



(See also d'Amico, Senatore, Lewandowski, Zhang)

Constraining Inflation with BOSS DR12

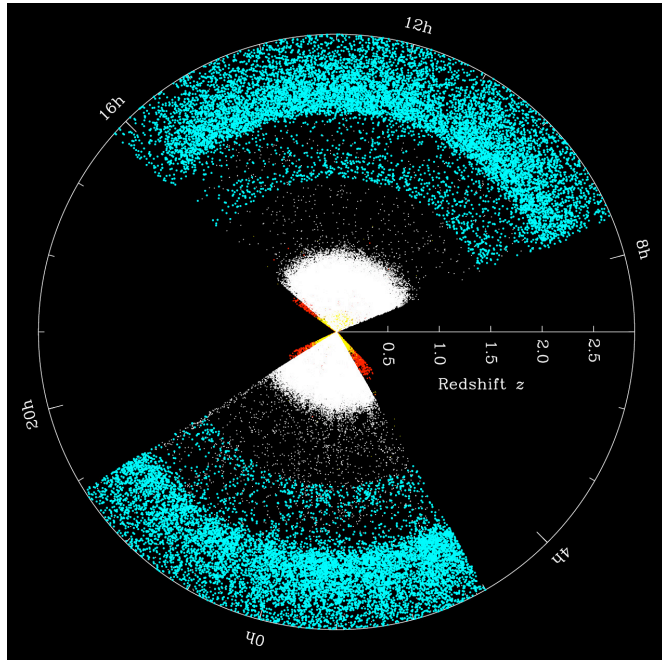
Oliver Philcox (Columbia / Simons Foundation)

PNG Workshop, September 2022

Collaborators:

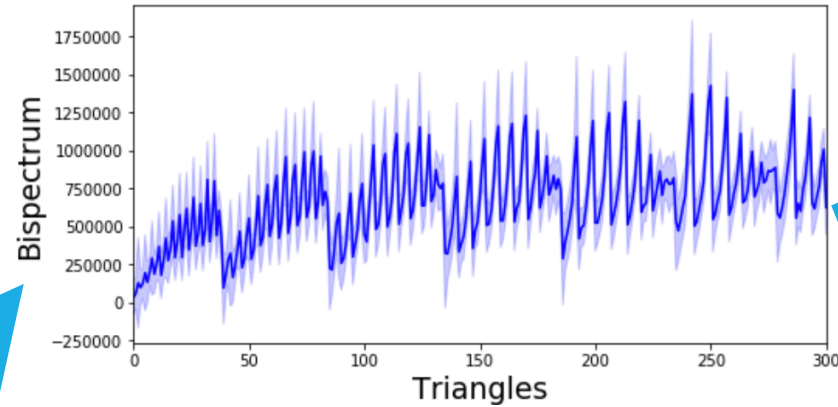
Mikhail Ivanov, Giovanni Cabass,
Marko Simonovic, Matias Zaldarriaga

FROM GALAXY SURVEYS TO INFLATION



Raw data

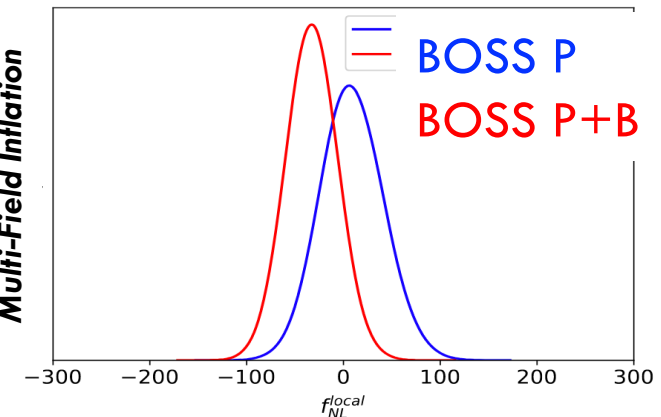
Summary statistics



Theory model

$$\begin{aligned}
 Z_1(q_1) &= K_1 + f\mu_1^2, & (A.3) \\
 Z_2(q_1, q_2) &= K_2(q_1, q_2) + f\mu_{12}^2 G_2(q_1, q_2) + \frac{f\mu_{12}q_{12}}{2} K_1 \left[\frac{\mu_1}{q_1} + \frac{\mu_2}{q_2} \right] + \frac{(f\mu_{12}q_{12})^2}{2} \frac{\mu_1 \mu_2}{q_1 q_2}, \\
 Z_3(q_1, q_2, q_3) &= K_3(q_1, q_2, q_3) + f\mu_{123}^2 G_3(q_1, q_2, q_3) \\
 &\quad + (f\mu_{123}q_{123}) \left[\frac{\mu_{12}}{q_{12}} K_1 G_2(q_1, q_2) + \frac{\mu_3}{q_3} K_2(q_1, q_2) \right] \\
 &\quad + \frac{(f\mu_{123}q_{123})^2}{2} \left[2 \frac{\mu_{12} \mu_3}{q_{12} q_3} G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_1 \right] + \frac{(f\mu_{123}q_{123})^3}{6} \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3}, \\
 Z_4(q_1, q_2, q_3, q_4) &= K_4(q_1, q_2, q_3, q_4) + f\mu_{1234}^2 G_4(q_1, q_2, q_3, q_4) \\
 &\quad + (f\mu_{1234}q_{1234}) \left[\frac{\mu_{123}}{q_{123}} K_1 G_3(q_1, q_2, q_3) + \frac{\mu_4}{q_4} K_3(q_1, q_2, q_3) \right. \\
 &\quad \quad \left. + \frac{\mu_{12}}{q_{12}} G_2(q_1, q_2) K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^2}{2} \left[2 \frac{\mu_{123} \mu_4}{q_{123} q_4} G_3(q_1, q_2, q_3) + \frac{\mu_{12} \mu_{34}}{q_{12} q_{34}} G_2(q_1, q_2) G_2(q_3, q_4) \right. \\
 &\quad \quad \left. + 2 \frac{\mu_{12} \mu_3}{q_{12} q_3} K_1 G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^3}{6} \left[3 \frac{\mu_{12} \mu_3 \mu_4}{q_{12} q_3 q_4} G_2(q_1, q_2) + \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3} K_1 \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^4}{24} \frac{\mu_1 \mu_2 \mu_3 \mu_4}{q_1 q_2 q_3 q_4},
 \end{aligned}$$

Multi-Field Inflation



f_{NL} bounds

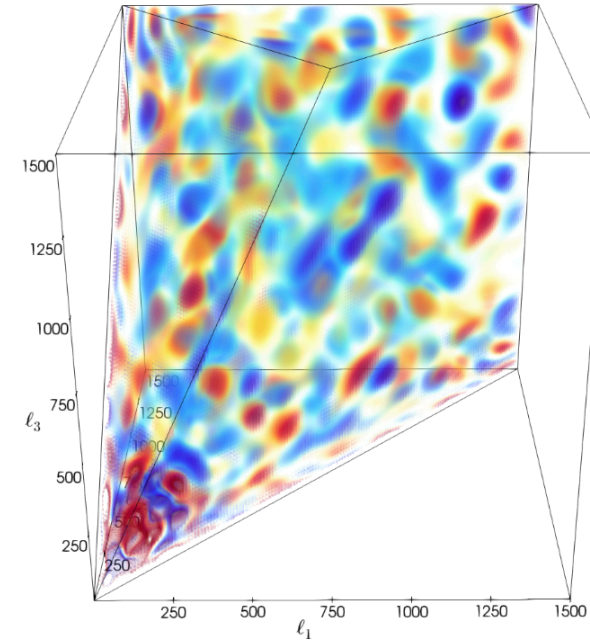
HOW CAN WE MEASURE f_{NL} ?

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

1. CMB Bispectrum

See Will's talk!

