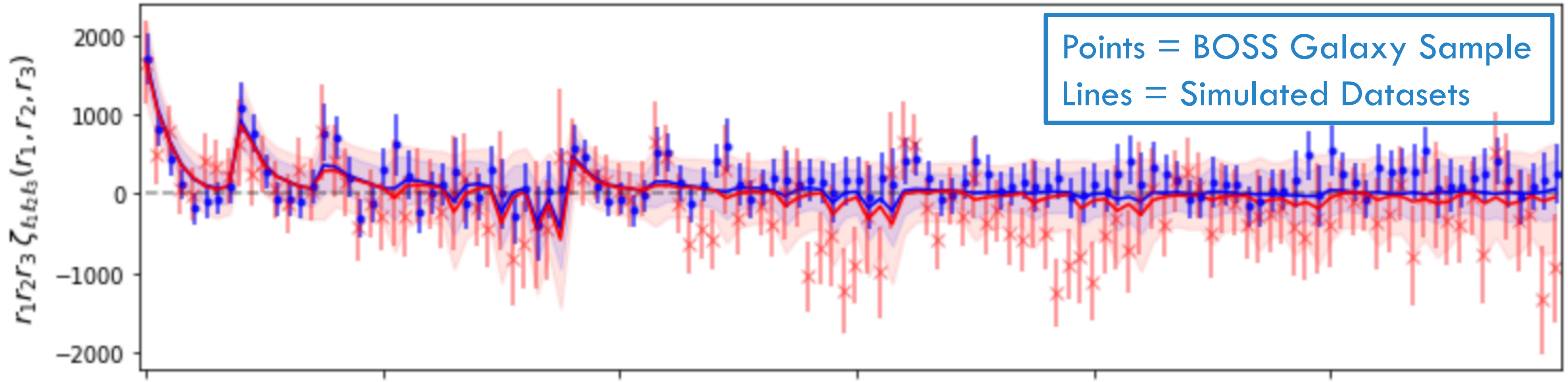
Philcox (in prep.)

MEASURING THE 4-POINT FUNCTION

Compute the 4PCF from $\sim 10^6$ **BOSS galaxies**

Do we detect a signal?

4PCF (one of 42 components)



