

The Cosmological Collider

An Observational Perspective

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Inflationary Q&A

Big Unanswered Questions:

• What is the **energy scale** of inflation?



$$E \lesssim 10^{13} \text{TeV}$$

• What sets the **potential**?



$$V(\phi) = ???$$

• Were there **other fields** during inflation?



$$\phi \rightarrow \phi, \chi, \psi_u, \dots$$

• Did the fields **interact**?



$$\text{Lagrangian} \supset \dot{\phi}^3 + \dots$$

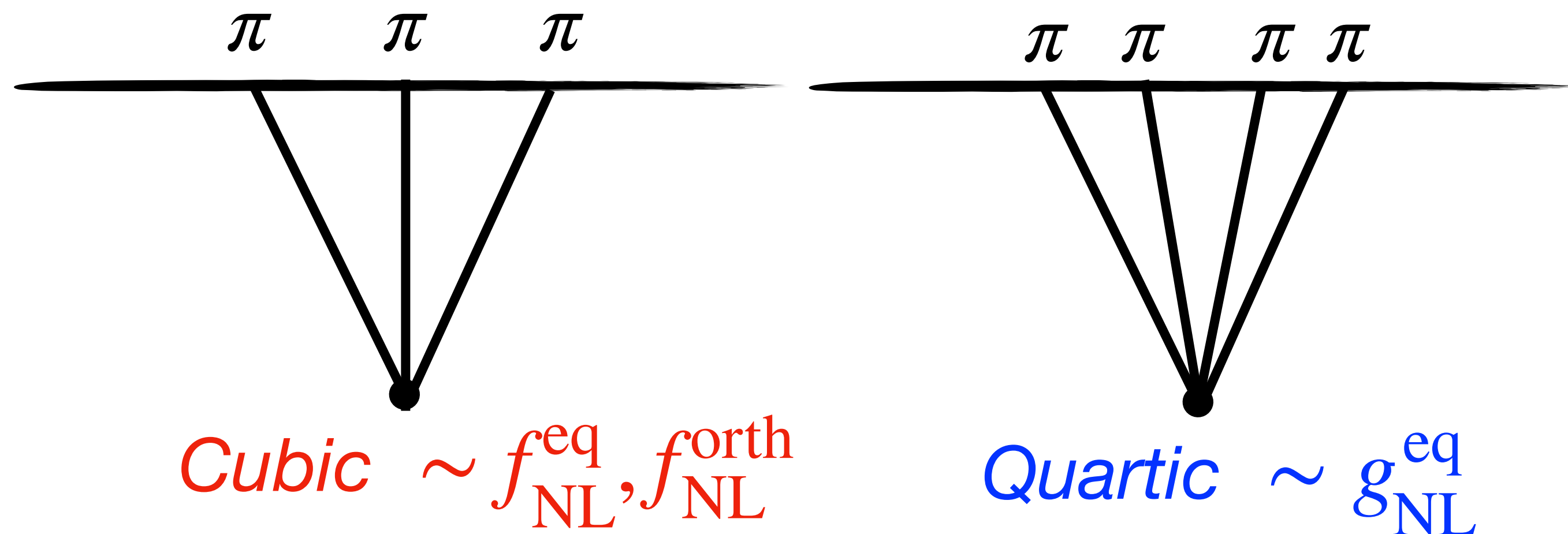
Primordial non-Gaussianity: $\zeta \sim e^{-\frac{1}{2}\zeta^2/P_\zeta} \left(1 + \zeta^3 B_\zeta + \zeta^4 T_\zeta + \dots \right)$

Self-Interactions

- Many models of inflation feature **self-interactions**:

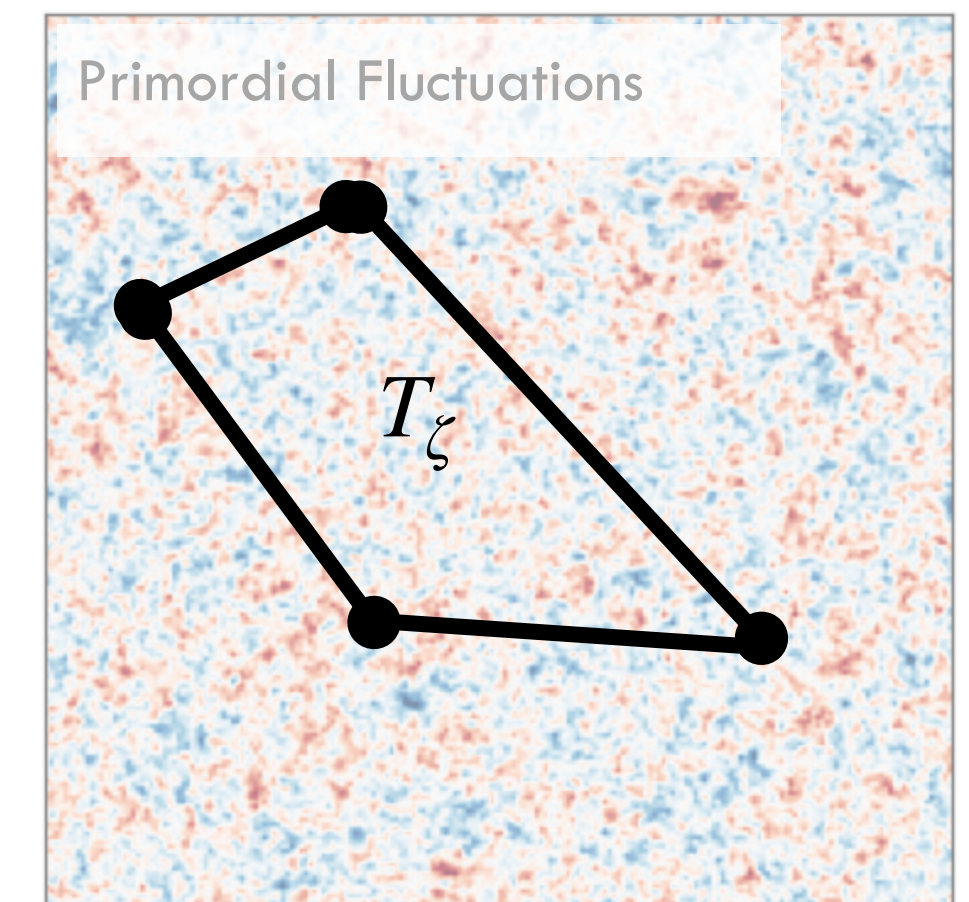
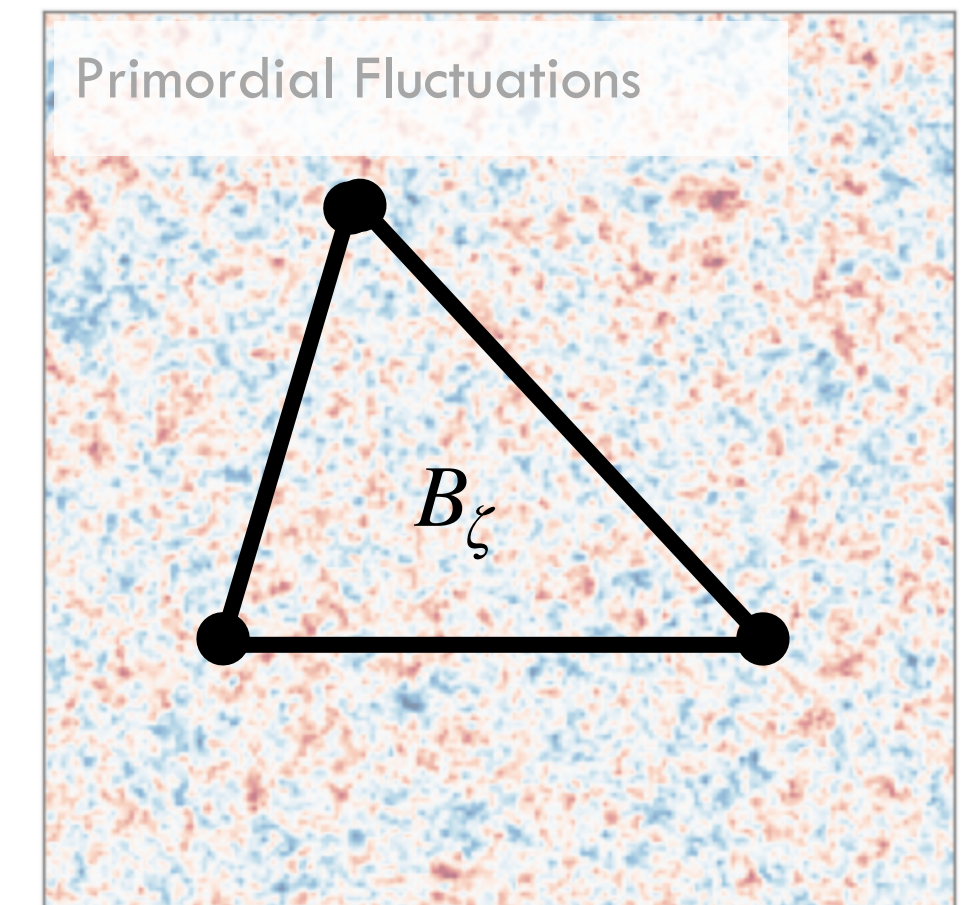
$$\mathcal{L} \supset \delta\dot{\pi}^3, \quad \delta\dot{\pi}(\partial\pi)^2, \quad \delta\dot{\pi}^4, \quad \dots$$

(π = Goldstone of broken time-translation)



- These lead to **three**- and **four**-point functions at the end of inflation
- The **shape** encodes the vertex, the **amplitude** encodes the microphysics

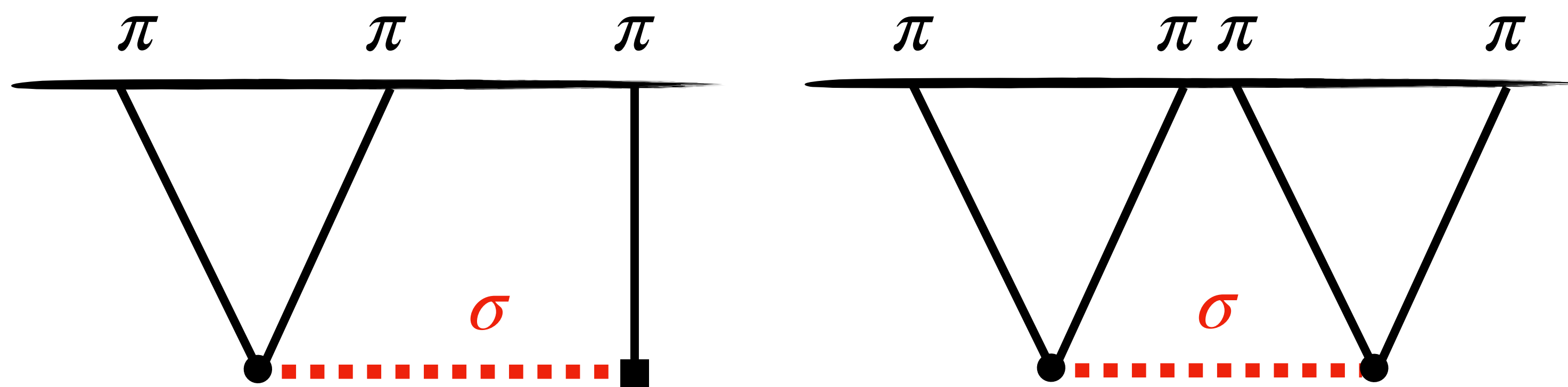
e.g. $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle \sim f_{\text{NL}}^{\text{eq}} \times \text{shape}$



New Degrees of Freedom

- Other models feature **new particles**, σ :

$$\mathcal{L} \supset \dot{\pi}\sigma, \quad \dot{\pi}^2\sigma, \quad (\partial_i\pi)^2\sigma, \quad \dots$$