Planck TTT Bispectrum



$\approx 2\times$ better
with CMB-S4!

f_{NL} Constraints

Local	6.7 ± 5.6
Equilateral	6 ± 66
Orthogonal	-38 ± 36

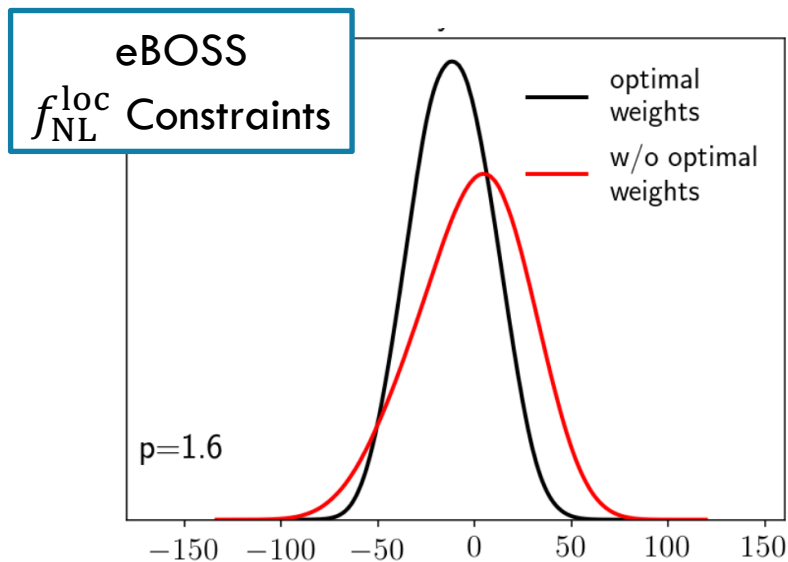
HOW CAN WE MEASURE f_{NL} ?

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

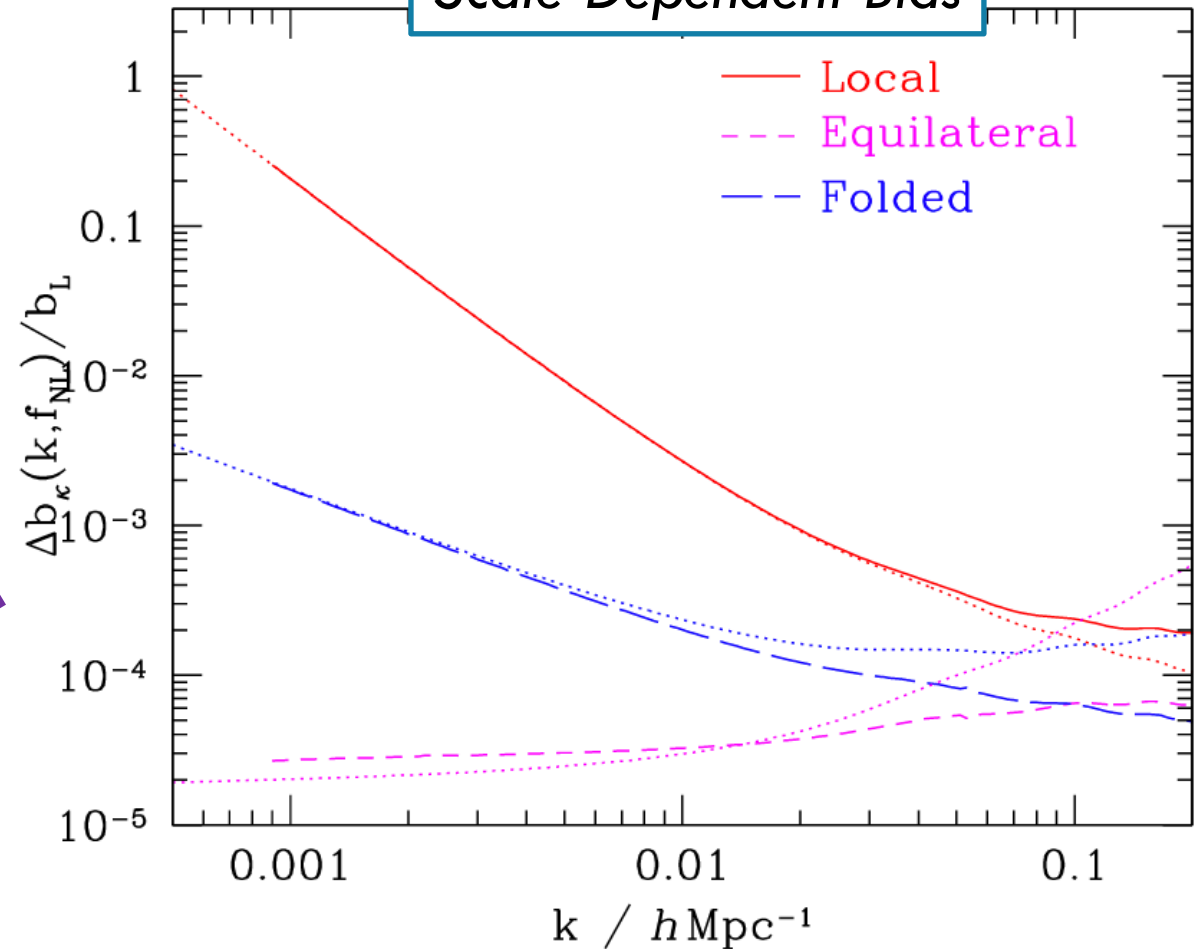
1. CMB Bispectrum

2. Galaxy Power Spectrum

See *Eva-Maria and others' talks!*



Scale-Dependent Bias



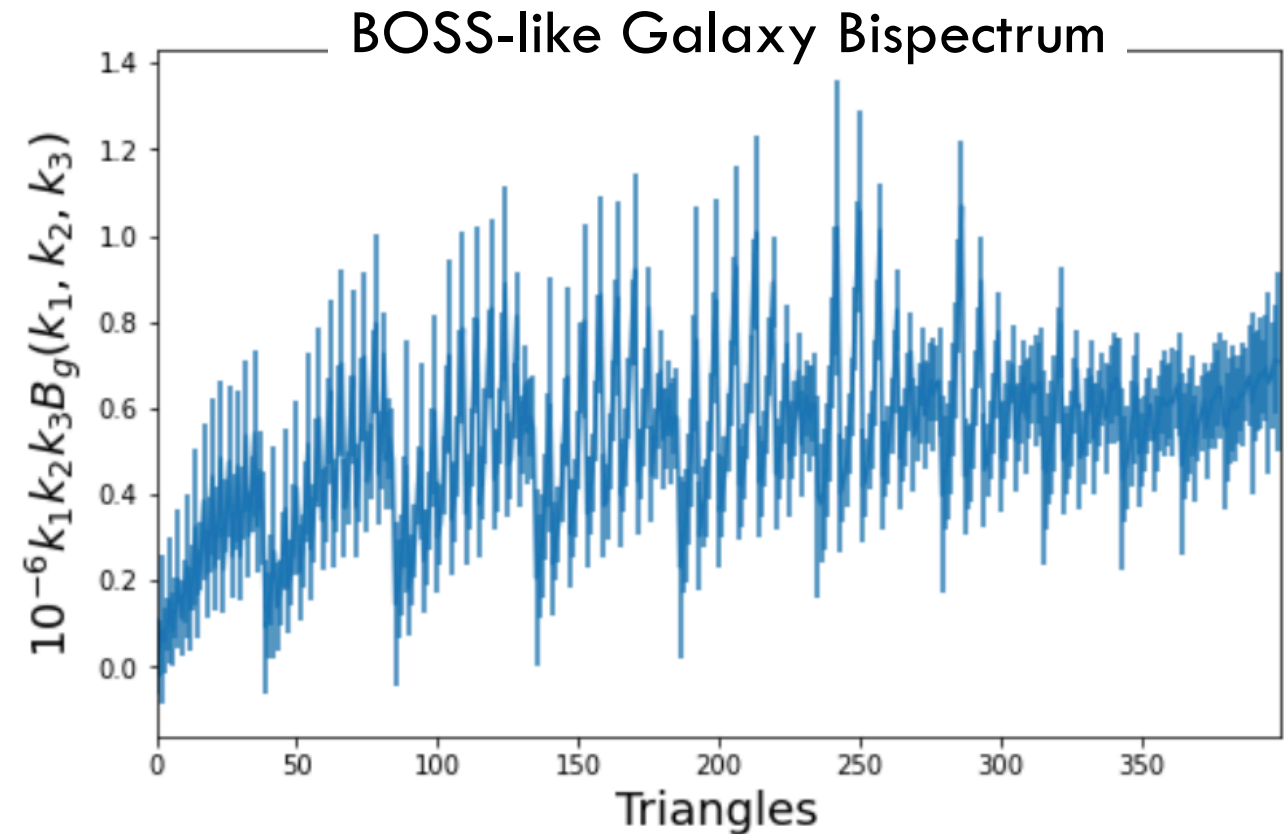
HOW CAN WE MEASURE f_{NL} ?

$$B_{\zeta}(\mathbf{k}_1, \mathbf{k}_2) \approx \frac{6}{5} f_{\text{NL}} P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.}$$

1. CMB Bispectrum
2. Galaxy Power Spectrum
3. Galaxy Bispectrum

See also Hector's talk!

We need a good theory model!