~ 20 Mpc$/h$

Increasing Radius

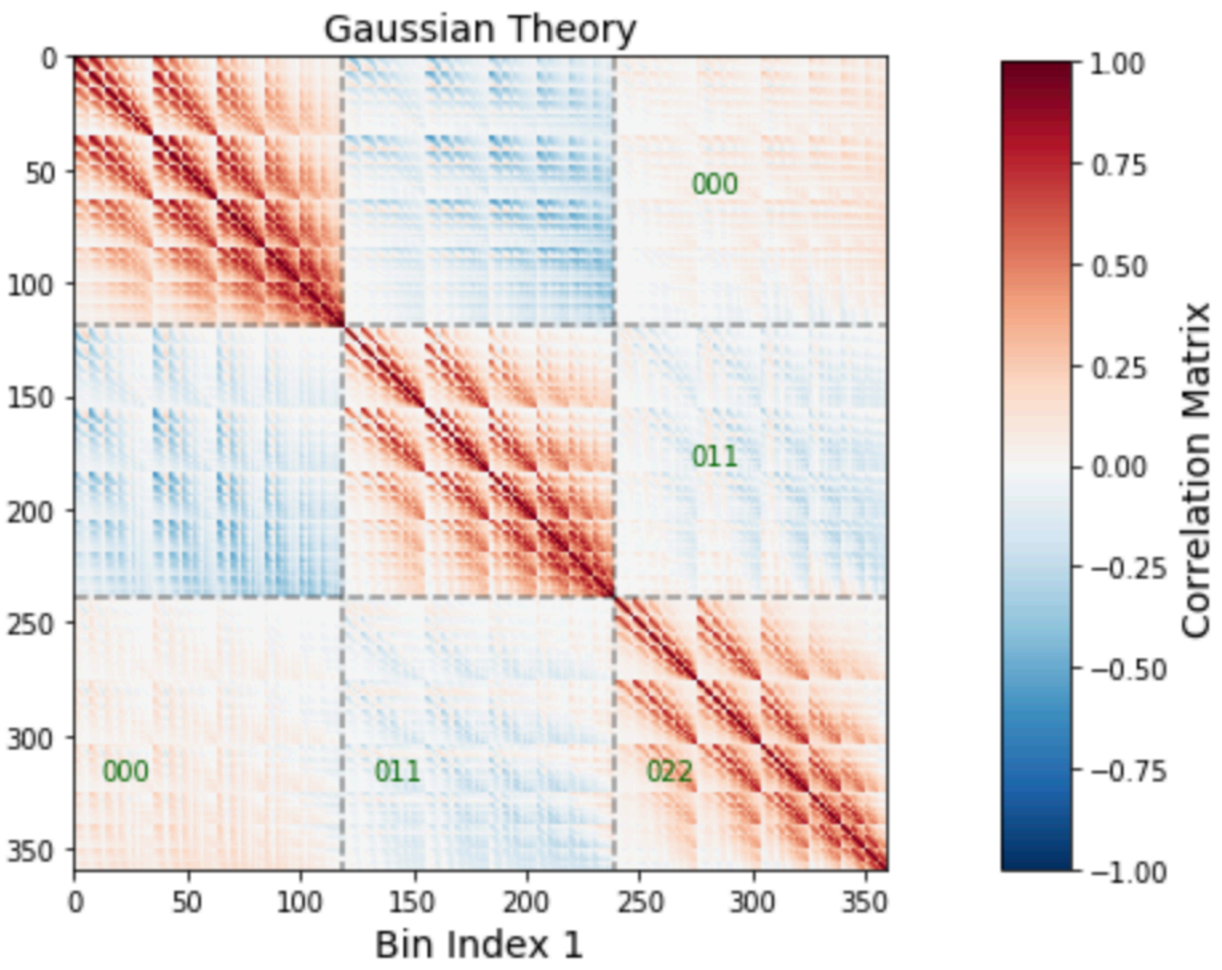
~ 150 Mpc$/h$

Philcox+21

COMPRESSION AND COVARIANCES

- The 4PCF is **high-dimensional**
- Use a **linear compression** scheme
- Compute covariance **analytically**

$$\mathrm{Cov}(\zeta_4) = \left\langle \hat{\zeta}_4 \hat{\zeta}_4' \right\rangle - \left\langle \hat{\zeta}_4 \right\rangle \left\langle \hat{\zeta}_4' \right\rangle$$