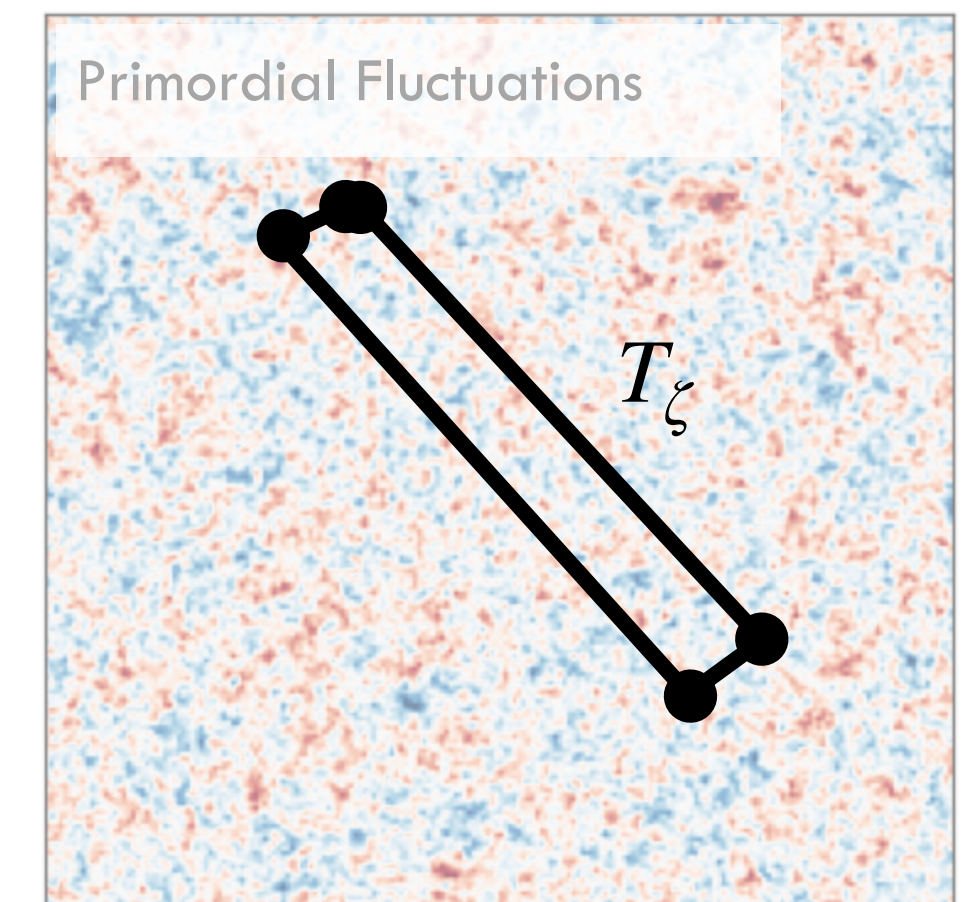
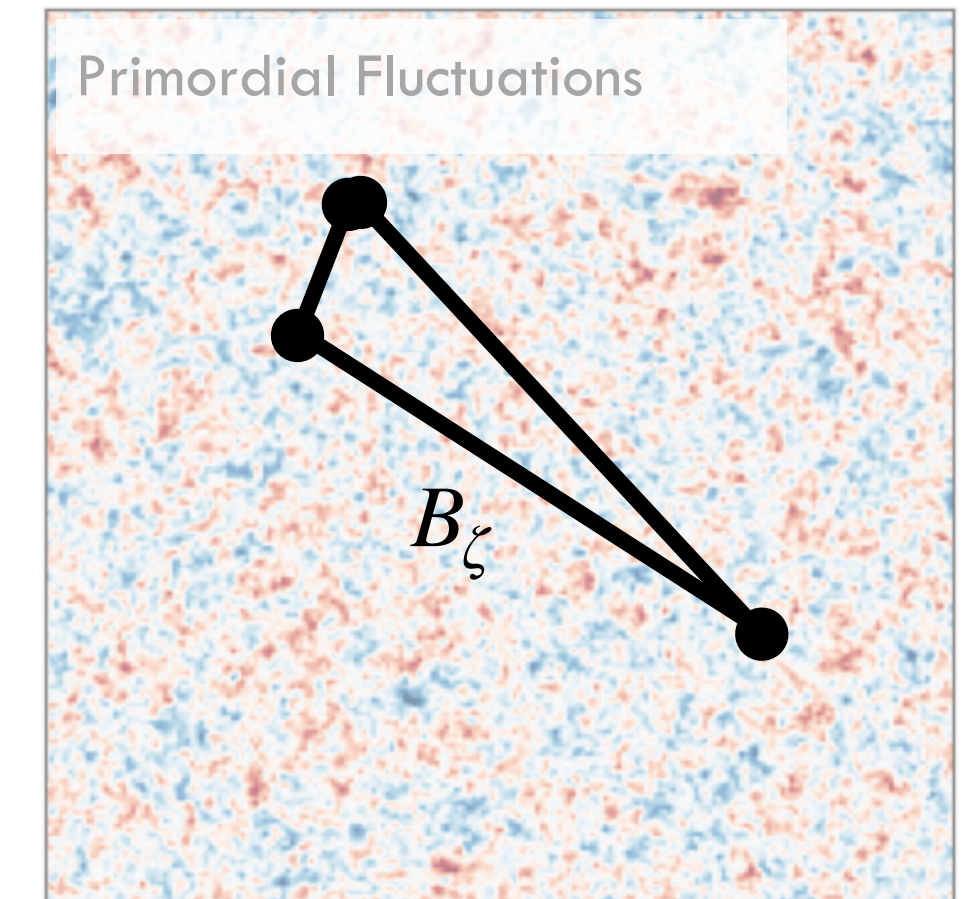


Linear-Quadratic $\sim f_{\text{NL}}^{\text{loc}}$

Quadratic² $\sim \tau_{\text{NL}}^{\text{loc}}$

- These lead to **three**- and **four**-point functions at the end of inflation
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e.g. $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle \sim f_{\text{NL}}^{\text{loc}} \times \text{shape}$



The Cosmological Collider

see Qianshu's talk!

- In the **squeezed limit** (low exchange momentum), the inflationary signatures are set by **symmetry**
- They depend **only** on the mass m_σ , the spin, s , and the speed c_σ *not* on the microphysical model!
- For the **bispectrum**:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle_{k_1 \ll k_3} \sim f_{\text{NL}}(m_\sigma, s, c_\sigma)$$

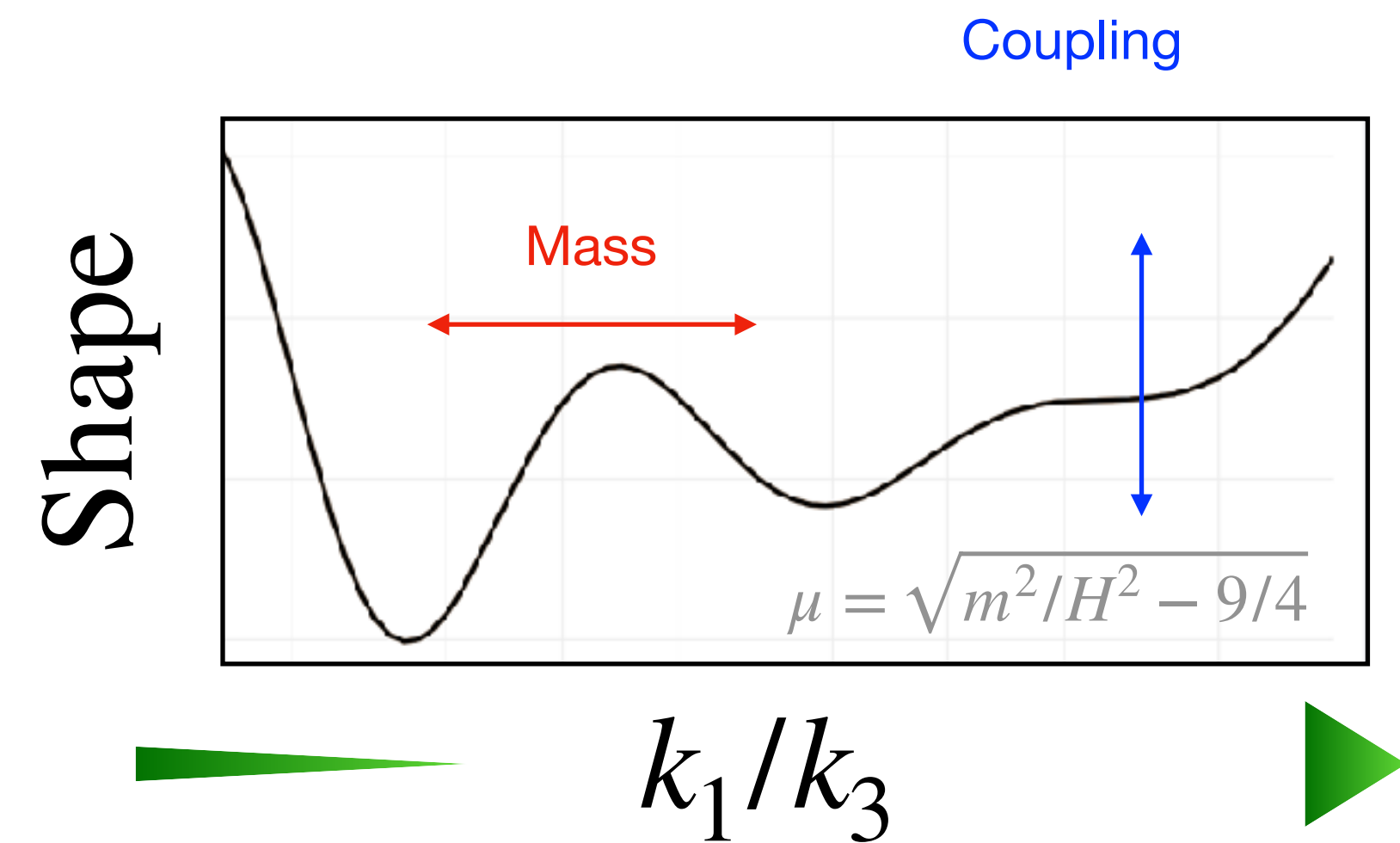
Amplitude $\sim e^{-\pi m_\sigma/H}$

$$\times \frac{1}{k_1^3 k_3^3} \left[\left(\frac{k_1}{k_3} \right)^{3/2+i\mu} + \left(\frac{k_1}{k_3} \right)^{3/2-i\mu} \right]$$

Shape (mass dependent)

$$\times \mathcal{L}_s(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3)$$

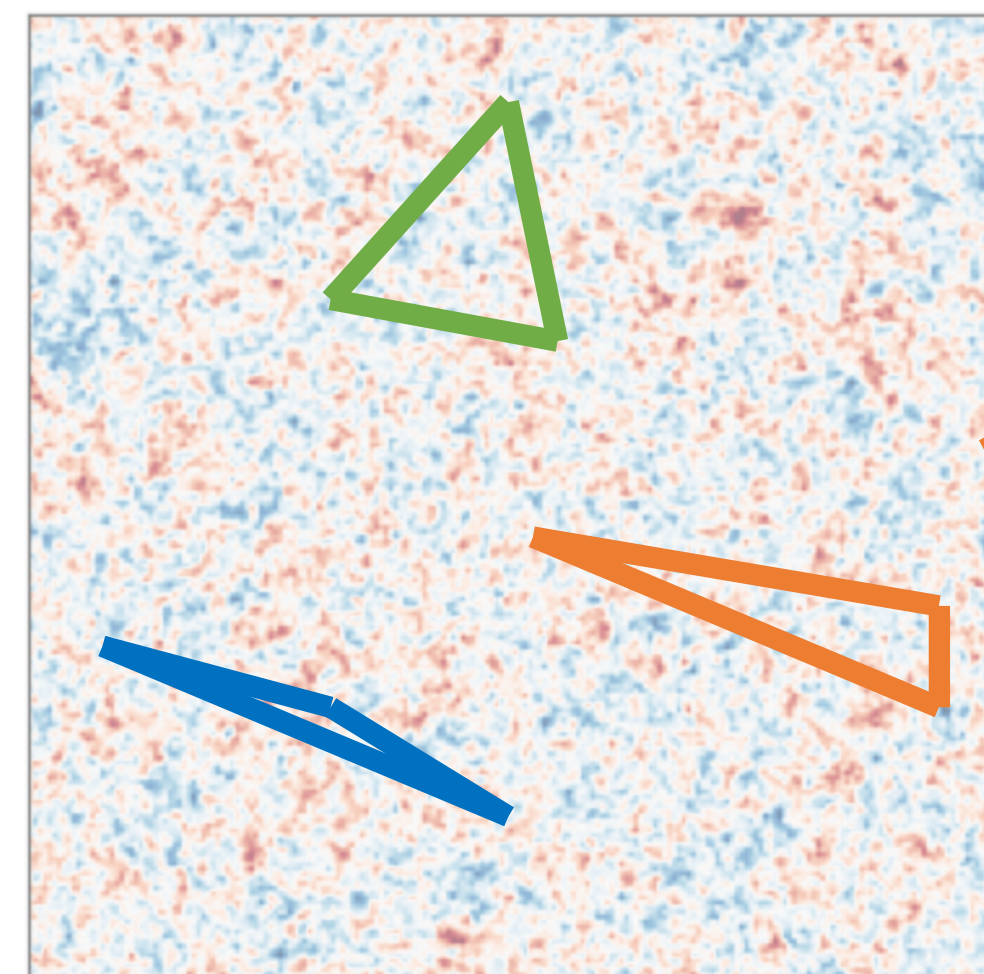
Angle (spin-dependent)



- These **oscillations** are a **smoking-gun** for new physics

How to Constrain Inflation

The **curvature perturbation** ζ sets the **initial conditions** for the late Universe!