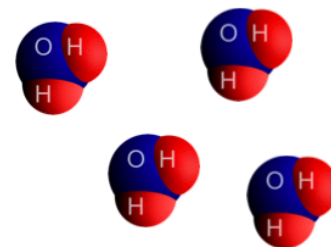
THE EFFECTIVE FIELD THEORY OF LSS

▷ **Analytic** theory for $\delta(\mathbf{x})$, based on the non-ideal **fluid equations**

$$\dot{v}^i + H v^i + v^j \delta_j v^i = \frac{1}{\rho} \delta_j \tau^{ij}$$

▷ A controlled Taylor series in k/k_{NL}
(or $k\sigma_{\text{FoG}}$, kR_{Halo})

▷ **Major Ingredient:** *Back-reaction* of small-scale physics on large-scale modes



large
scales



MODELLING PNG

Theory model requires:

- ▷ Primordial bispectrum:

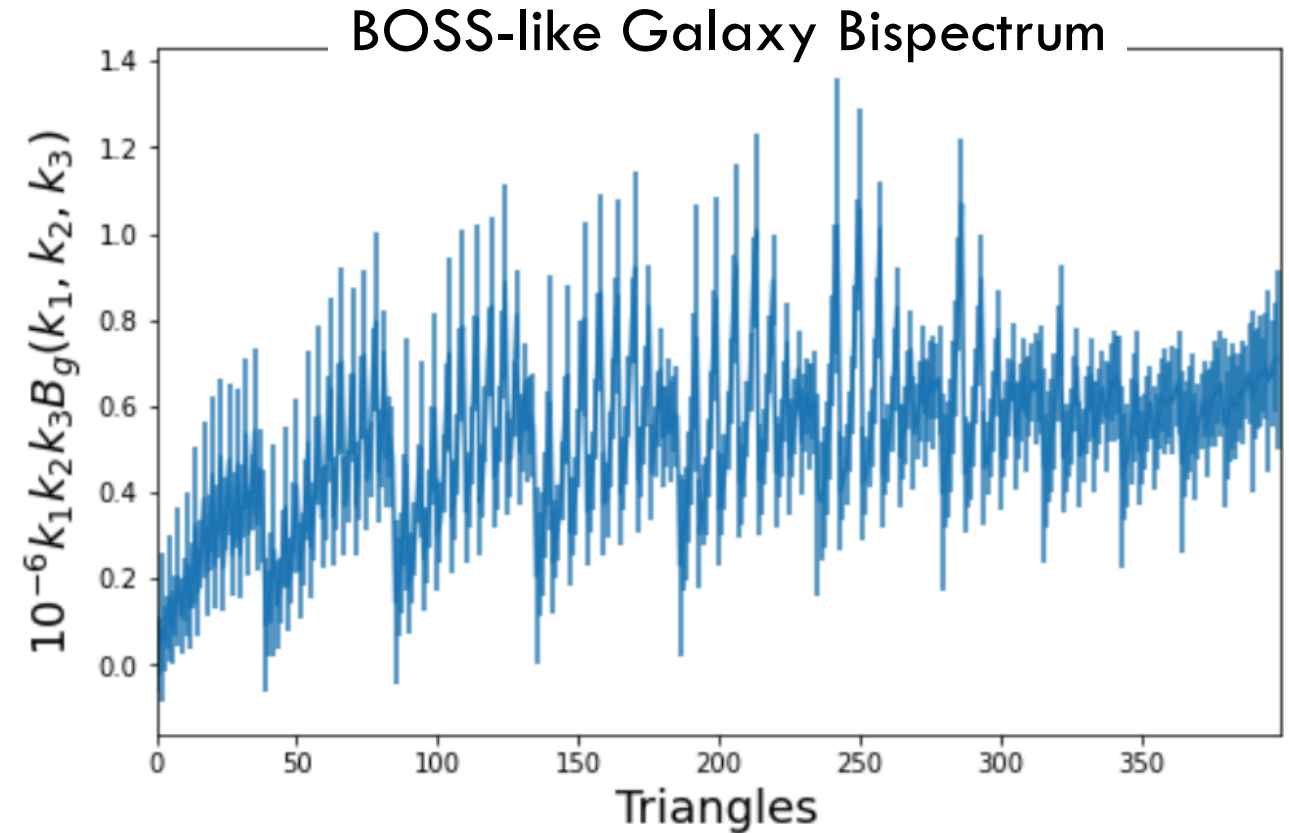
$$\langle \delta^{(1)} \delta^{(1)} \delta^{(1)} \rangle \sim f_{\text{NL}} P^2(k)$$

- ▷ Scale dependent bias:

$$b_1(f_{\text{NL}}) \rightarrow b_1 + (b_\phi f_{\text{NL}})/k^2$$

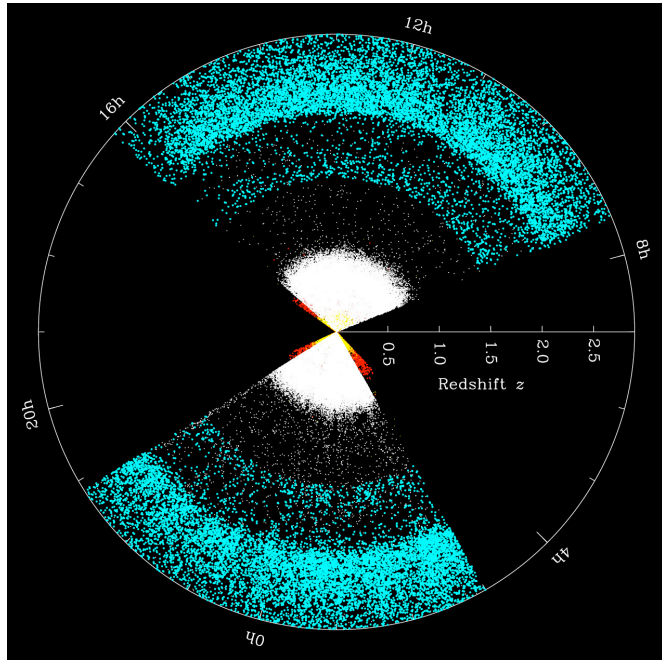
- ▷ Loop corrections:

$$P_{gg}(\mathbf{k}) \rightarrow P_{gg}(\mathbf{k}) + f_{\text{NL}} \int d\mathbf{q} \alpha P(\mathbf{q})P(\mathbf{k} - \mathbf{q})$$

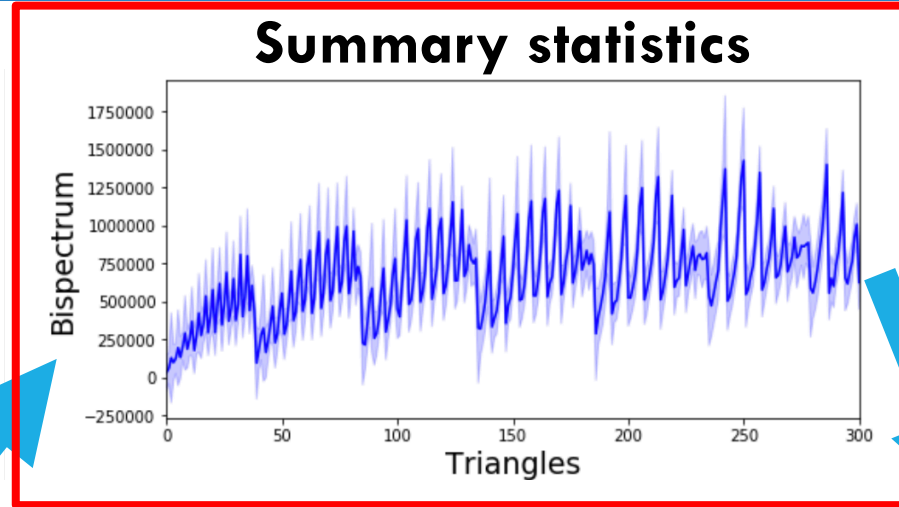


$$B_g = B_g(f_{\text{NL}}^{\text{eq}}, f_{\text{NL}}^{\text{orth}}, f_{\text{NL}}^{\text{loc}})$$