Philcox+21, Hou+21

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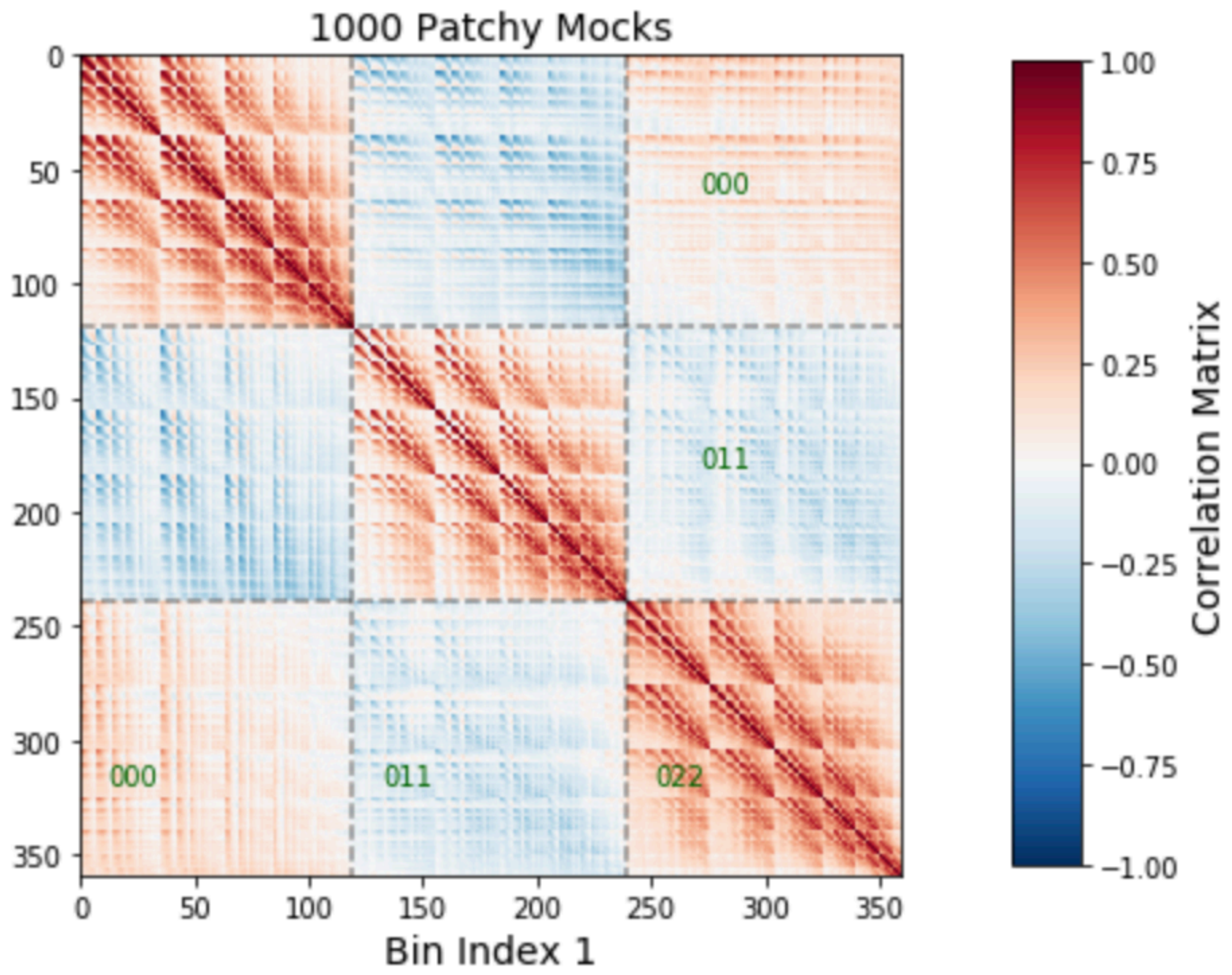
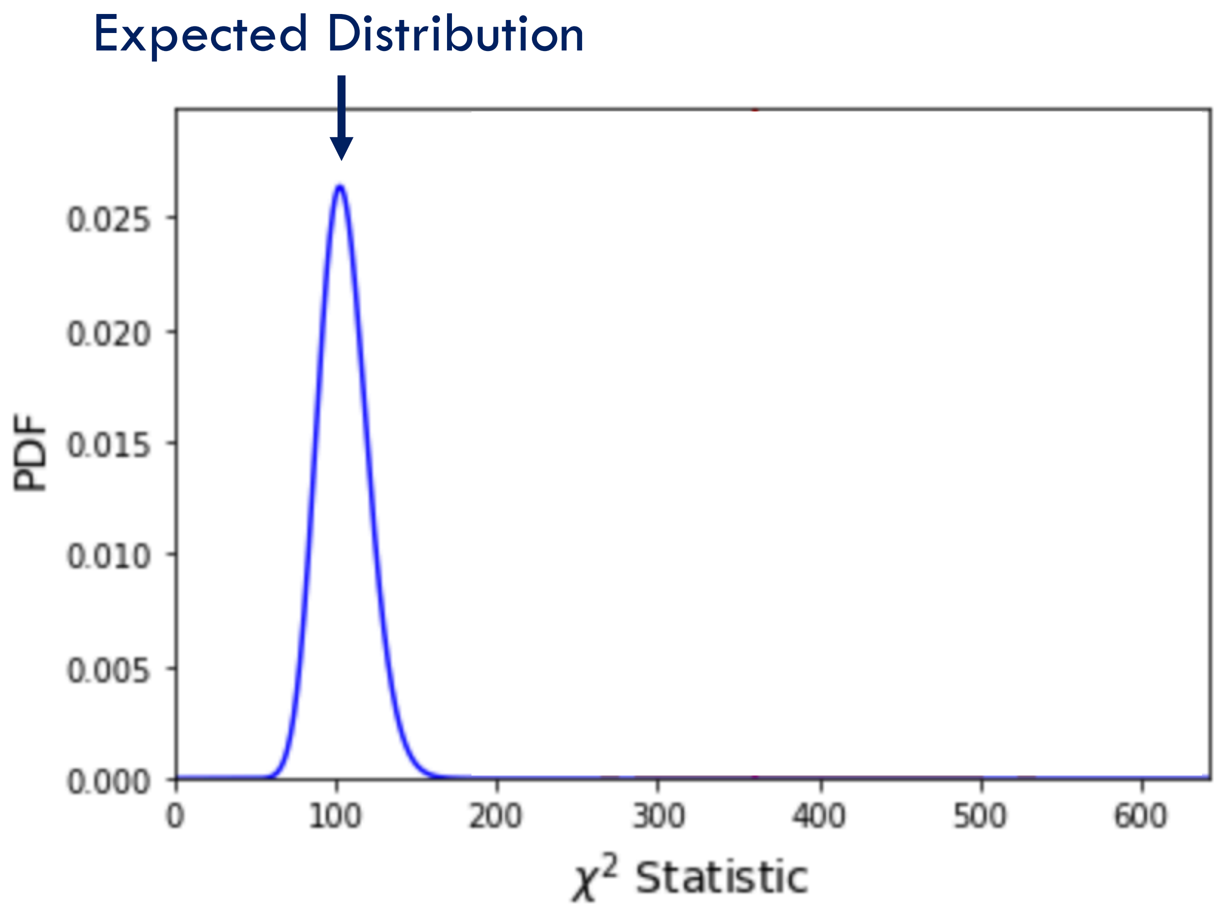