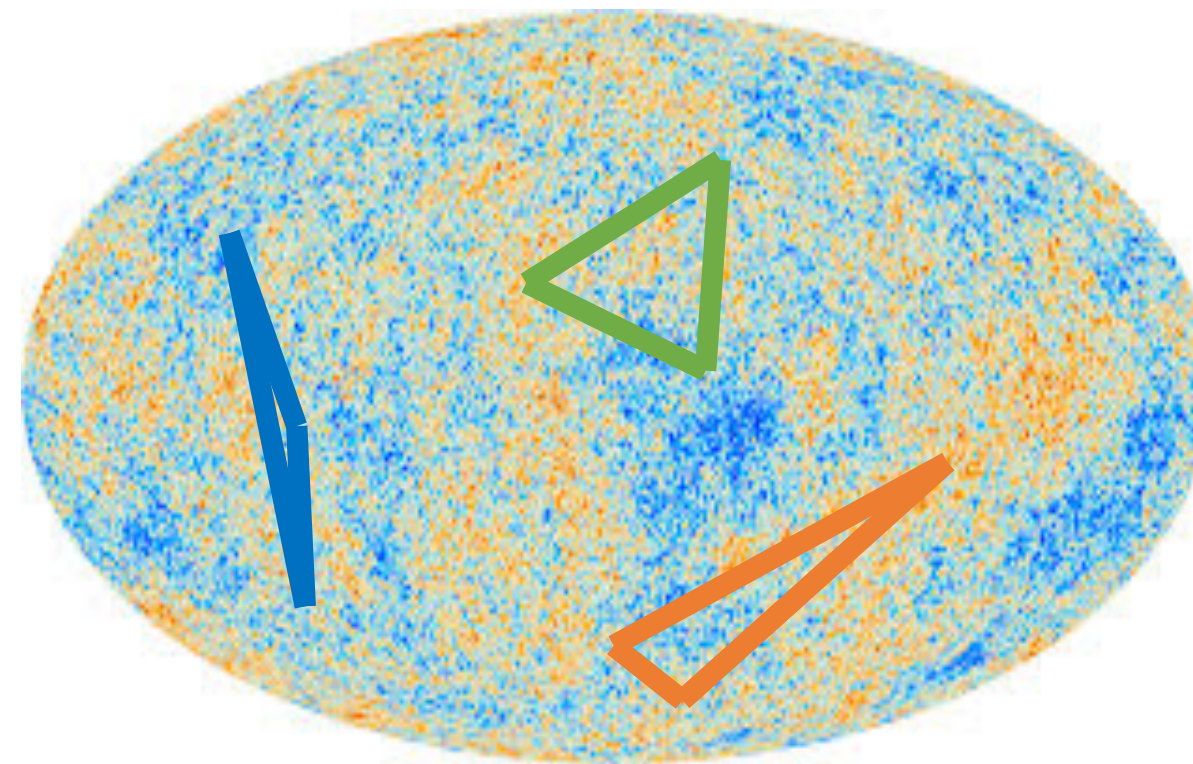


Primordial Curvature

$$\langle \zeta^n \rangle \neq 0?$$

Linear Physics

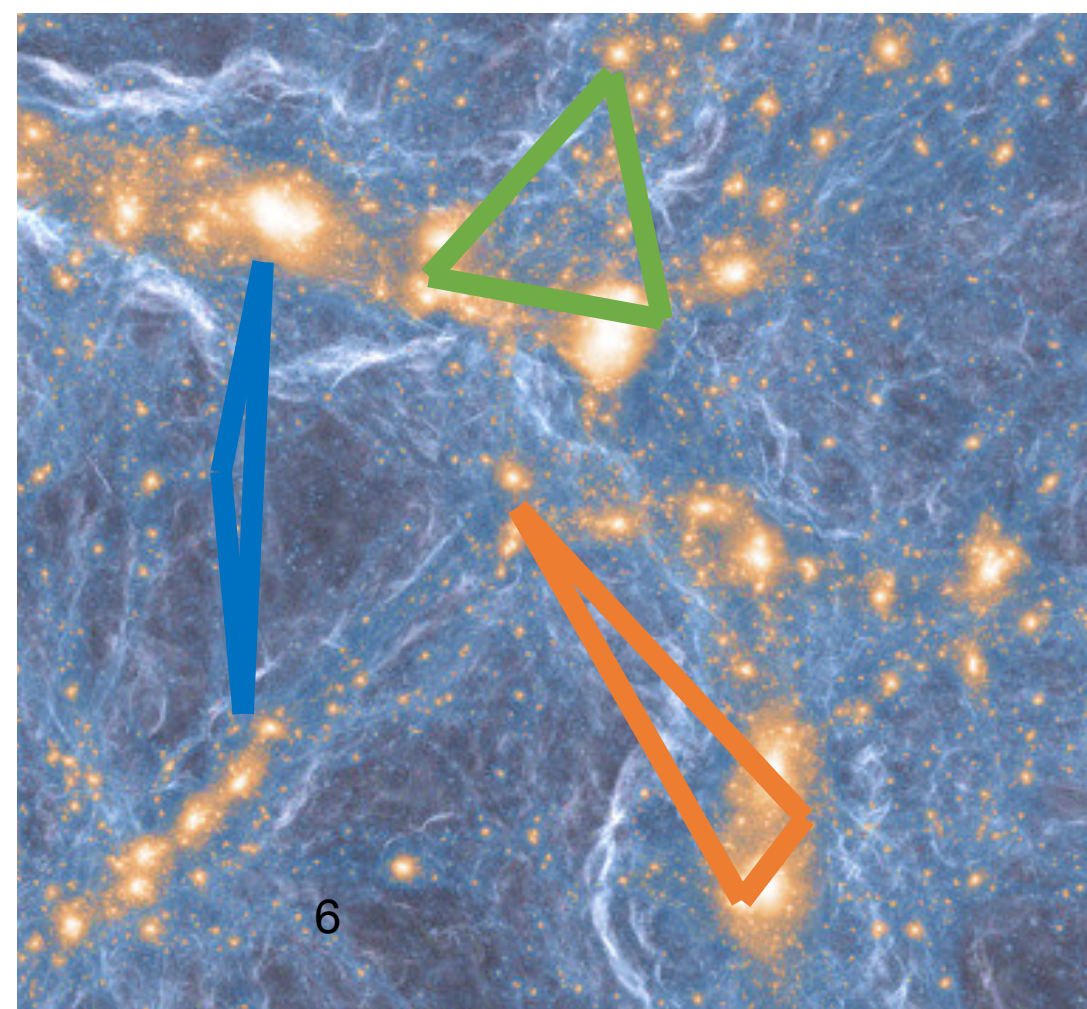
Non-Linear Physics



Cosmic Microwave Background fluctuations

$$\langle \delta T^n \rangle \neq 0?$$

(tracing **photon energies**)



Galaxy Number Density Fluctuations

$$\langle \delta n_{\text{galaxy}}^n \rangle \neq 0?$$

(tracing **dark matter**)

How to Constrain Inflation

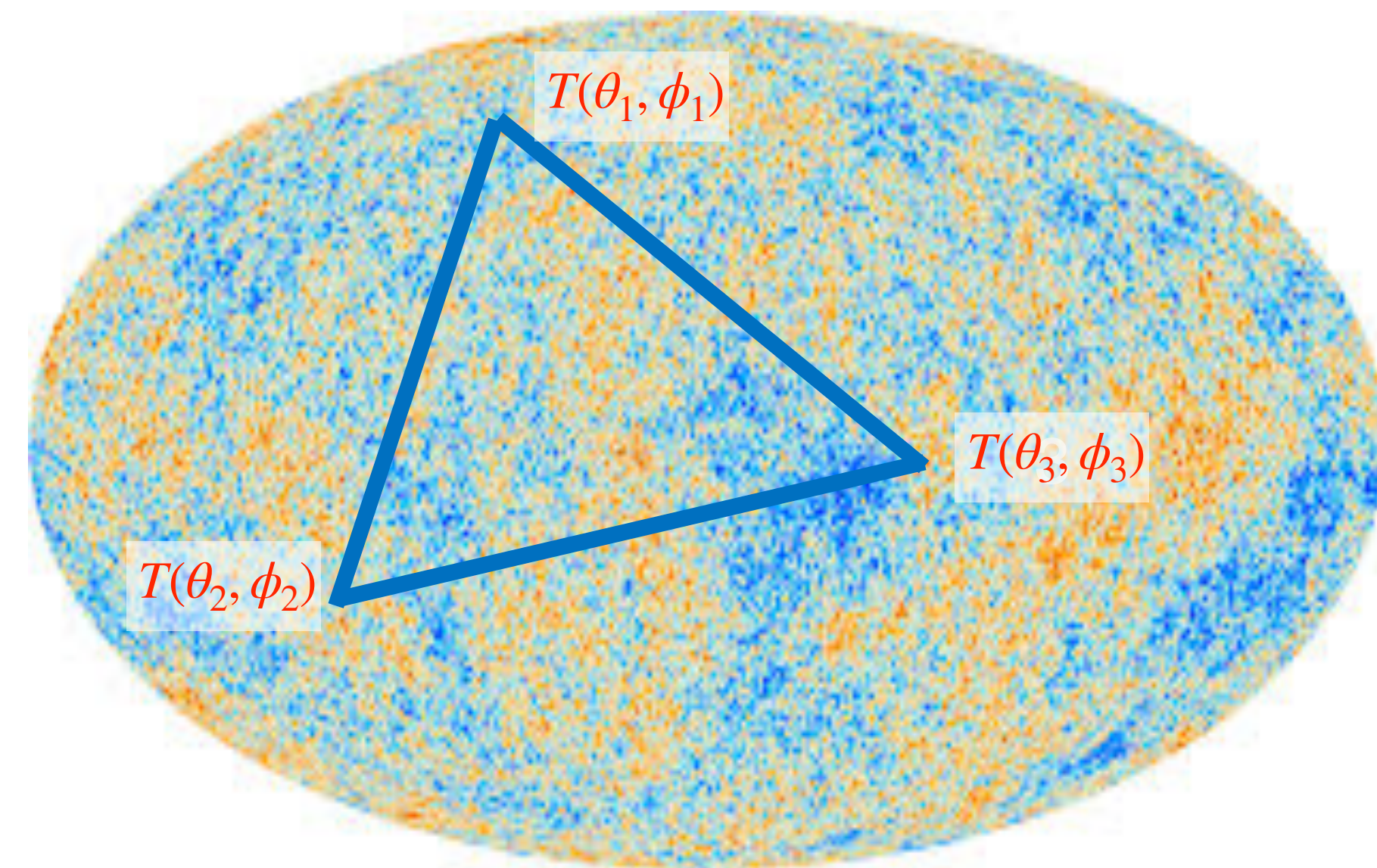
- CMB experiments measure the **temperature** and **polarization** across (part of) the whole sky

$$T(\theta, \phi), E(\theta, \phi) \leftrightarrow a_{\ell m}^T, a_{\ell m}^E$$

- Since the physics is **linear** we just need to correlate the CMB at **three** angles

$$\langle T(\theta_1, \phi_1) T(\theta_2, \phi_2) T(\theta_3, \phi_3) \rangle \leftrightarrow \langle a_{\ell_1 m_1}^T a_{\ell_2 m_2}^T a_{\ell_3 m_3}^T \rangle$$

- This is computationally **expensive**:
 - The bispectrum is **3-dimensional** [after symmetries]
 - There's $N_{\text{pix}}^3 \sim 10^{21}$ combinations of points!



How to Constrain Inflation

Most CMB analyses use two tricks:

1. Compression:

- We compress all 10^{21} elements into a single number, encoding the amplitude of a **specific** model, e.g., **self-interactions**

$$\widehat{f_{\text{NL}}} \sim \sum_{\ell_1 m_1 \ell_2 m_2 \ell_3 m_3} \underbrace{\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle_{\text{theory}}^\dagger}_{\text{Model}} \times \underbrace{(C^{-1}a)_{\ell_1 m_1} (C^{-1}a)_{\ell_2 m_2} (C^{-1}a)_{\ell_3 m_3}}_{\text{Data}}$$

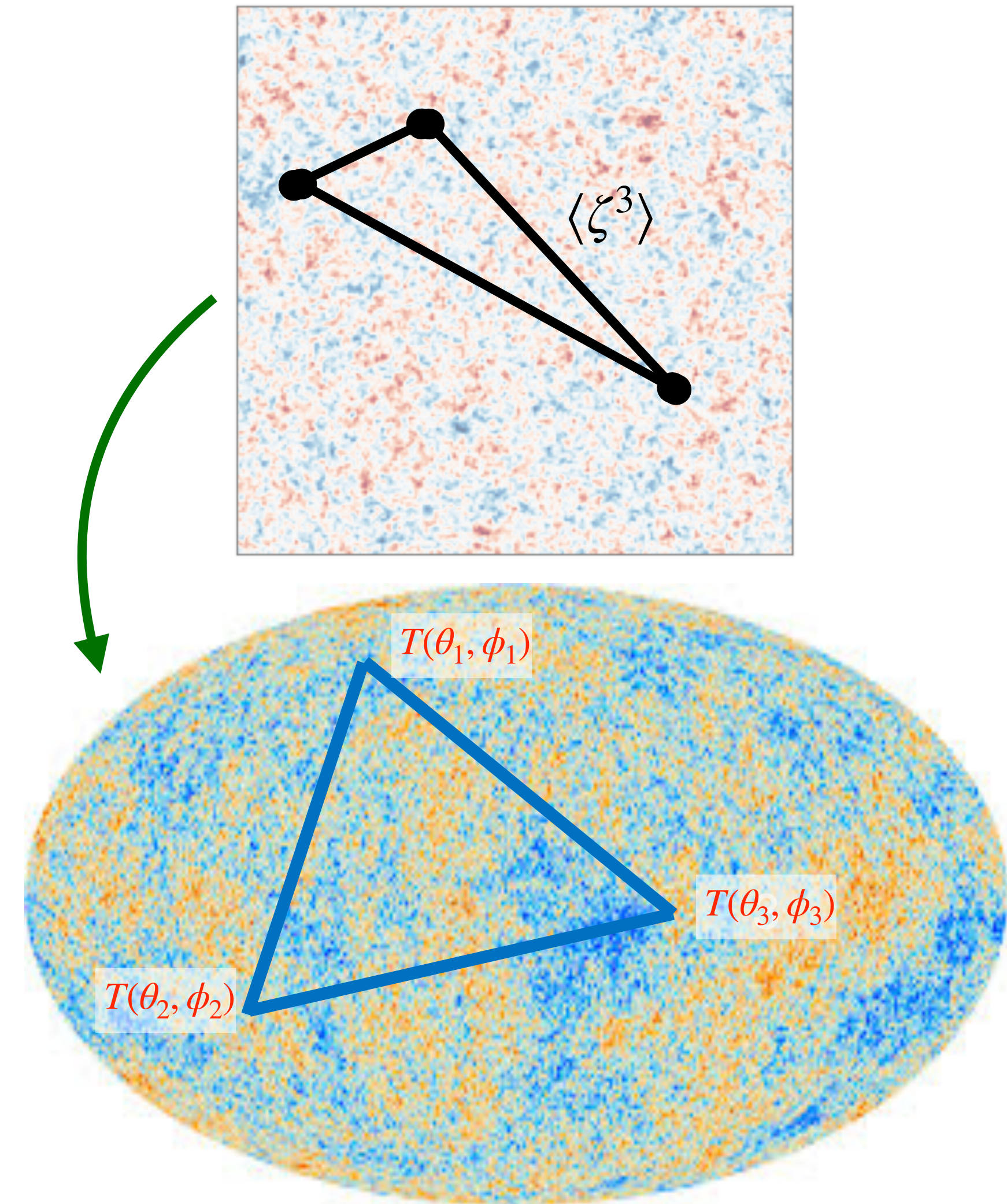
- This is an **optimal estimator** for f_{NL} , *i.e.* it is **lossless**

2. Factorizable:

- If the **theory model** is **factorizable**, we can rewrite the ℓ, m sum using spherical harmonic transforms!