FROM GALAXY SURVEYS TO INFLATION



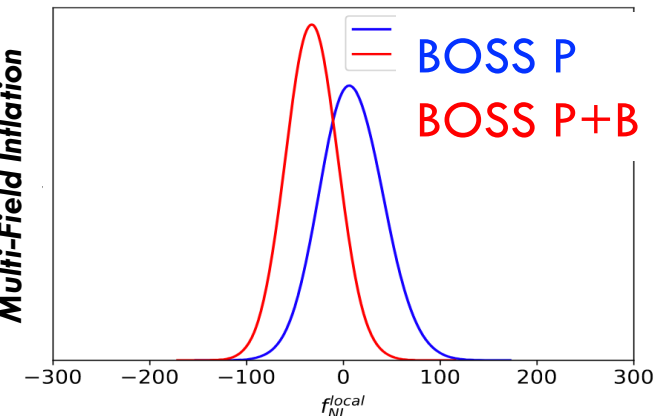
Raw data



Theory model

$$\begin{aligned}
 Z_1(q_1) &= K_1 + f\mu_1^2, & (A.3) \\
 Z_2(q_1, q_2) &= K_2(q_1, q_2) + f\mu_1^2 G_2(q_1, q_2) + \frac{f\mu_{12}q_{12}}{2} K_1 \left[\frac{\mu_1}{q_1} + \frac{\mu_2}{q_2} \right] + \frac{(f\mu_{12}q_{12})^2}{2} \frac{\mu_1 \mu_2}{q_1 q_2}, \\
 Z_3(q_1, q_2, q_3) &= K_3(q_1, q_2, q_3) + f\mu_{123}^2 G_3(q_1, q_2, q_3) \\
 &\quad + (f\mu_{123}q_{123}) \left[\frac{\mu_{12}}{q_{12}} K_1 G_2(q_1, q_2) + \frac{\mu_3}{q_3} K_2(q_1, q_2) \right] \\
 &\quad + \frac{(f\mu_{123}q_{123})^2}{2} \left[2 \frac{\mu_{12} \mu_3}{q_{12} q_3} G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_1 \right] + \frac{(f\mu_{123}q_{123})^3}{6} \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3}, \\
 Z_4(q_1, q_2, q_3, q_4) &= K_4(q_1, q_2, q_3, q_4) + f\mu_{1234}^2 G_4(q_1, q_2, q_3, q_4) \\
 &\quad + (f\mu_{1234}q_{1234}) \left[\frac{\mu_{123}}{q_{123}} K_1 G_3(q_1, q_2, q_3) + \frac{\mu_4}{q_4} K_3(q_1, q_2, q_3) \right. \\
 &\quad \quad \left. + \frac{\mu_{12}}{q_{12}} G_2(q_1, q_2) K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^2}{2} \left[2 \frac{\mu_{123} \mu_4}{q_{123} q_4} G_3(q_1, q_2, q_3) + \frac{\mu_{12} \mu_{34}}{q_{12} q_{34}} G_2(q_1, q_2) G_2(q_3, q_4) \right. \\
 &\quad \quad \left. + 2 \frac{\mu_{12} \mu_3}{q_{12} q_3} K_1 G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^3}{6} \left[3 \frac{\mu_{12} \mu_3 \mu_4}{q_{12} q_3 q_4} G_2(q_1, q_2) + \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3} K_1 \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^4}{24} \frac{\mu_1 \mu_2 \mu_3 \mu_4}{q_1 q_2 q_3 q_4},
 \end{aligned}$$

Multi-Field Inflation



f_{NL} bounds

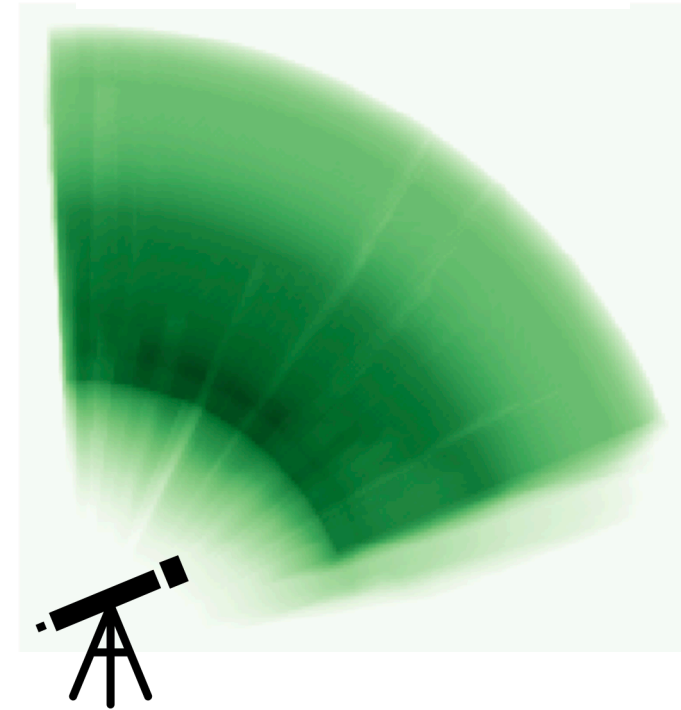
HOW TO MEASURE A BISPECTRUM

We usually measure the **window-convolved** bispectrum

$$B_g(\mathbf{k}_1, \mathbf{k}_2) \rightarrow \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2) B_g(\mathbf{p}_1, \mathbf{p}_2)$$

Understanding this is **crucial** for getting robust f_{NL} bounds

Survey Mask, $W(\mathbf{r})$



HOW TO MEASURE A BISPECTRUM

We usually measure the **window-convolved** bispectrum

$$B_g(\mathbf{k}_1, \mathbf{k}_2) \rightarrow \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2) B_g(\mathbf{p}_1, \mathbf{p}_2)$$

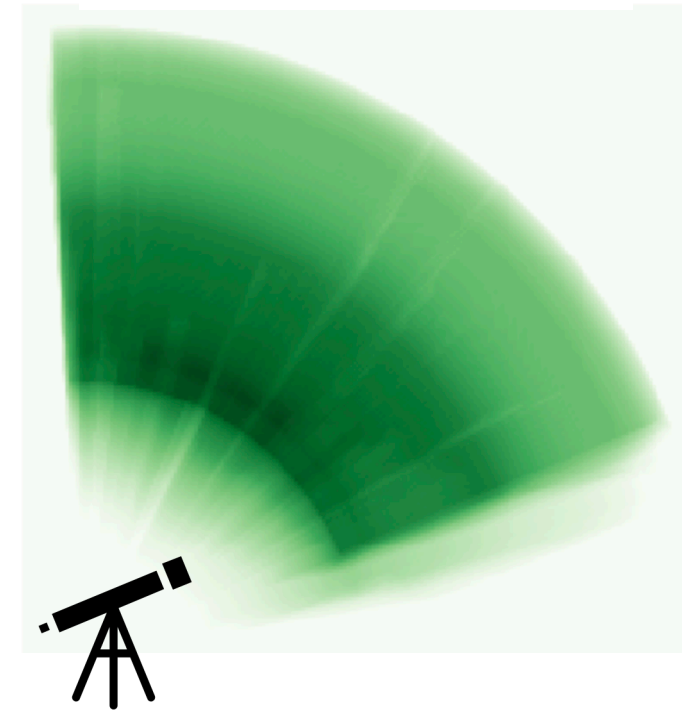
Understanding this is **crucial** for getting robust f_{NL} bounds

Three options:

1. Explicitly perform convolution integral [very expensive!]
2. Make approximations [robust?]
3. Circumvent the problem!

See Kevin's talk!