Philcox+21, Hou+21

CAN WE DETECT THE GRAVITATIONAL 4PCF?

- Perform a χ^2-test to search for a **gravitational** 4PCF

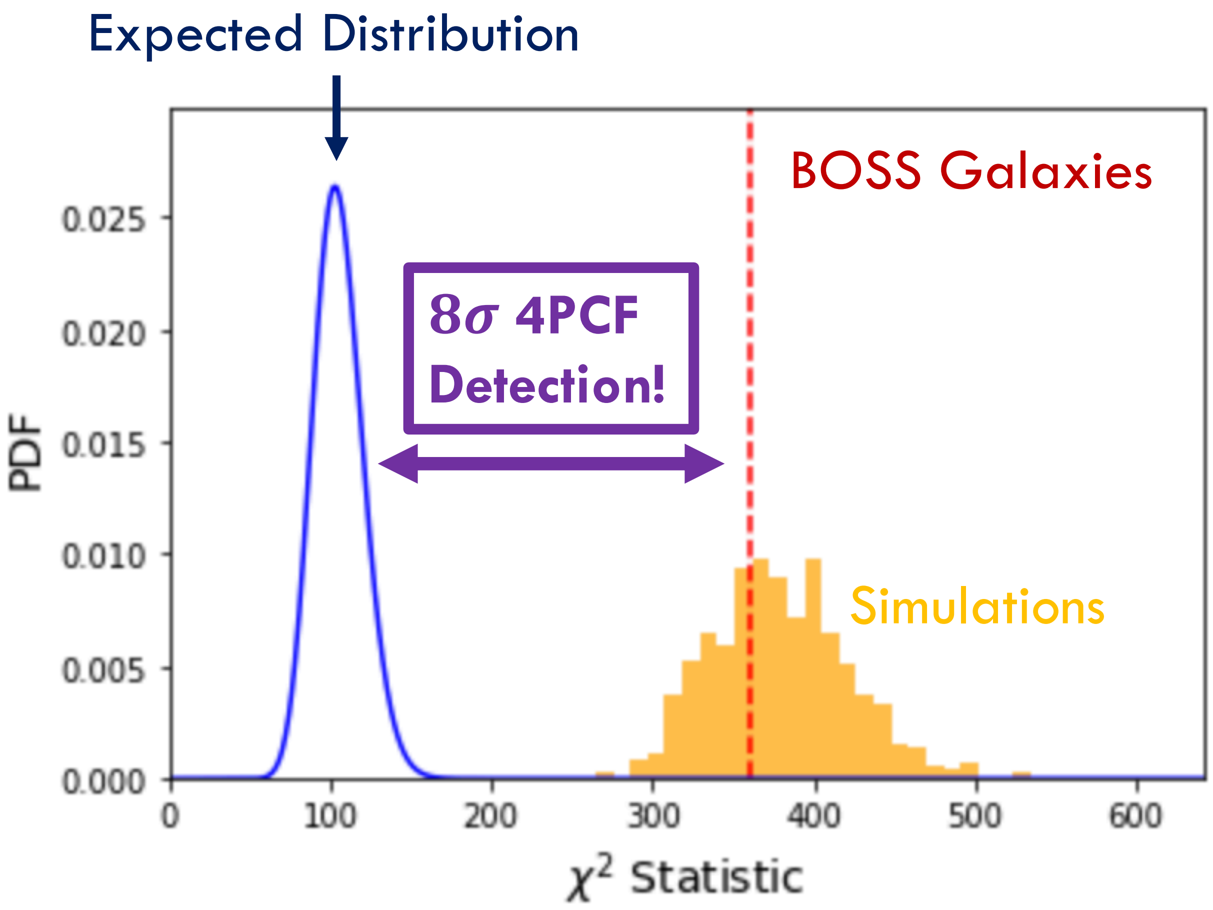
- Null Hypothesis: **4PCF = 0.**



Philcox+21

CAN WE DETECT THE GRAVITATIONAL 4PCF?

- Perform a χ^2-test to search for a **gravitational** 4PCF
- Null Hypothesis: **4PCF = 0.**
- **Strong** detection of non-Gaussianity!



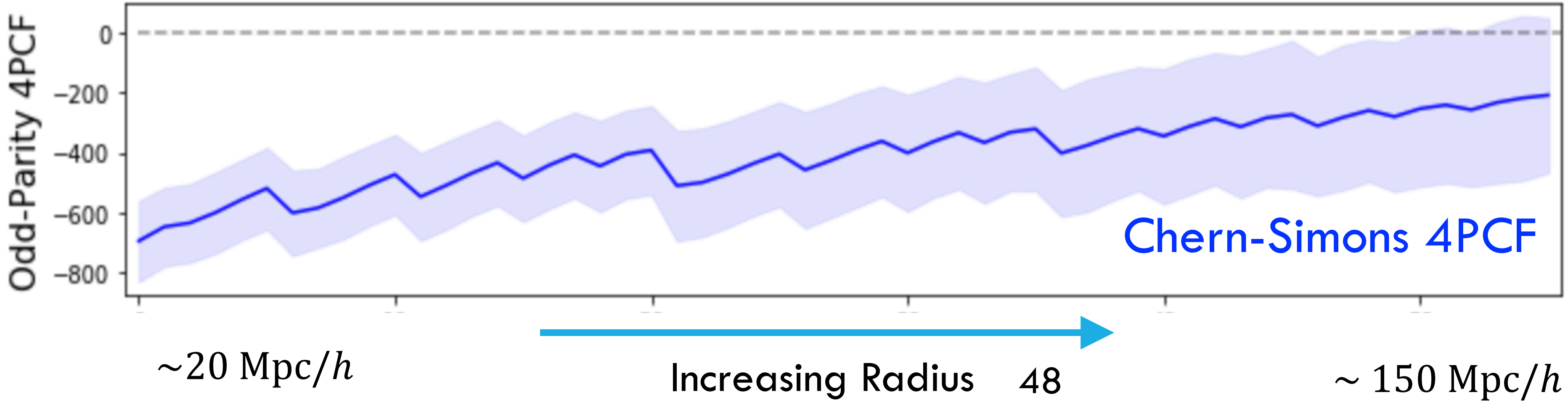
Philcox+21

WHAT'S NEXT FOR THE 4-POINT FUNCTION?