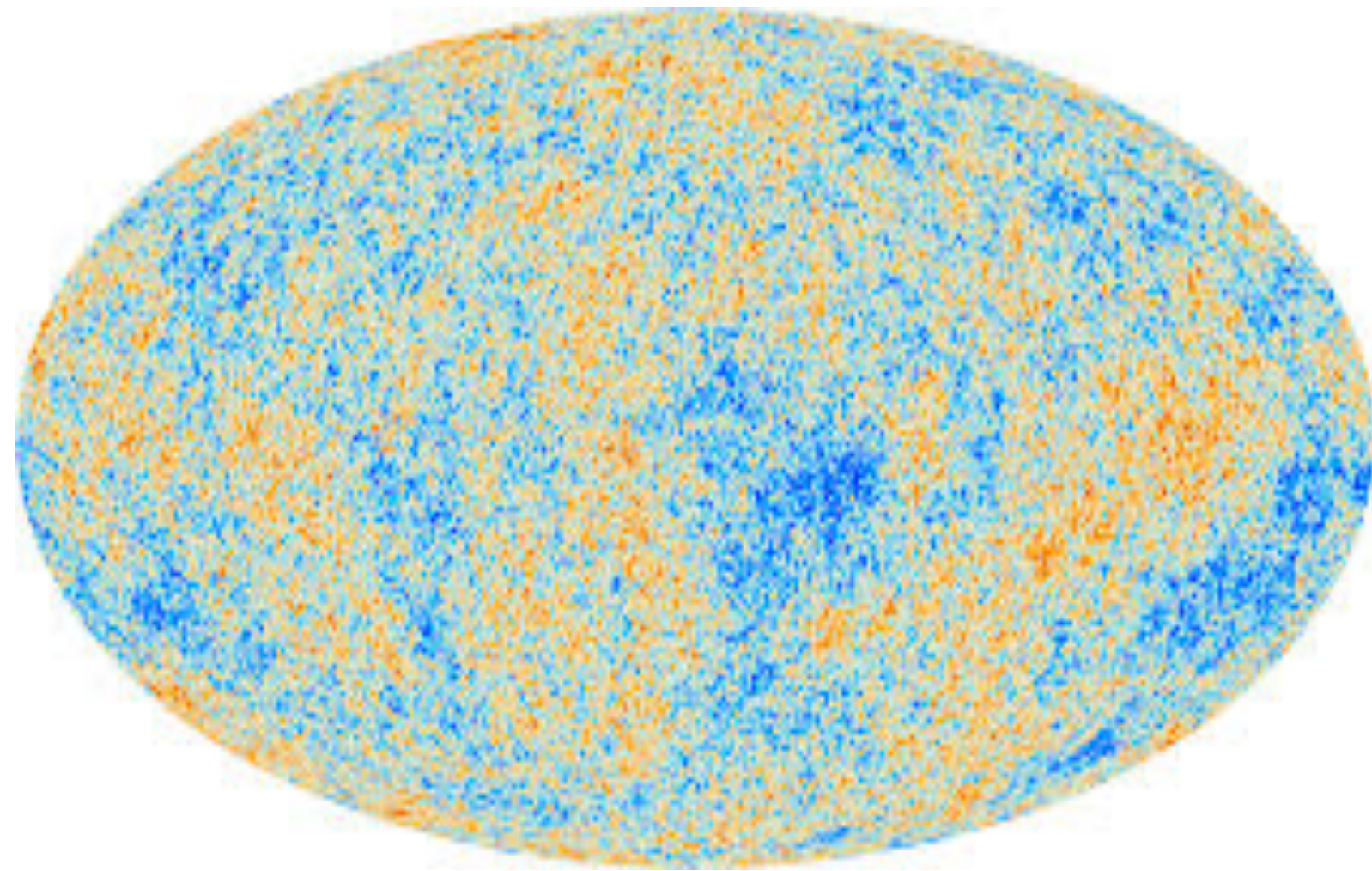
$$B_\zeta(k_1, k_2, k_3) \rightarrow a(k_1)b(k_2)c(k_3)$$

- This reduces the complexity from $\mathcal{O}(N_{\text{pix}}^3)$ to $\mathcal{O}(N_{\text{pix}} \log N_{\text{pix}})$



How to Constrain Inflation

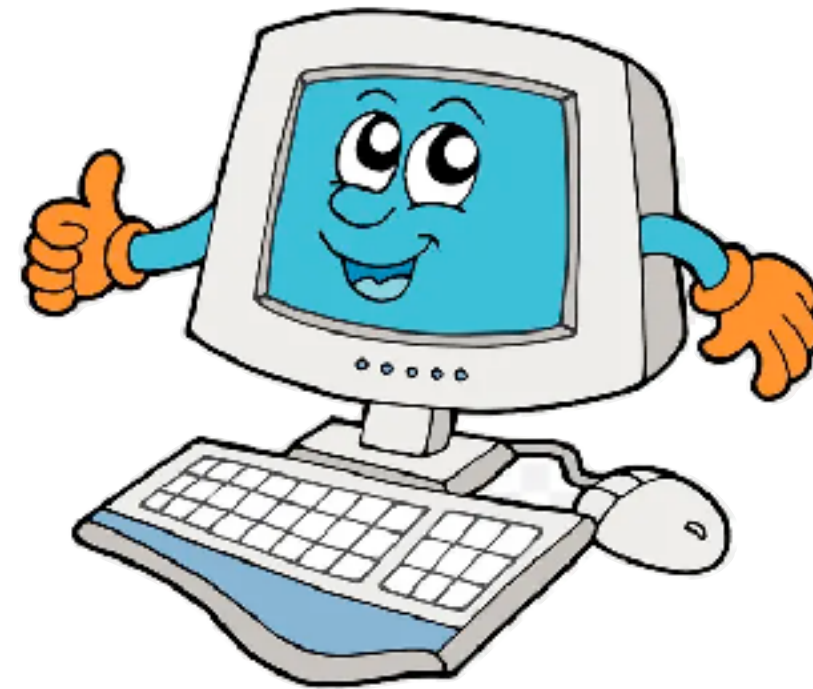
Data



CMB Fluctuations

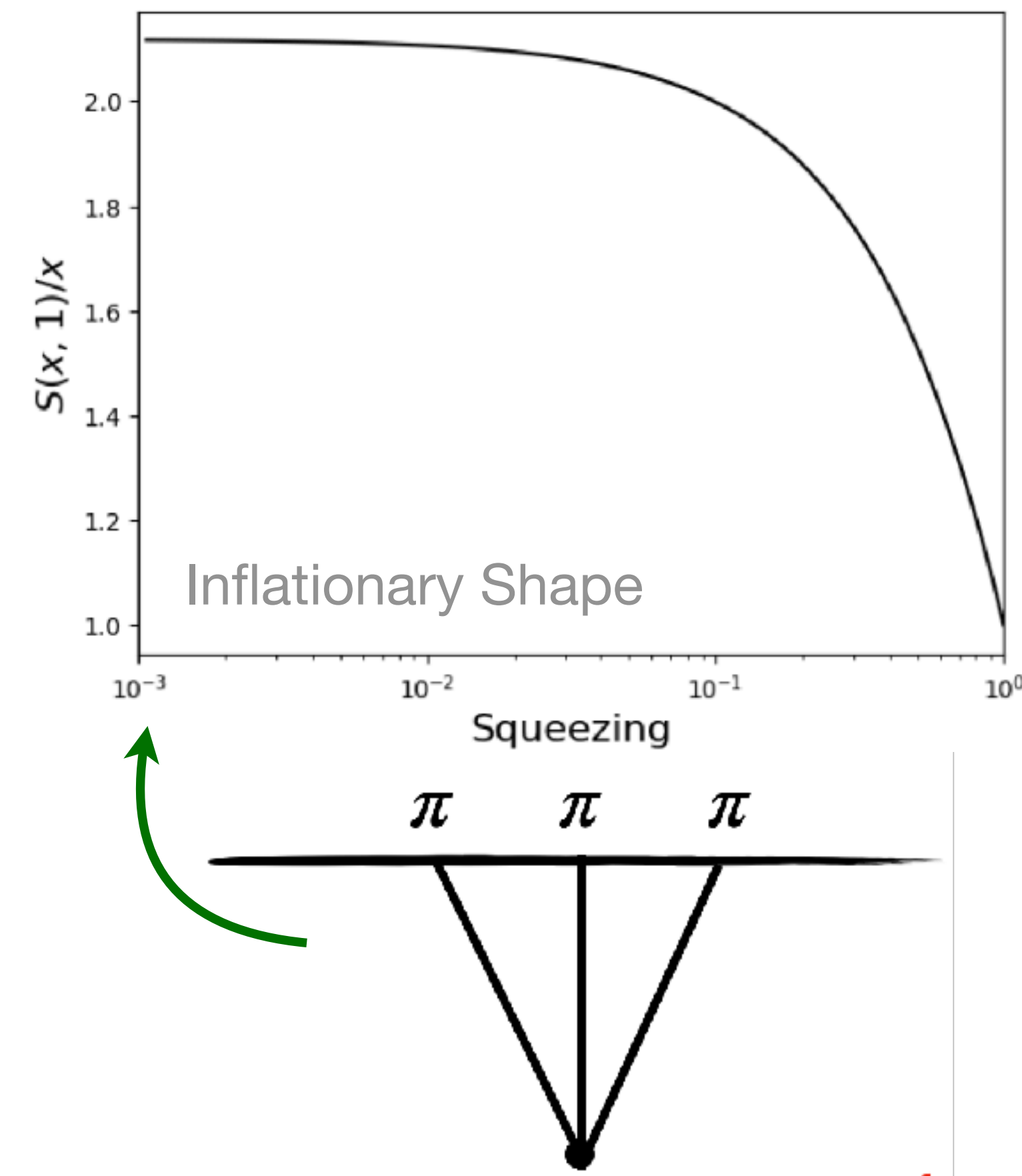
This is hard

- We need a **factorizable** template
- Every shape needs its **own** analysis



Amplitude

Theoretical Template



Constraining Factorizable Templates

- Previous CMB experiments have placed **strong** constraints on factorizable **three-point** functions

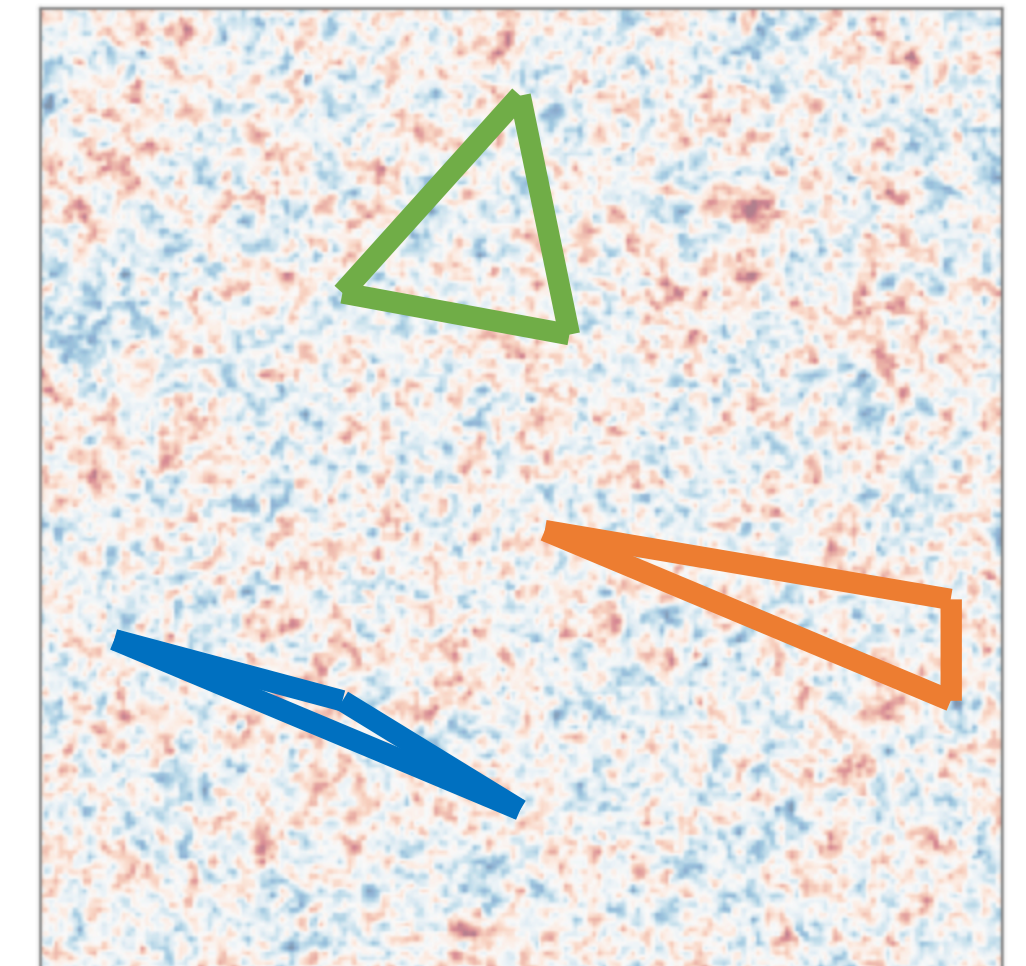
see Jo's talk!

$$\langle \delta T^3 \rangle \sim \langle \zeta^3 \rangle \sim f_{\text{NL}} \times \text{Shape}$$

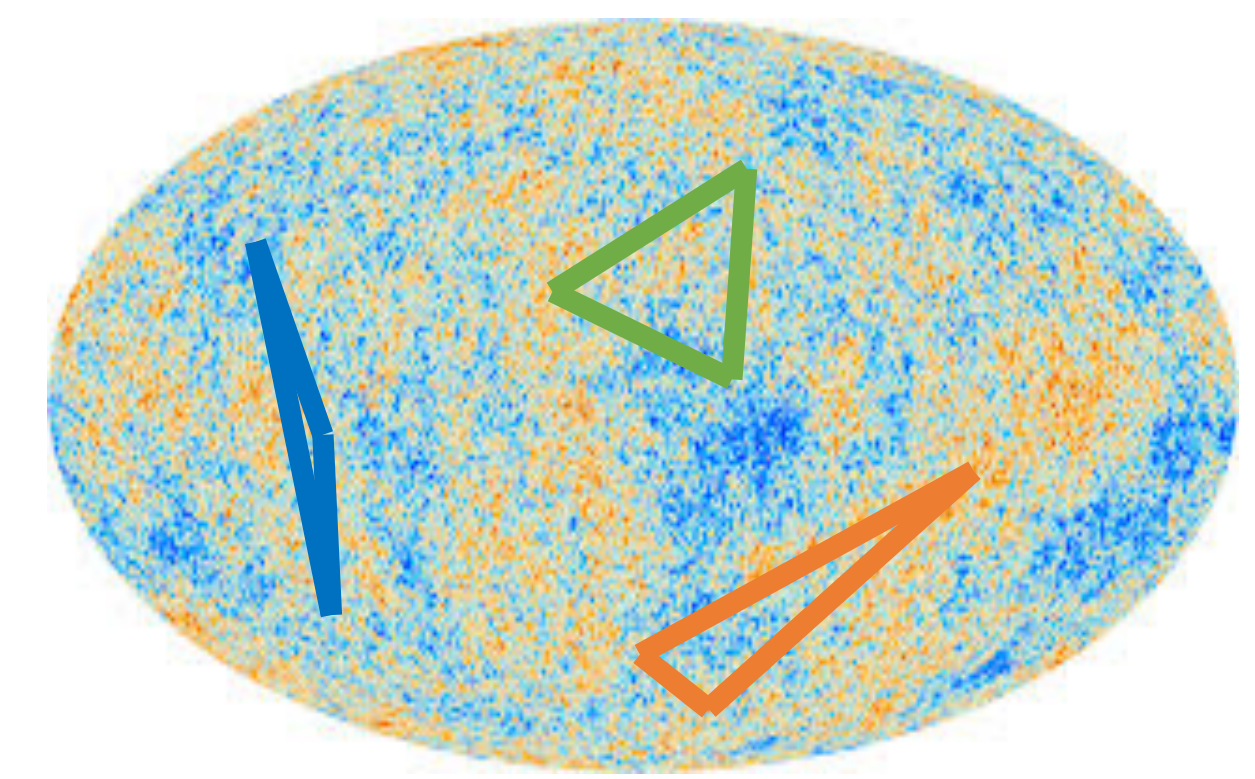
Planck 2018	Local	-0.9 ± 5.1
	Equilateral	-26 ± 47
	Orthogonal	-38 ± 24

Light field exchange

Self interactions



Linear Physics



Conclusion: Primordial non-Gaussianity is **small!**

$$10^{-5} |f_{\text{NL}}| \ll 1$$

We are still far from the (vaguely defined) theory targets:

$$f_{\text{NL}} \sim 1$$

Constraining Non-Factorizable Templates

General inflationary models are not factorizable so how do we analyze them?