Survey Mask, $W(\mathbf{r})$



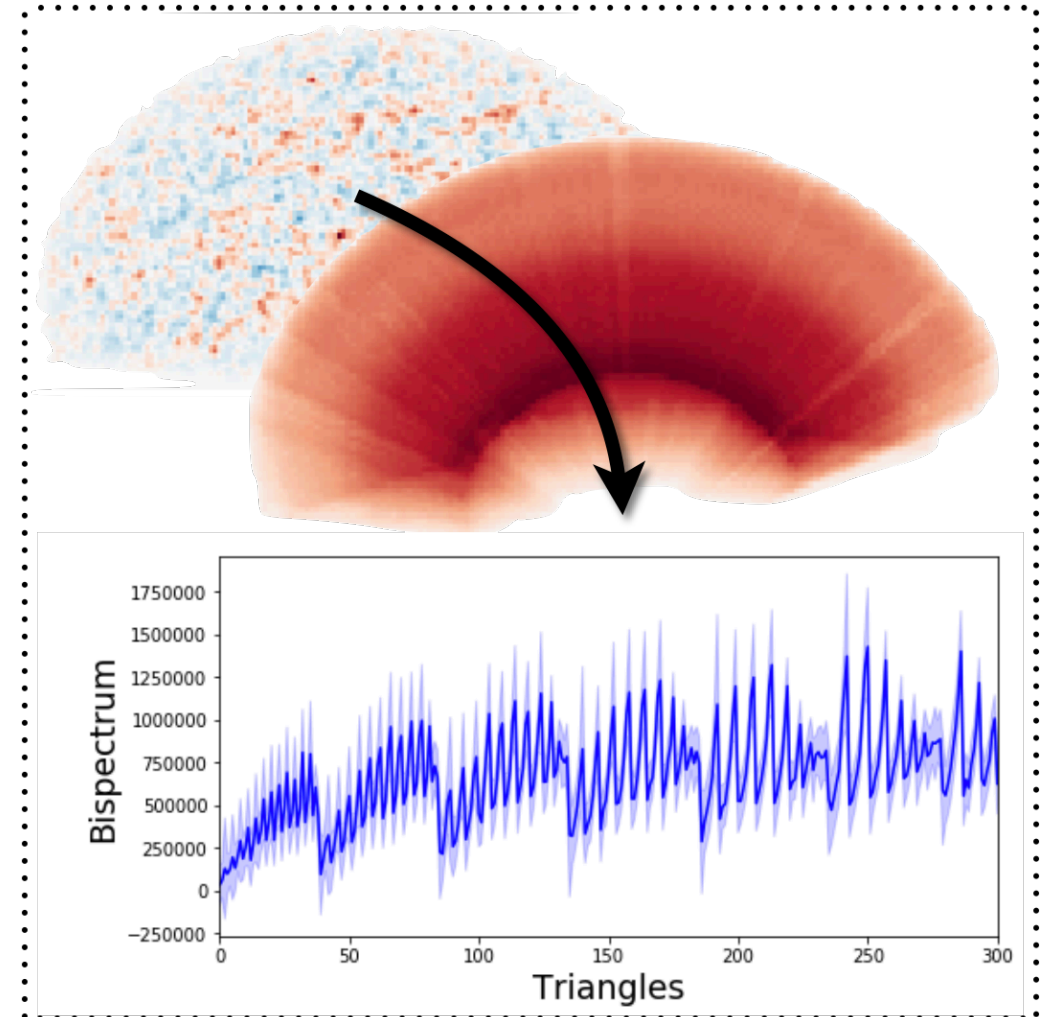
BISPECTRA WITHOUT WINDOWS

Estimate the **unwindowed** bispectrum directly

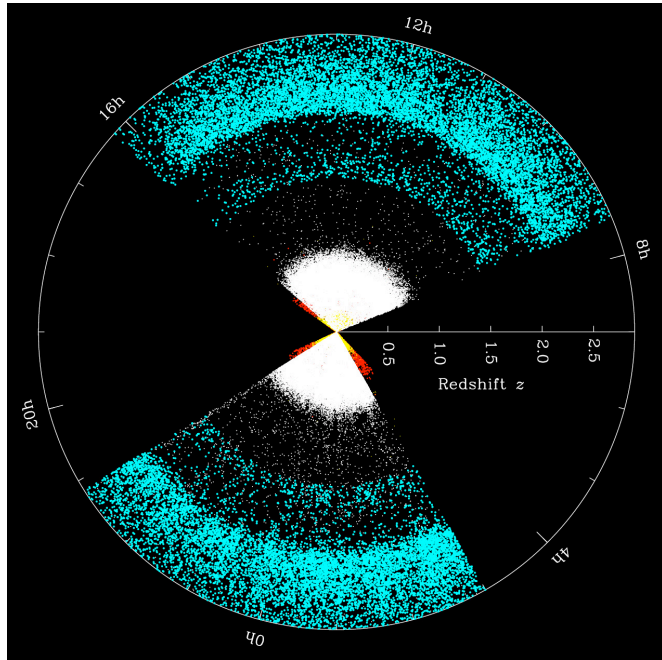
$$B_g^{\text{win}}(\mathbf{k}_1, \mathbf{k}_2) = \int_{\mathbf{p}_1 \mathbf{p}_2} W(\mathbf{k}_1 - \mathbf{p}_1) W(\mathbf{k}_2 - \mathbf{p}_2) W(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}_1 - \mathbf{k}_2) \boxed{B_g(\mathbf{p}_1, \mathbf{p}_2)}$$

We use a **maximum-likelihood** estimator for the **true** bispectrum

$$\nabla_{B_g} L[\text{data} | B_g] = 0 \quad \Rightarrow \quad \hat{B}_g = \dots$$