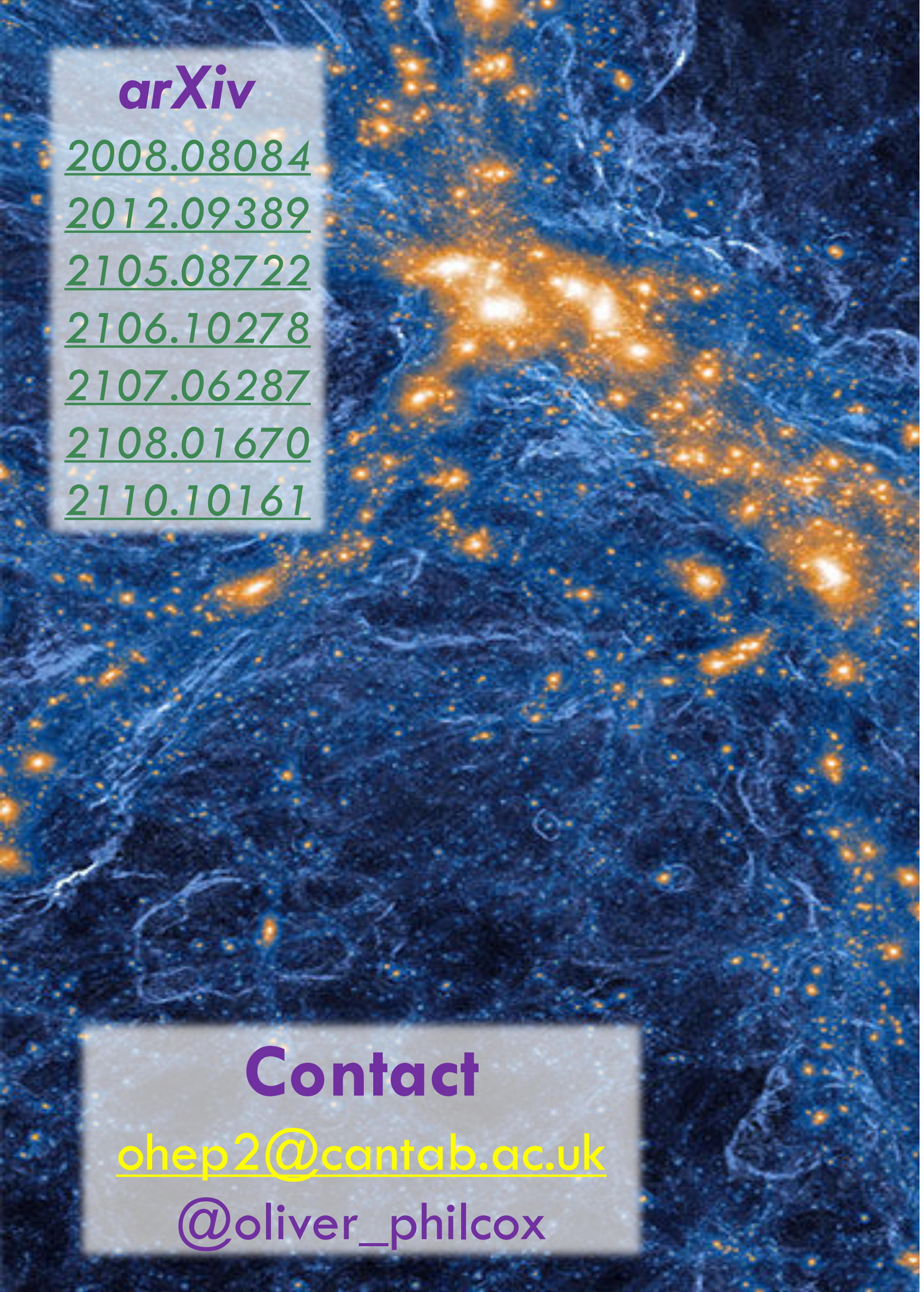
- Create a **theory** model and **quantify** information content:
  - Allows **ΛCDM information** to be extracted
- Search for **parity-violating** physics in the BOSS 4PCF

$$\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) - \mathbb{P}[\zeta_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)]$$

- Apply to **DESI** data [2× higher precision] and combine with the **CMB**



Cahn+21, **Philcox** (in prep.)



CONCLUSIONS

- Non-Gaussian statistics:
  1. **Sharpen** cosmological constraints
  2. Probe **non-standard** physics in the early Universe

- **Fast** and **accurate** estimators now available

- Extract **more** information from LSS surveys **without** additional cost