1. Measure the binned bispectrum

$$B(\bar{\ell}_1, \bar{\ell}_2, \bar{\ell}_3) \sim \int dk_1 dk_2 dk_3 dr B_\zeta(k_1, k_2, k_3) j_{\ell_1}(k_1 r) j_{\ell_2}(k_2 r) j_{\ell_3}(k_3 r) \dots$$

Expensive to compare theory and data

2. Measure the compressed modal bispectrum

$$B_\zeta(k_1, k_2, k_3) \sim \sum_{abc} \omega_{abc} k_1^a k_2^b k_3^c \Leftrightarrow \omega_{abc} \sim \int dk_1 dk_2 dk_3 k_1^a k_2^b k_3^c B_\zeta(k_1, k_2, k_3)$$

Powerful but **high-dimensional** and only as good as the **basis**

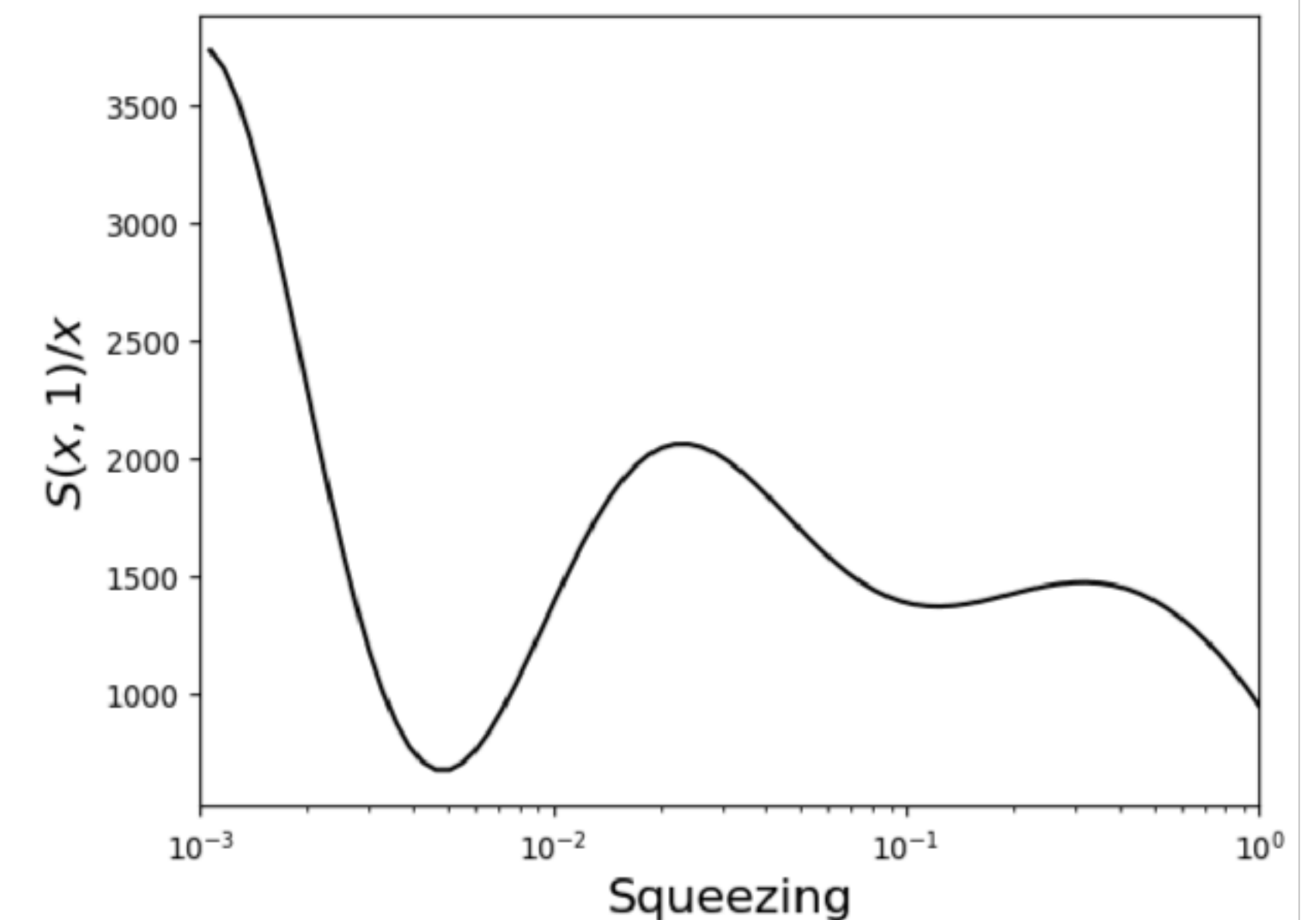
3. Measure the compressed bispectrum using **machine learning**

$$B_\zeta(k_1, k_2, k_3) \sim f_{\text{NL}} \sum \text{Neural}_1(k_1) \text{Neural}_2(k_2) \text{Neural}_3(k_3)$$

Easy to find the **best basis** for a particular model

$$B_\zeta(k_1, k_2, k_3) \neq a(k_1)b(k_2)c(k_3)$$

Massive particle exchange (non-factorizable)



CMB Bispectrum Constraints

These have placed strong constraints on novel physics

1. Measure the binned bispectrum

- Correlations with **gravitational waves** ($\langle \zeta \zeta h \rangle + \dots$)

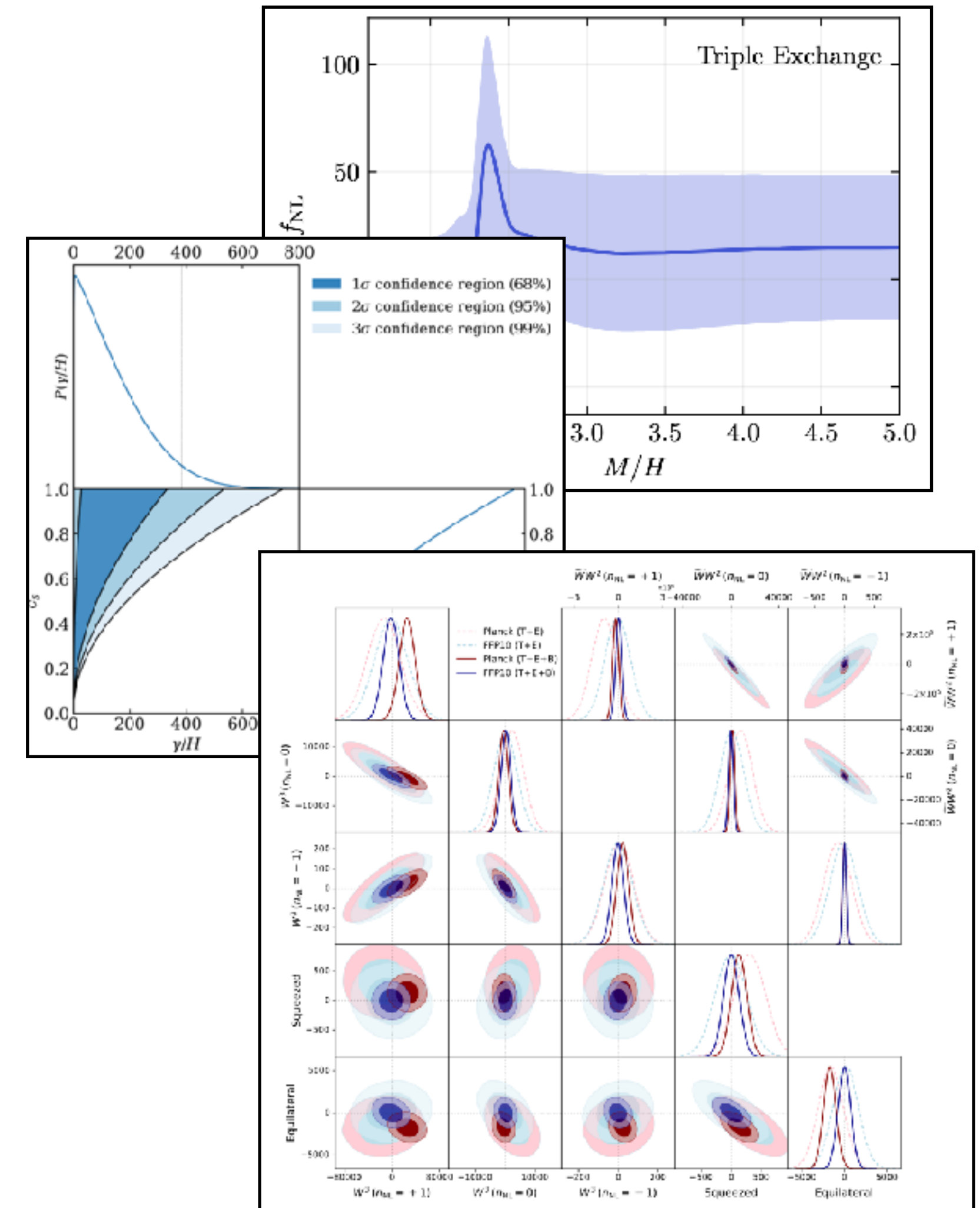
2. Measure the compressed modal bispectrum

- Oscillations and approximate shape of **massive particles** with $m_\sigma \sim H$
- Extensions with **triple-exchange**, **dissipation** and **chemical potential** (exact shapes)

3. Measure the compressed bispectrum using **machine learning**

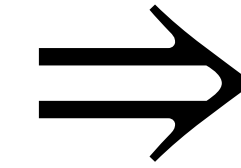
- **Strongly-coupled** systems (“unparticles”)

So far, there have been no detections

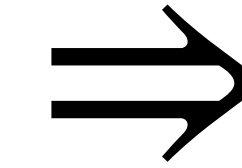


CMB Bispectrum Constraints

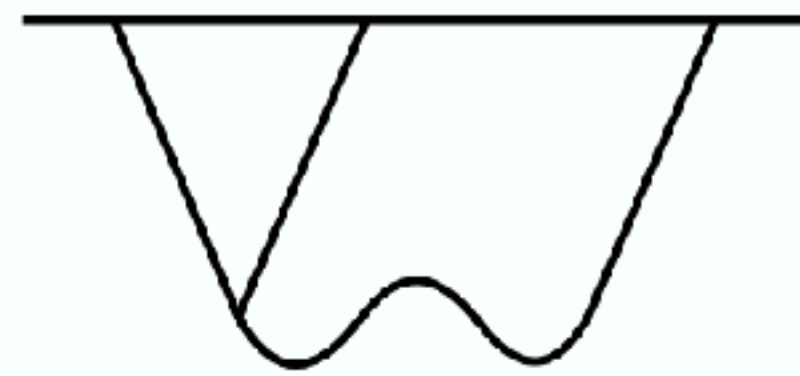
Models



Pipeline



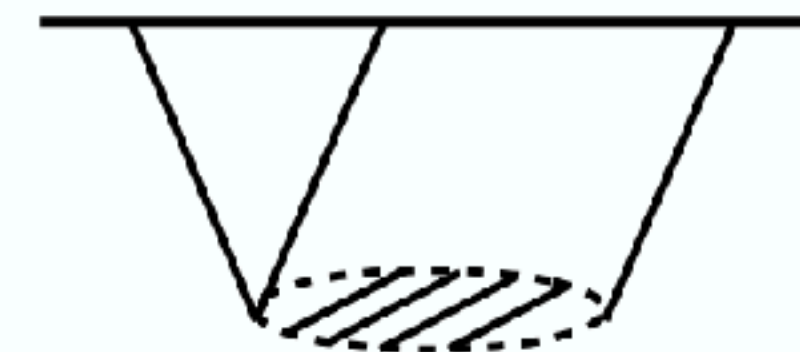
Constraints



Weakly coupled (M, J)

Massive particle exchange

$$\mathcal{L} \sim \dot{\pi}\sigma + (\partial_\mu\pi)^2\sigma$$



Strongly coupled (M, Δ)