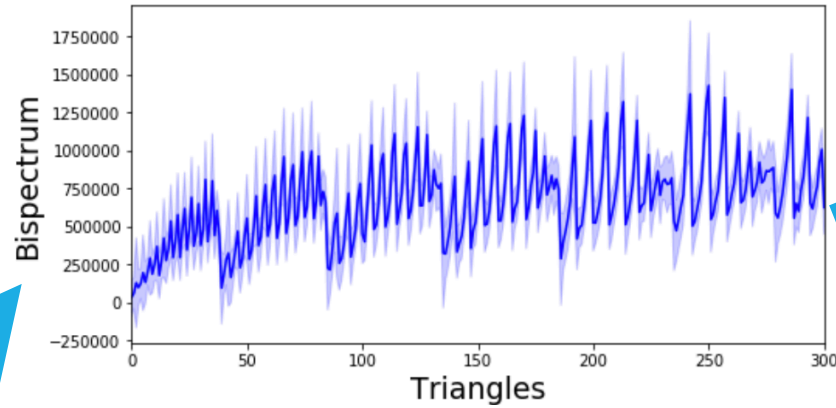


FROM GALAXY SURVEYS TO INFLATION



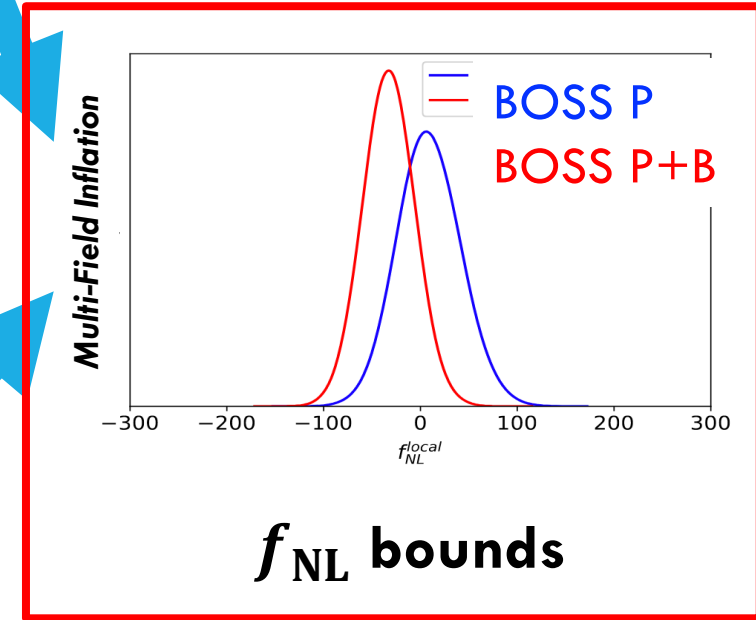
Raw data

Summary statistics



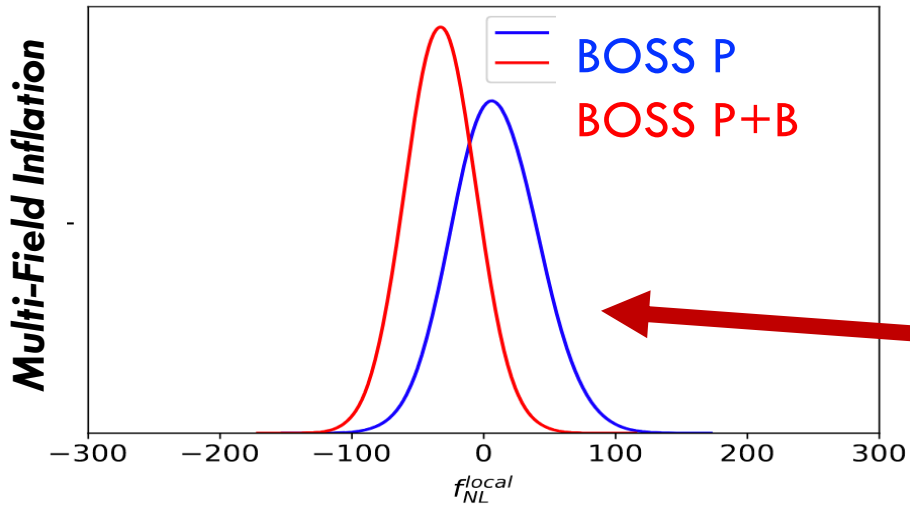
Theory model

$$\begin{aligned}
 Z_1(q_1) &= K_1 + f\mu_1^2, & (A.3) \\
 Z_2(q_1, q_2) &= K_2(q_1, q_2) + f\mu_{12}^2 G_2(q_1, q_2) + \frac{f\mu_{12}q_{12}}{2} K_1 \left[\frac{\mu_1}{q_1} + \frac{\mu_2}{q_2} \right] + \frac{(f\mu_{12}q_{12})^2}{2} \frac{\mu_1 \mu_2}{q_1 q_2}, \\
 Z_3(q_1, q_2, q_3) &= K_3(q_1, q_2, q_3) + f\mu_{123}^2 G_3(q_1, q_2, q_3) \\
 &\quad + (f\mu_{123}q_{123}) \left[\frac{\mu_{12}}{q_{12}} K_1 G_2(q_1, q_2) + \frac{\mu_3}{q_3} K_2(q_1, q_2) \right] \\
 &\quad + \frac{(f\mu_{123}q_{123})^2}{2} \left[2 \frac{\mu_{12} \mu_3}{q_{12} q_3} G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_1 \right] + \frac{(f\mu_{123}q_{123})^3}{6} \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3}, \\
 Z_4(q_1, q_2, q_3, q_4) &= K_4(q_1, q_2, q_3, q_4) + f\mu_{1234}^2 G_4(q_1, q_2, q_3, q_4) \\
 &\quad + (f\mu_{1234}q_{1234}) \left[\frac{\mu_{123}}{q_{123}} K_1 G_3(q_1, q_2, q_3) + \frac{\mu_4}{q_4} K_3(q_1, q_2, q_3) \right. \\
 &\quad \quad \left. + \frac{\mu_{12}}{q_{12}} G_2(q_1, q_2) K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^2}{2} \left[2 \frac{\mu_{123} \mu_4}{q_{123} q_4} G_3(q_1, q_2, q_3) + \frac{\mu_{12} \mu_{34}}{q_{12} q_{34}} G_2(q_1, q_2) G_2(q_3, q_4) \right. \\
 &\quad \quad \left. + 2 \frac{\mu_{12} \mu_3}{q_{12} q_3} K_1 G_2(q_1, q_2) + \frac{\mu_1 \mu_2}{q_1 q_2} K_2(q_3, q_4) \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^3}{6} \left[3 \frac{\mu_{12} \mu_3 \mu_4}{q_{12} q_3 q_4} G_2(q_1, q_2) + \frac{\mu_1 \mu_2 \mu_3}{q_1 q_2 q_3} K_1 \right] \\
 &\quad + \frac{(f\mu_{1234}q_{1234})^4}{24} \frac{\mu_1 \mu_2 \mu_3 \mu_4}{q_1 q_2 q_3 q_4},
 \end{aligned}$$



f_{NL} bounds

CONSTRAINTS ON f_{NL}



BOSS Power Spectrum + Bispectrum + $\mathcal{O}(f_{\text{NL}})$ Theory Model

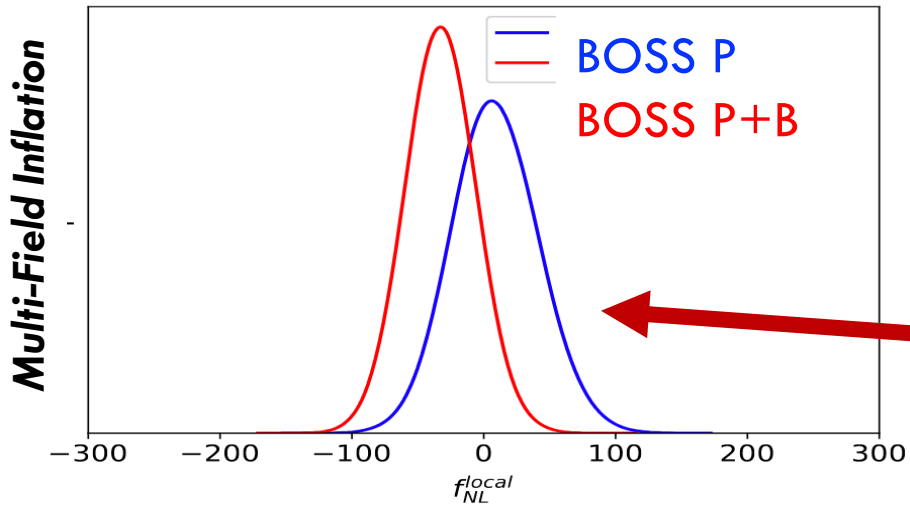
$$f_{\text{NL}}^{\text{local}} = -33 \pm 28$$

Really measuring $b_{\phi} f_{\text{NL}}$ - see Alex's talk!

All analysis is public:

github.com/oliverphilcox/full_shape_likelihoods

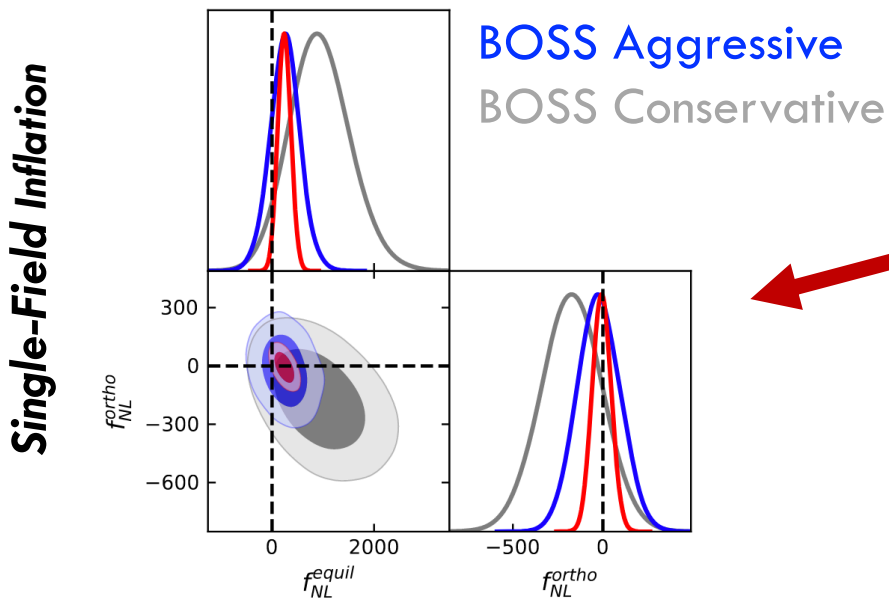
CONSTRAINTS ON f_{NL}



BOSS Power Spectrum + Bispectrum + $\mathcal{O}(f_{\text{NL}})$ Theory Model

$$f_{\text{NL}}^{\text{local}} = -33 \pm 28$$

Really measuring $b_\phi f_{\text{NL}}$ - see Alex's talk!



$$f_{\text{NL}}^{\text{equil}} = 260 \pm 300$$

$$f_{\text{NL}}^{\text{orth}} = -23 \pm 120$$

*- First measurement without CMB!
- Needs bispectrum!*

All analysis is public:

github.com/oliverphilcox/full_shape_likelihoods

CONSTRAINING INFLATION

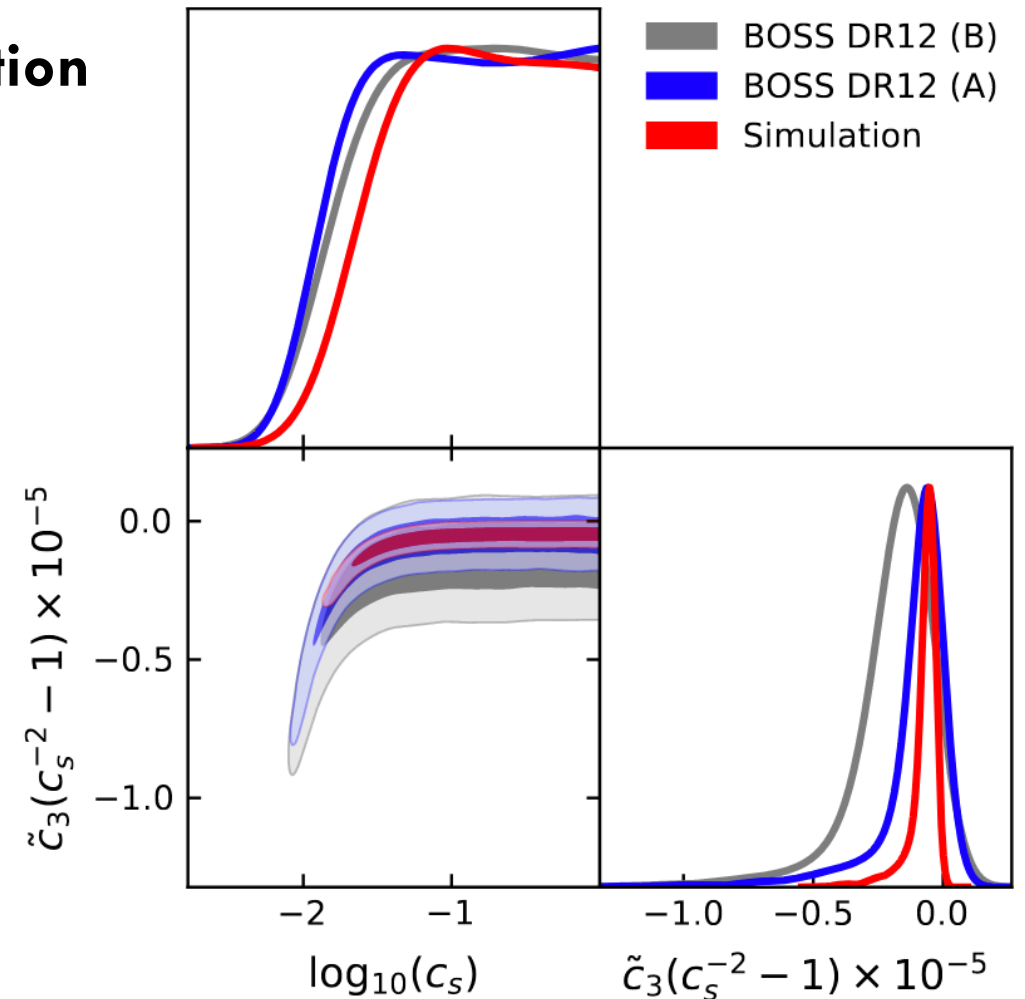
We can relate f_{NL} to the **couplings** in the **EFT of Inflation**

Most general 3rd order single-field action:

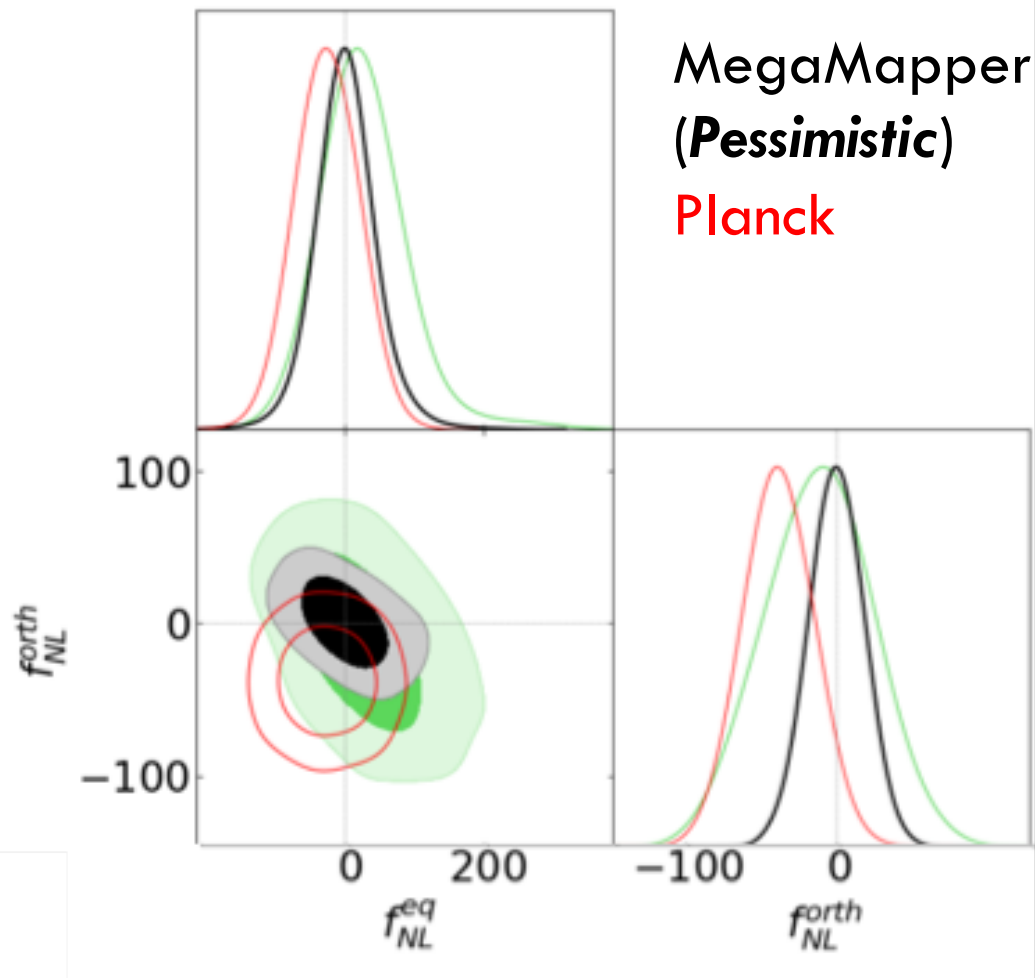
$$S_{\text{EFT}} = \int d^4x \sqrt{-g} \left[-\frac{M_{\text{P}}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\nabla \pi)^2}{a^2} \right) + \frac{M_{\text{P}}^2 \dot{H}}{c_s^2} (1 - c_s^2) \left(\frac{\dot{\pi} (\nabla \pi)^2}{a^2} - \left(1 + \frac{2 \tilde{c}_3}{3 c_s^2} \right) \dot{\pi}^3 \right) \right]$$

We find $c_s^2 \geq 0.013$ at 95% CL

See Ana's talk!



FUTURE PROSPECTS



- MegaMapper gets **better** non-local PNG constraints than *Planck*
- **Actual** constraints will use **higher** k_{max} :
 - Higher redshift
 - Better modelling