Conformal Field Theory exchange

(Unparticles)

$$\mathcal{L} \sim \dot{\pi}\mathcal{O} + (\partial_\mu\pi)^2\mathcal{O}$$

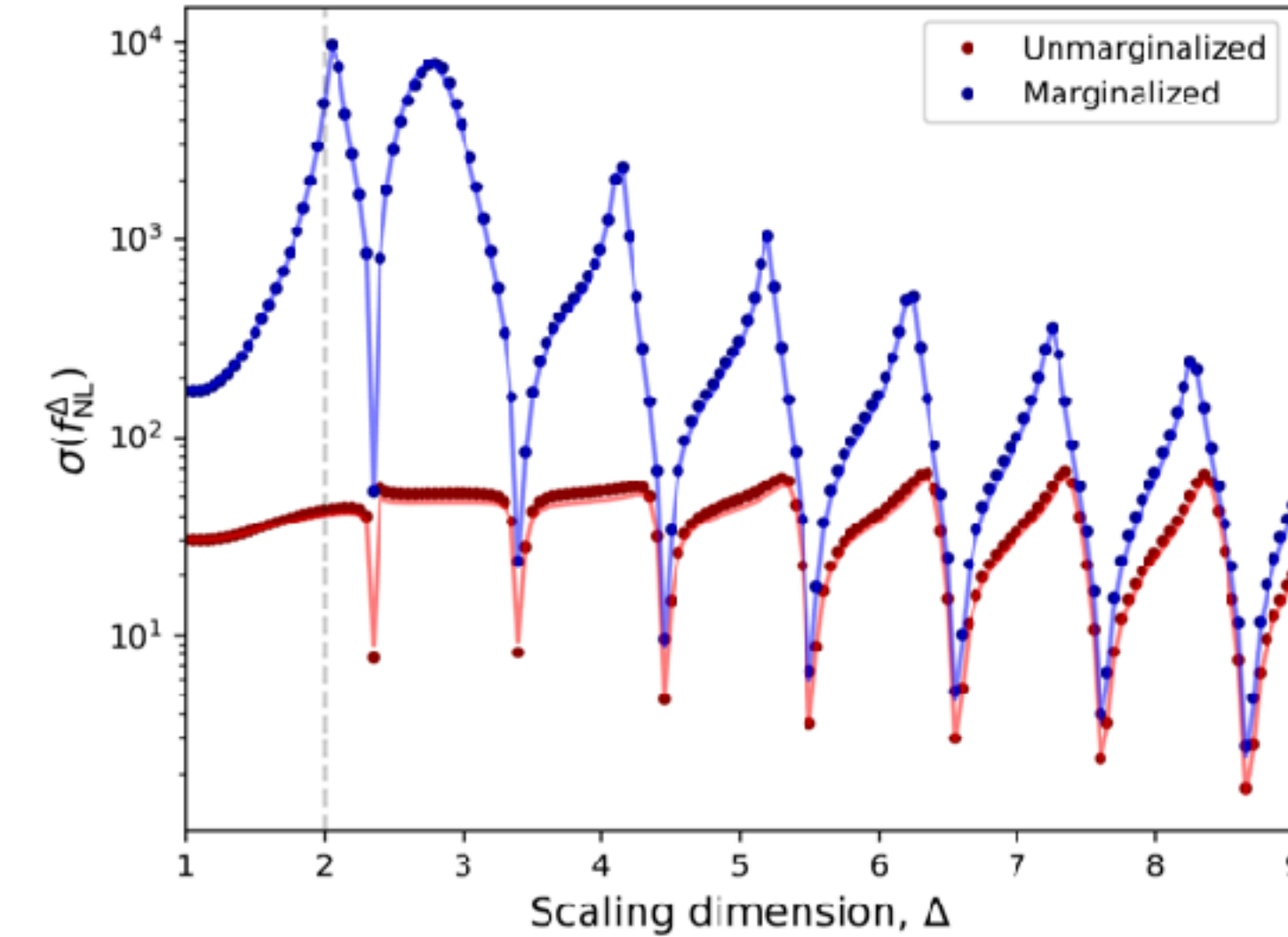
$$\langle \tilde{\mathcal{O}}_\Delta(\eta_1, \mathbf{x}) \tilde{\mathcal{O}}_\Delta(\eta_2, \mathbf{y}) \rangle_{\text{dS}_4} = \frac{(H^2\eta_1\eta_2)^\Delta}{(-(\eta_1 - \eta_2)^2 + (\mathbf{x} - \mathbf{y})^2)^\Delta}$$

Theoretical Model
 $S_\Delta(x, y)$

Orthogonal
Components
 $S^{(i)}(k_1, k_2, k_3)$

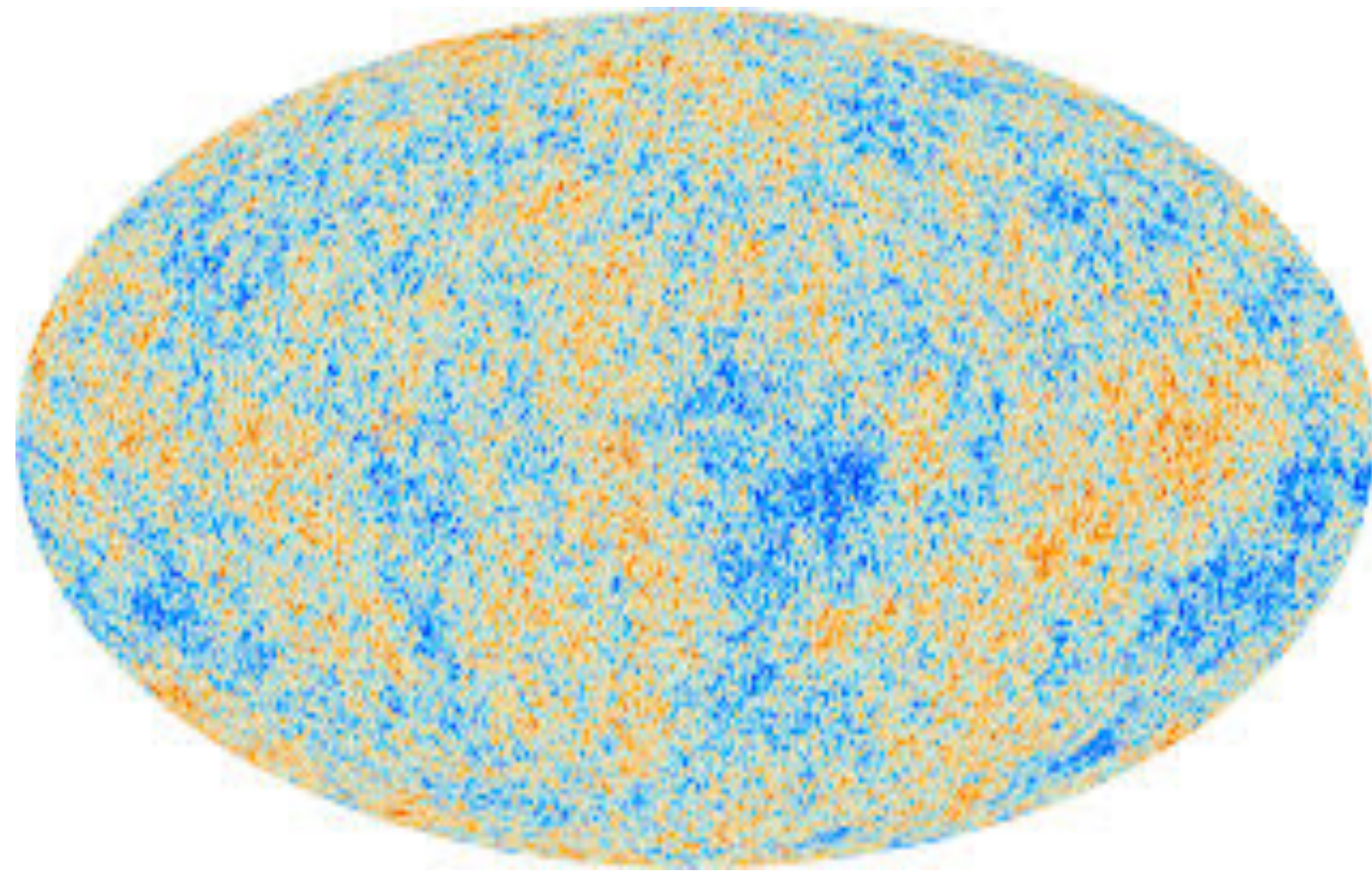
Separable Basis
 $\alpha_n^{(i)}(k), \beta_n^{(i)}(k), \gamma_n^{(i)}(k)$

PNG Constraints
 f_{NL}^Δ

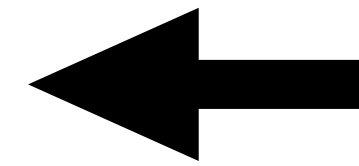
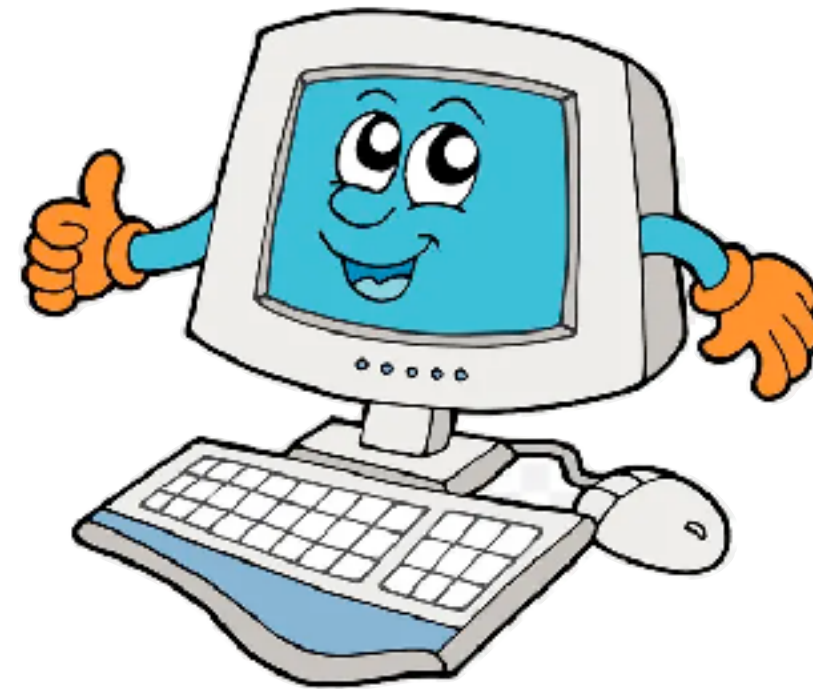
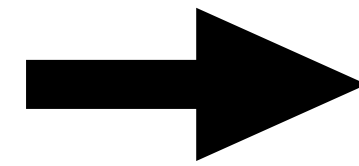