OTHER IDEAS: BEYOND PERTURBATION THEORY

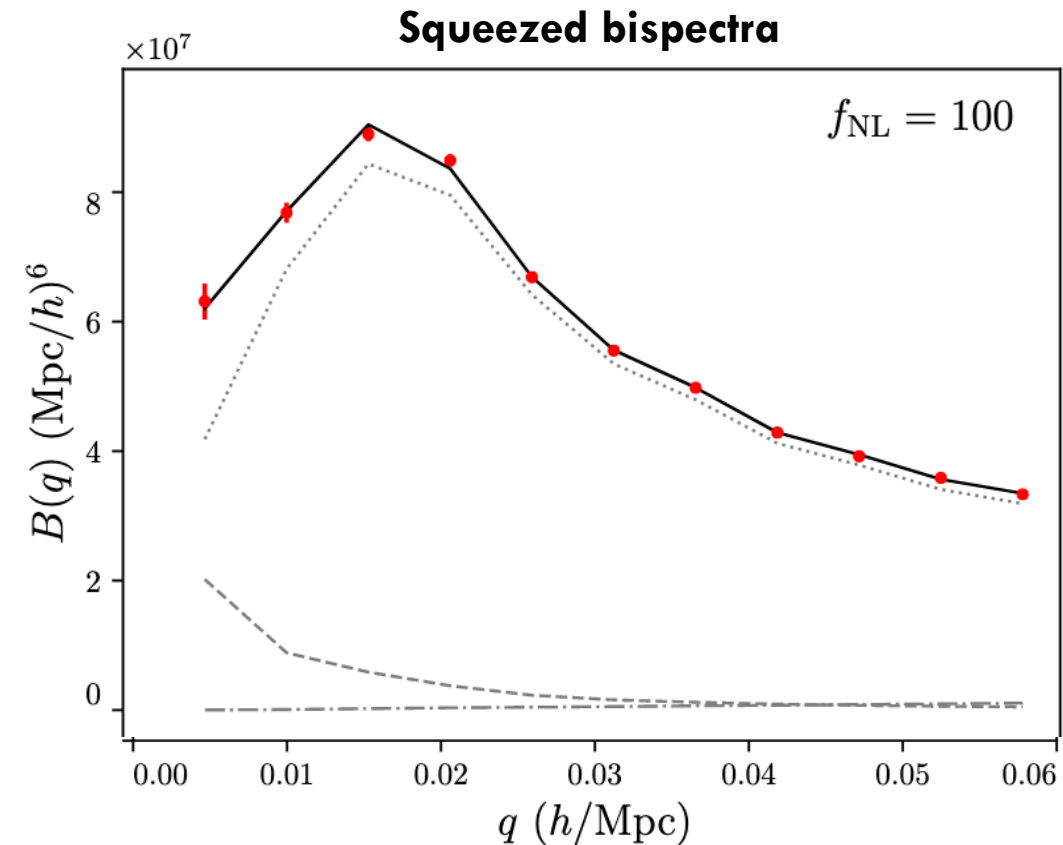


Sam Goldstein

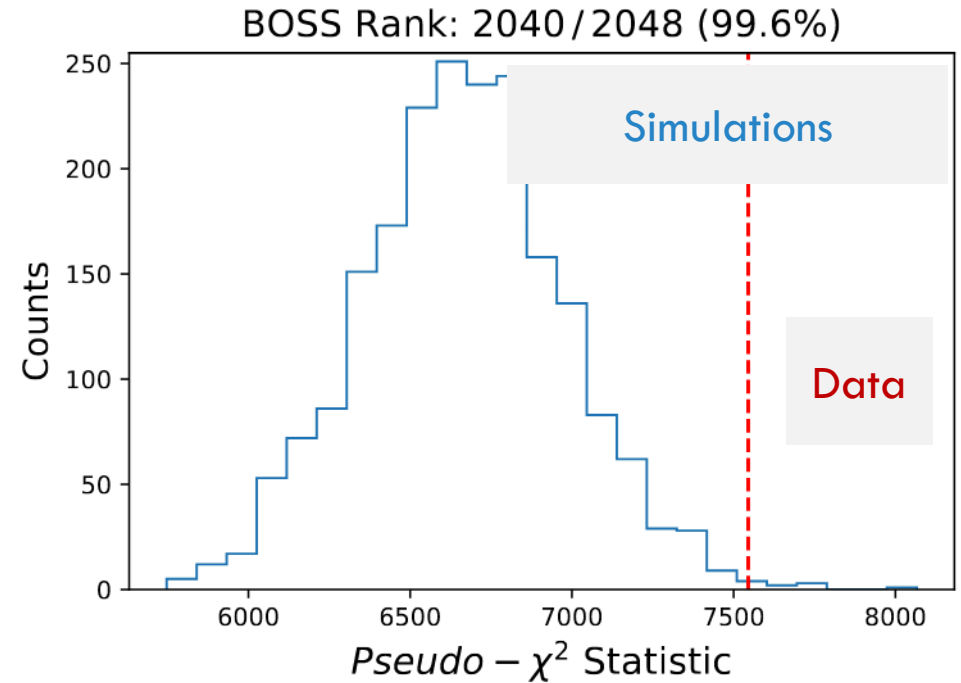
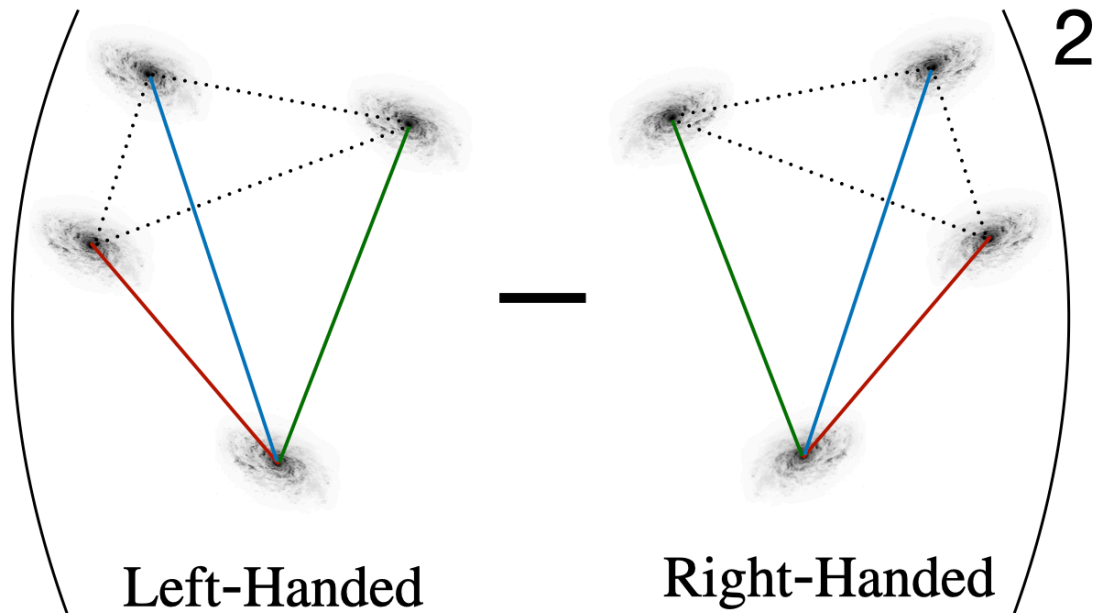
- ▶ We can measure **local** f_{NL} from the **non-linear** bispectrum using **consistency relations**

$$B(\mathbf{q}, \mathbf{k}) = \frac{6f_{\text{NL}}\Omega_{m,0}H_0^2}{D_{\text{md}}(z)} \frac{\partial P(k)}{\partial \log \sigma_8^2} \frac{P(q)}{q^2 T(q)} + \mathcal{O}(f_{\text{NL}}^2)$$

- ▶ This gives **accurate** $f_{\text{NL}}^{\text{loc}}$ constraints for matter at $k = 0.6h/\text{Mpc}$



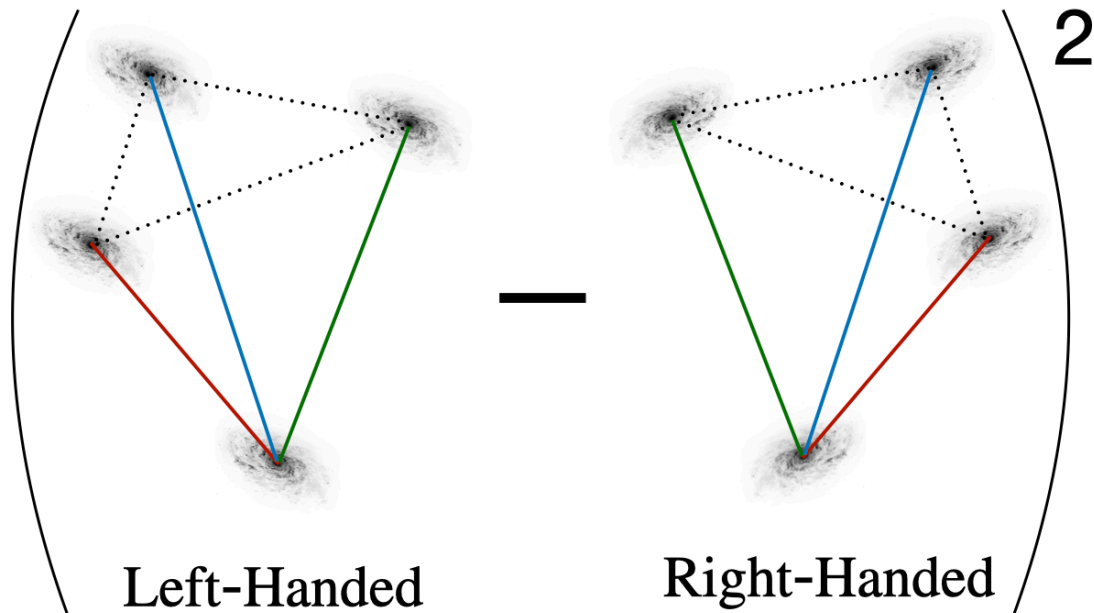
OTHER IDEAS: PARITY-ODD 4-POINT FUNCTIONS



Conclusions

- Simulations do not capture noise properties of the data
- Or we have detected *parity-violating inflation* at 3σ ???

BONUS: PARITY-ODD 4-POINT FUNCTIONS



NEWSLETTERS

Sign up to read our regular email newsletters

NewScientist

[News](#) [Podcasts](#) [Video](#) [Technology](#) [Space](#) [Physics](#) [Health](#) [More](#) [Shop](#) [Courses](#) [Events](#)

The universe is surprisingly lopsided and we don't know why

Two analyses of a million galaxies show that their distribution may not be symmetrical, which may mean that our understandings of gravity and the early universe are incorrect

Conclusions

- Simulations do not capture noise properties of the data
- Or we have detected *parity-violating inflation* at 3σ ???

CONCLUSIONS

- We can measure **any** type of 3-point PNG using the galaxy **power spectrum** and **bispectrum**
- Constraints are **weak** compared to the CMB but will get much stronger soon!
- We can learn more from non-perturbative physics and higher-point functions!

arXiv

[2107.06287](#)

[2201.07238](#)

[2204.01781](#)

[2206.04227](#)

[2209.06228](#)

Contact

ohp2@cantab.ac.uk

[@oliver_philcox](#)