How to Constrain Inflation

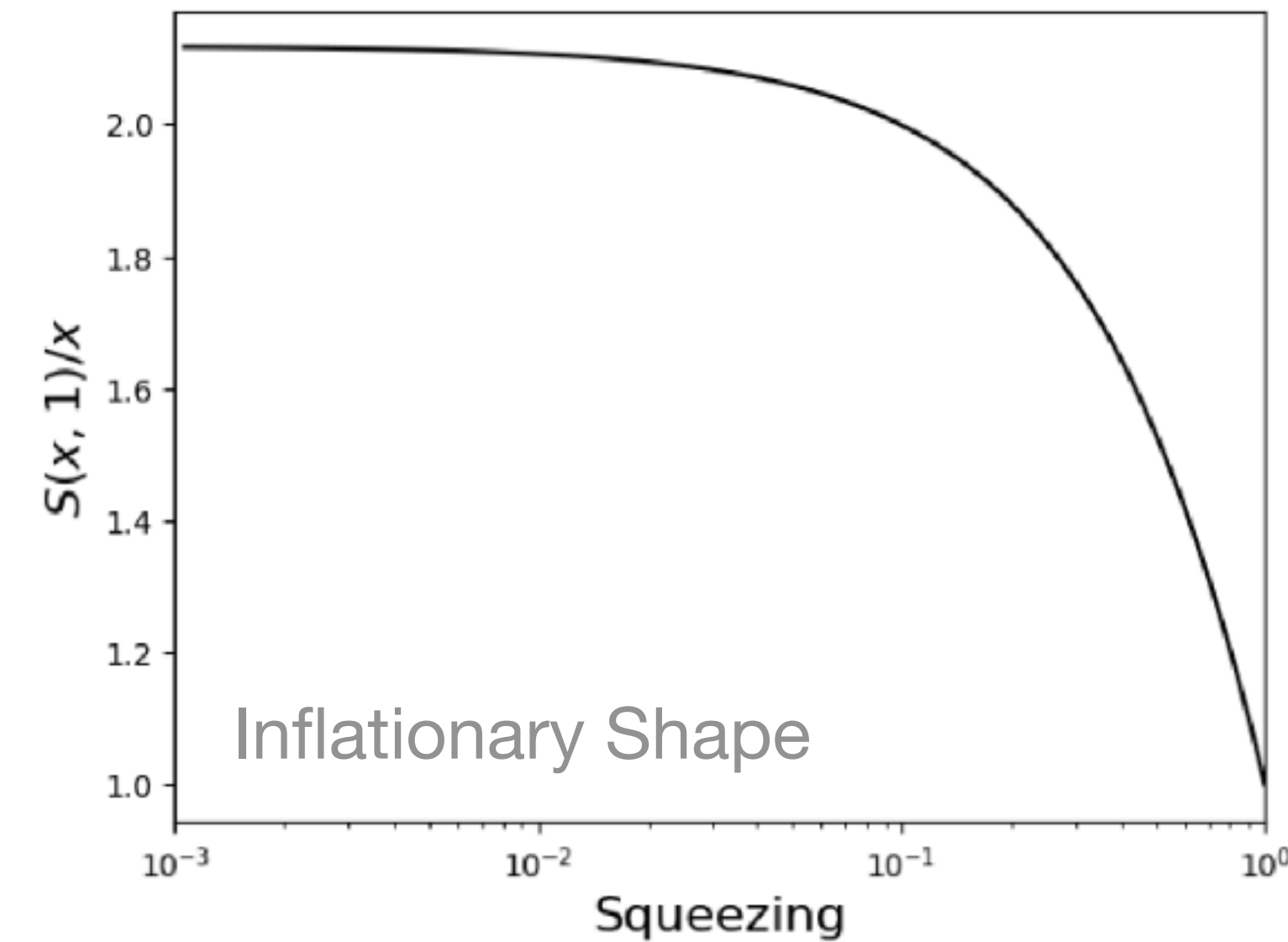
Data



CMB Fluctuations



Theoretical Template



This is **hard**

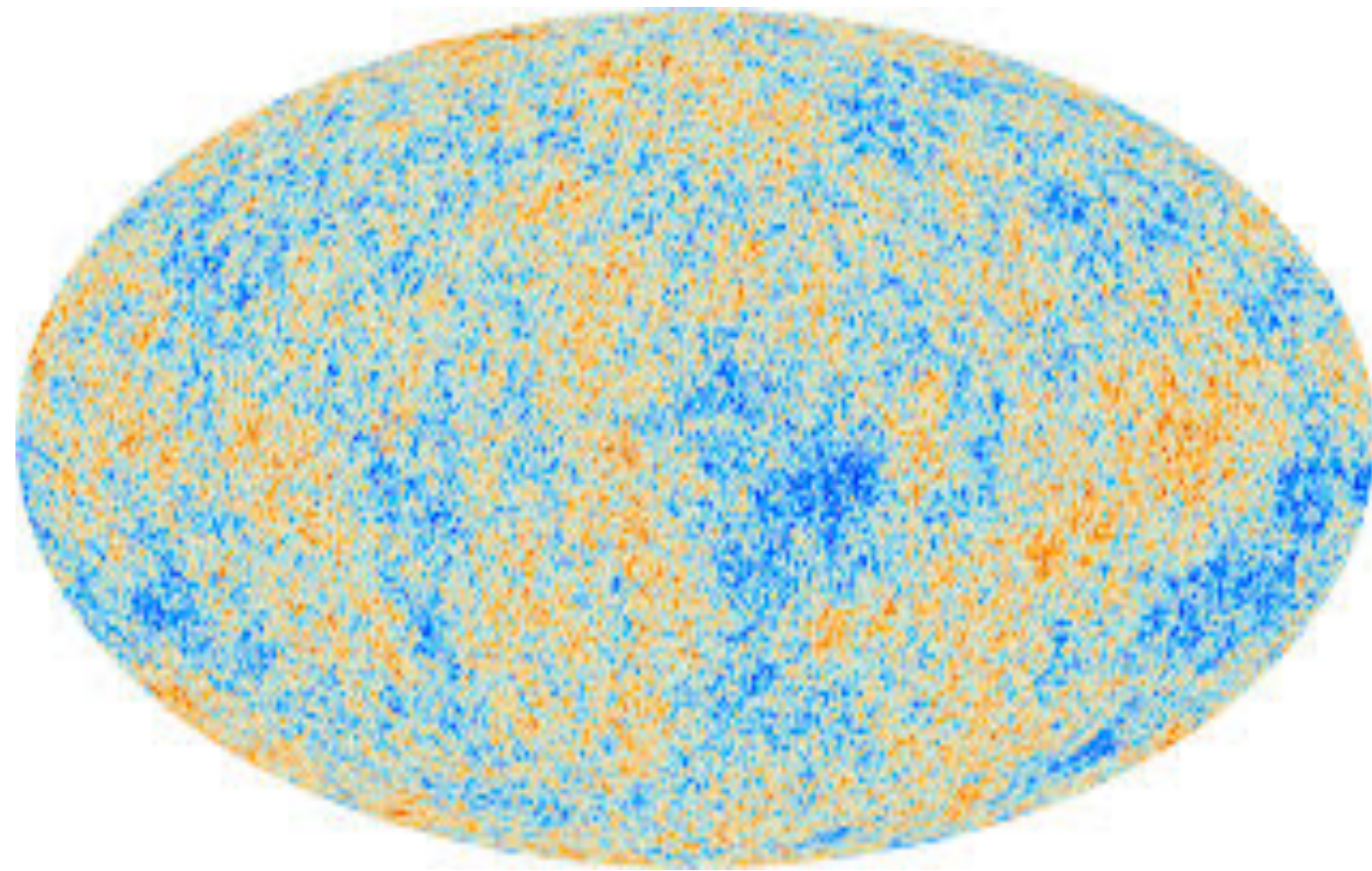
- We need a **factorizable** template
- Every shape needs its **own** analysis



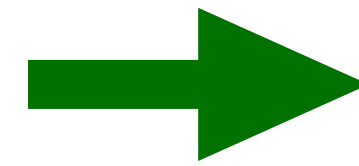
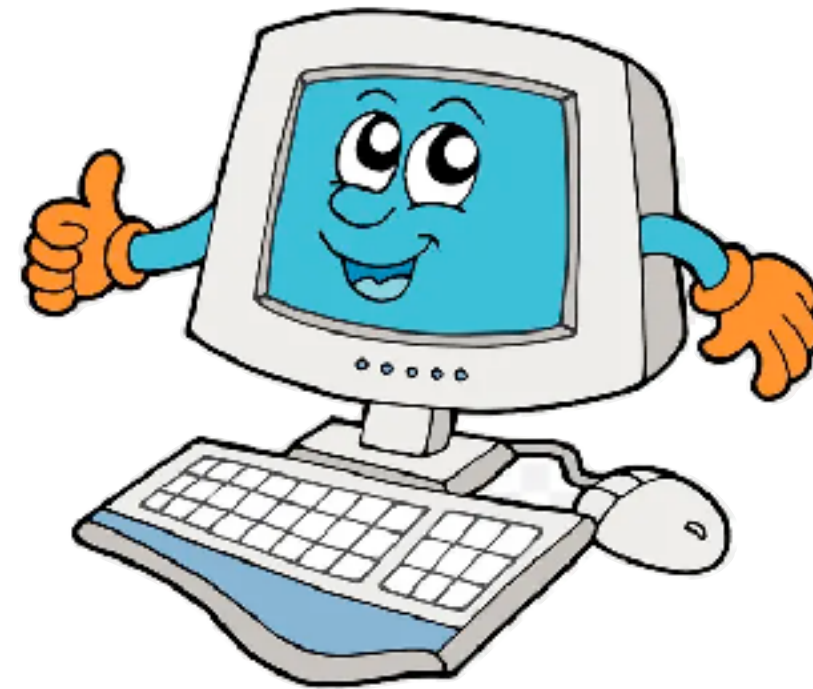
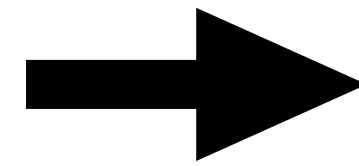
Amplitude

How to Constrain Inflation

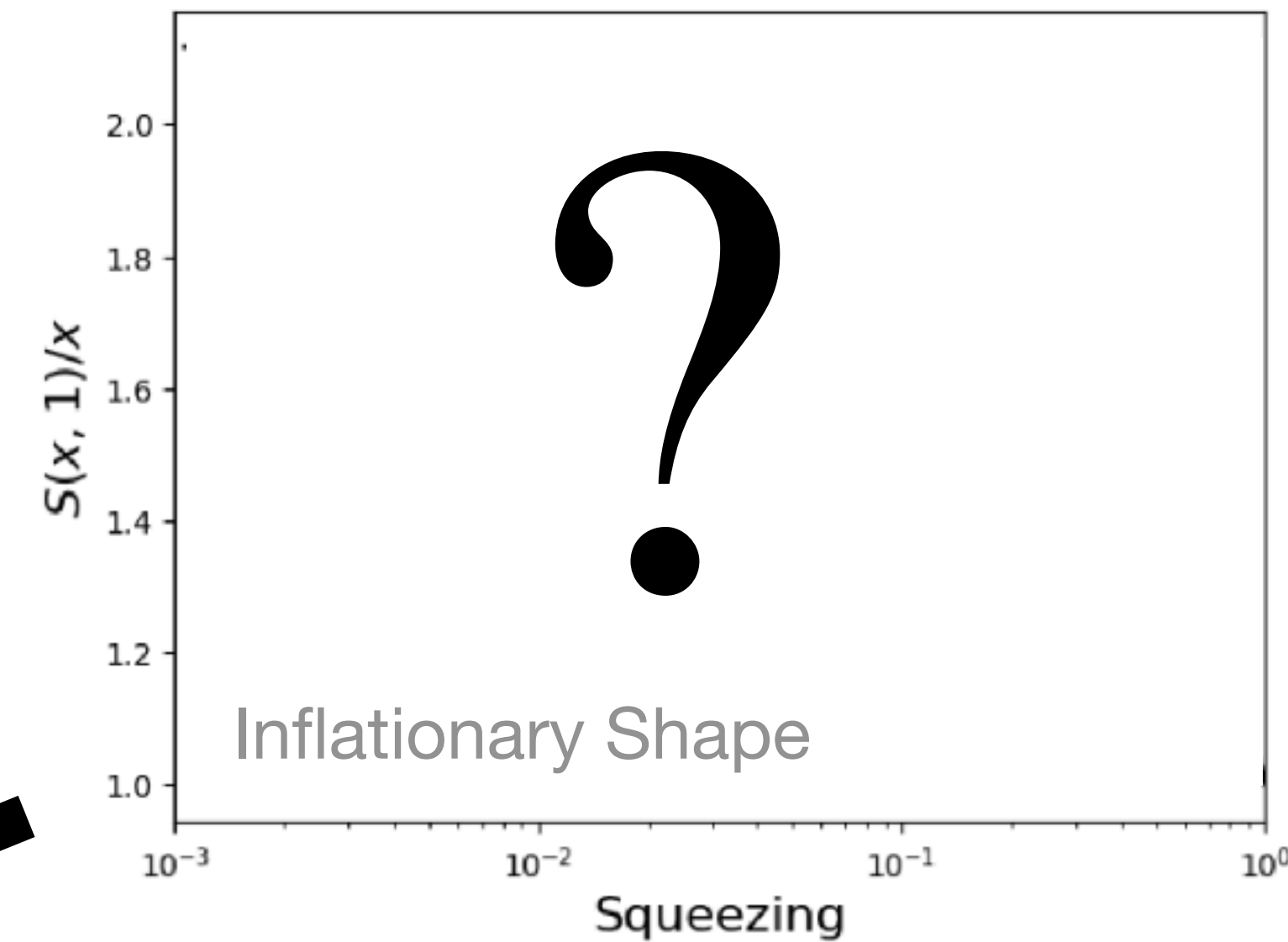
Data



CMB Fluctuations



Observed Shape



This is **easy**

- We can analyze *any* shape
- The analysis is (almost) **free**



Amplitude

What Shape is the Bispectrum?

Can we reconstruct the **inflationary bispectrum**?

$$B(\ell_1, \ell_2, \ell_3) \sim \int dk_1 dk_2 dk_3 dr B_\zeta(k_1, k_2, k_3) j_{\ell_1}(k_1 r) j_{\ell_2}(k_2 r) j_{\ell_3}(k_3 r) \dots$$

\uparrow
Observer-friendly
 \uparrow
Theory-friendly

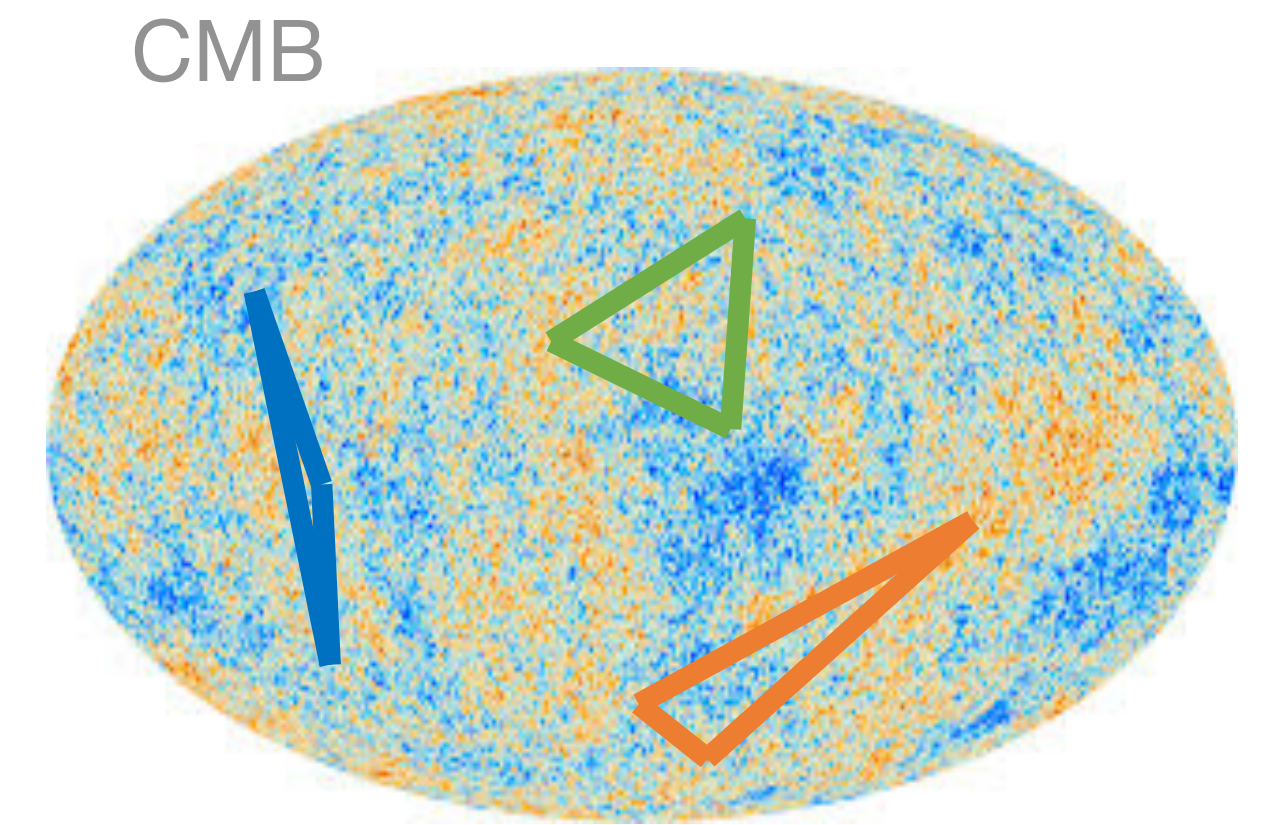
- Let's assume **scale-invariance** (i.e. dilatation invariance) \Rightarrow we have a 2D **shape function**

$$B_\zeta(k, k, k) \sim k^{-6} \quad S(x = k_1/k_3, y = k_2/k_3) \propto (k_1 k_2 k_3)^2 B_\zeta(k_1, k_2, k_3)$$

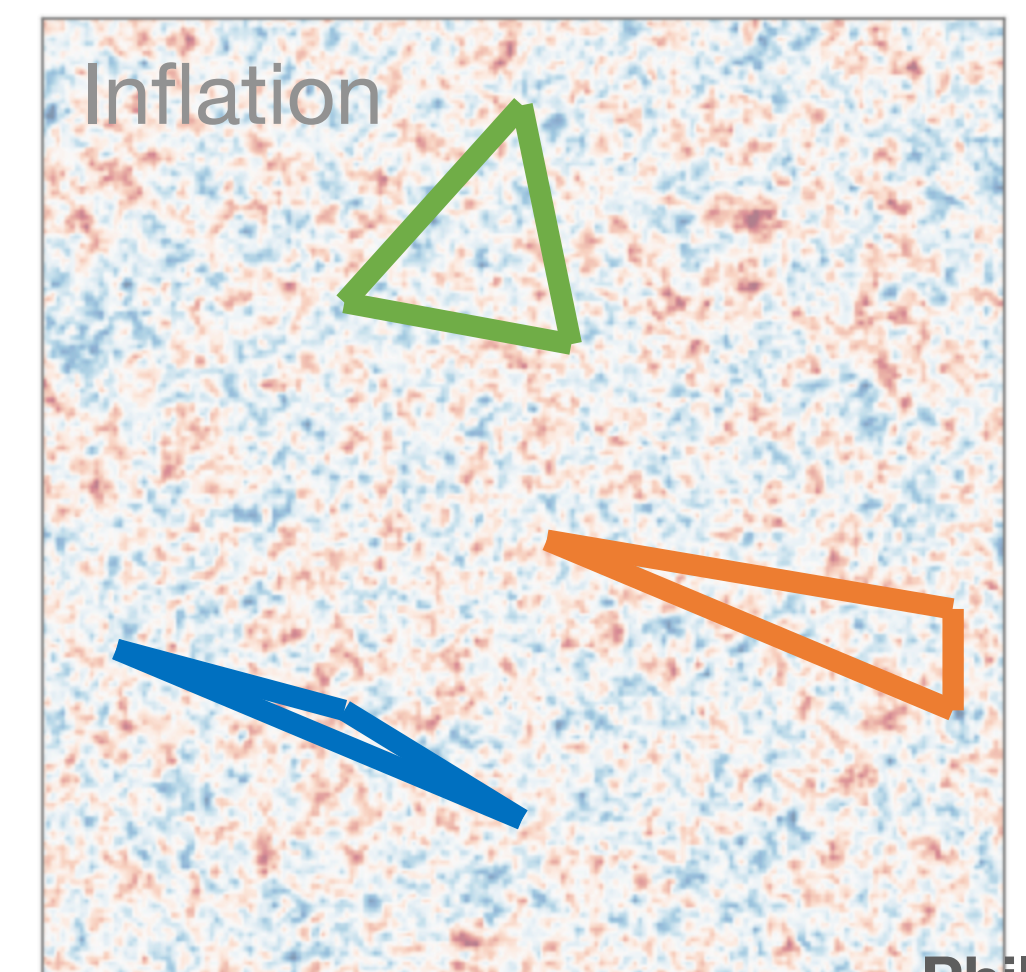
- Build a **binned estimator** for $S(x, y)$ from the CMB!

$$\widehat{S(x, y)} \sim \sum_{\ell_1 m_1 \ell_2 m_2 \ell_3 m_3} \frac{\partial \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle_{\text{theory}}^\dagger}{\partial S(x, y)} \times (C^{-1} a)_{\ell_1 m_1} (C^{-1} a)_{\ell_2 m_2} (C^{-1} a)_{\ell_3 m_3}$$

(In practice, build a 3D estimator binned in $\log k$, then linearly transform to $\log x / \log y$ bins)



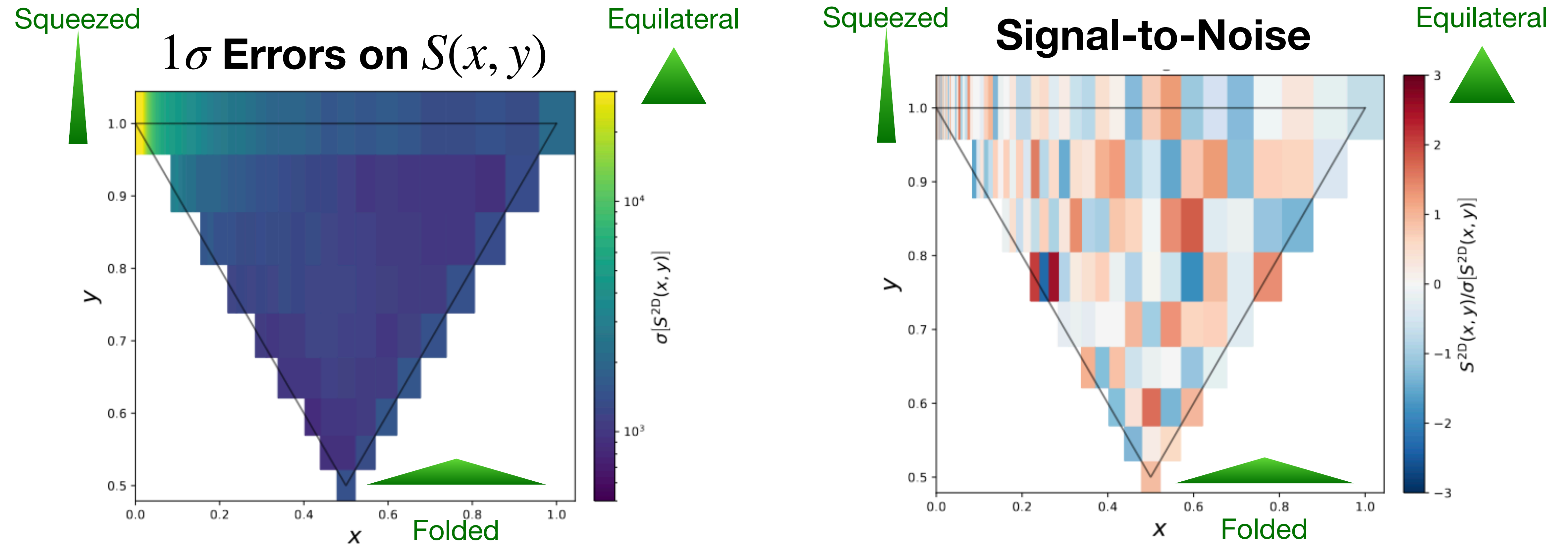
\downarrow
 Linear Algebra



The *Planck* Bispectrum Shape



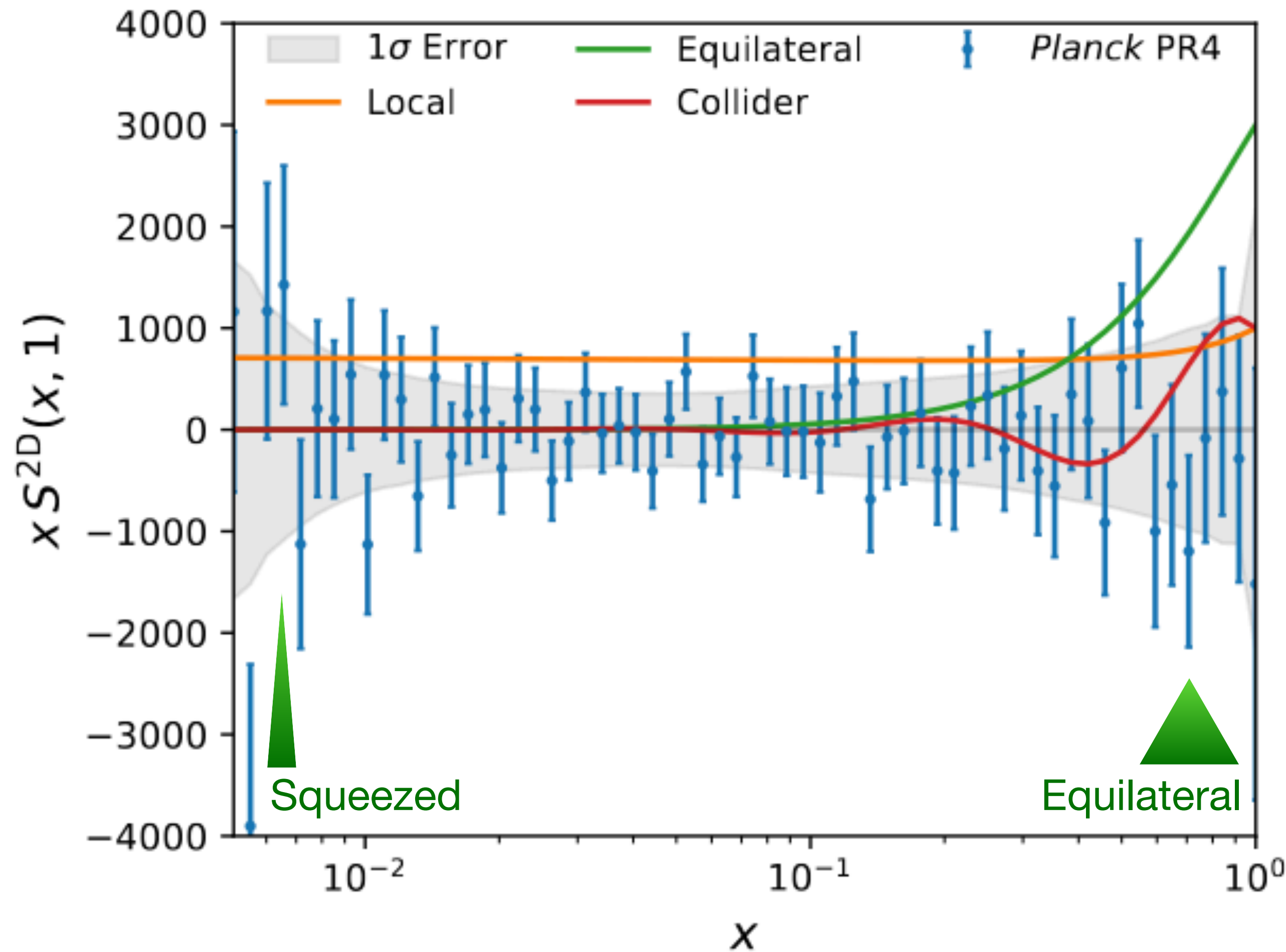
We build **efficient**, quasi-optimal, mask/beam-corrected, **cubic estimators** for the underlying primordial shape





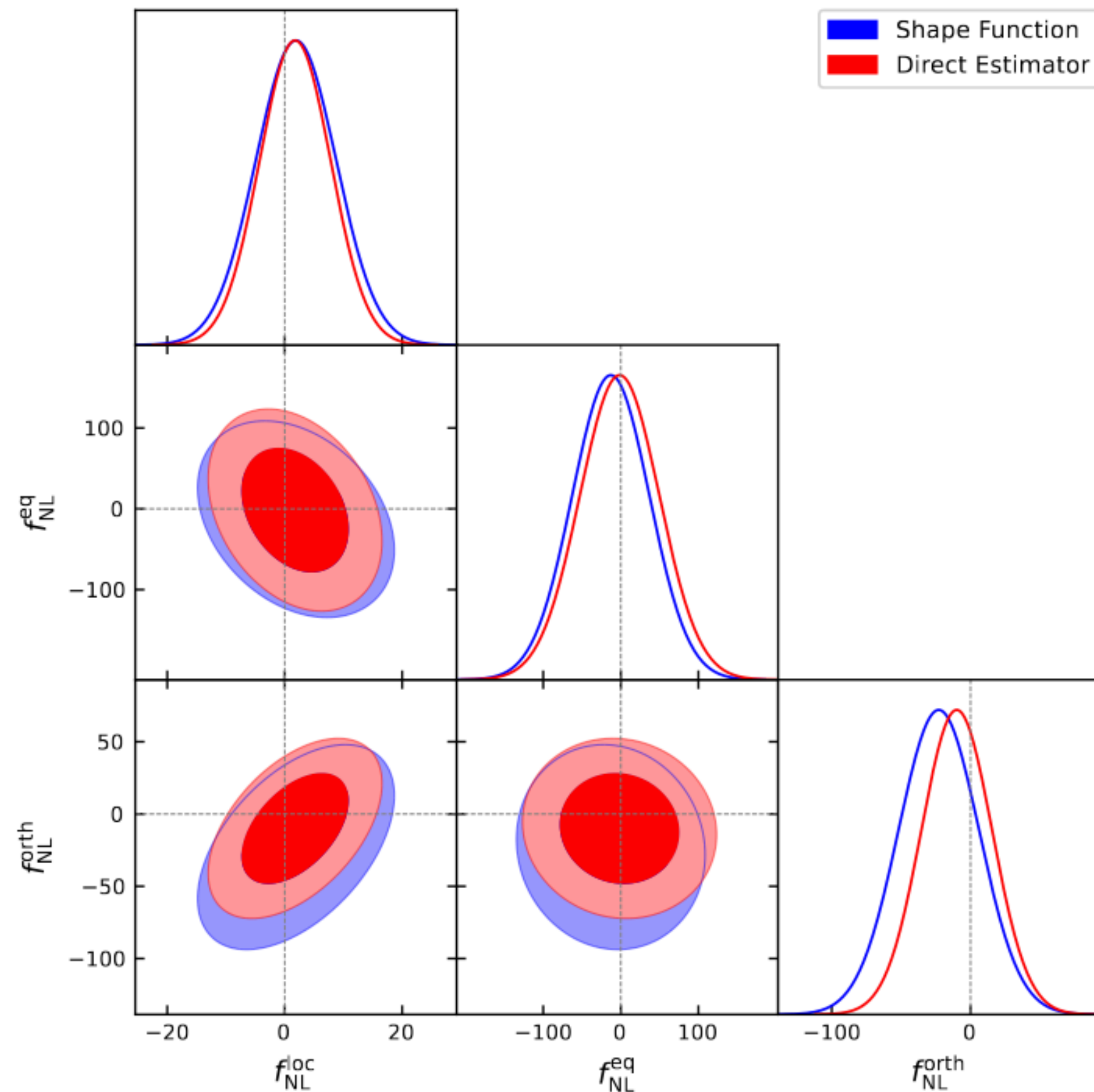
The *Planck* Bispectrum Shape

We can directly probe the **squeezed limit** to **directly** compare to models!



← Collider oscillations are hard to detect!

Does this actually work?



We are **compressing** the full bispectrum into a set of **binned** $S(x, y)$ measurements.

Is this a sensible approach?

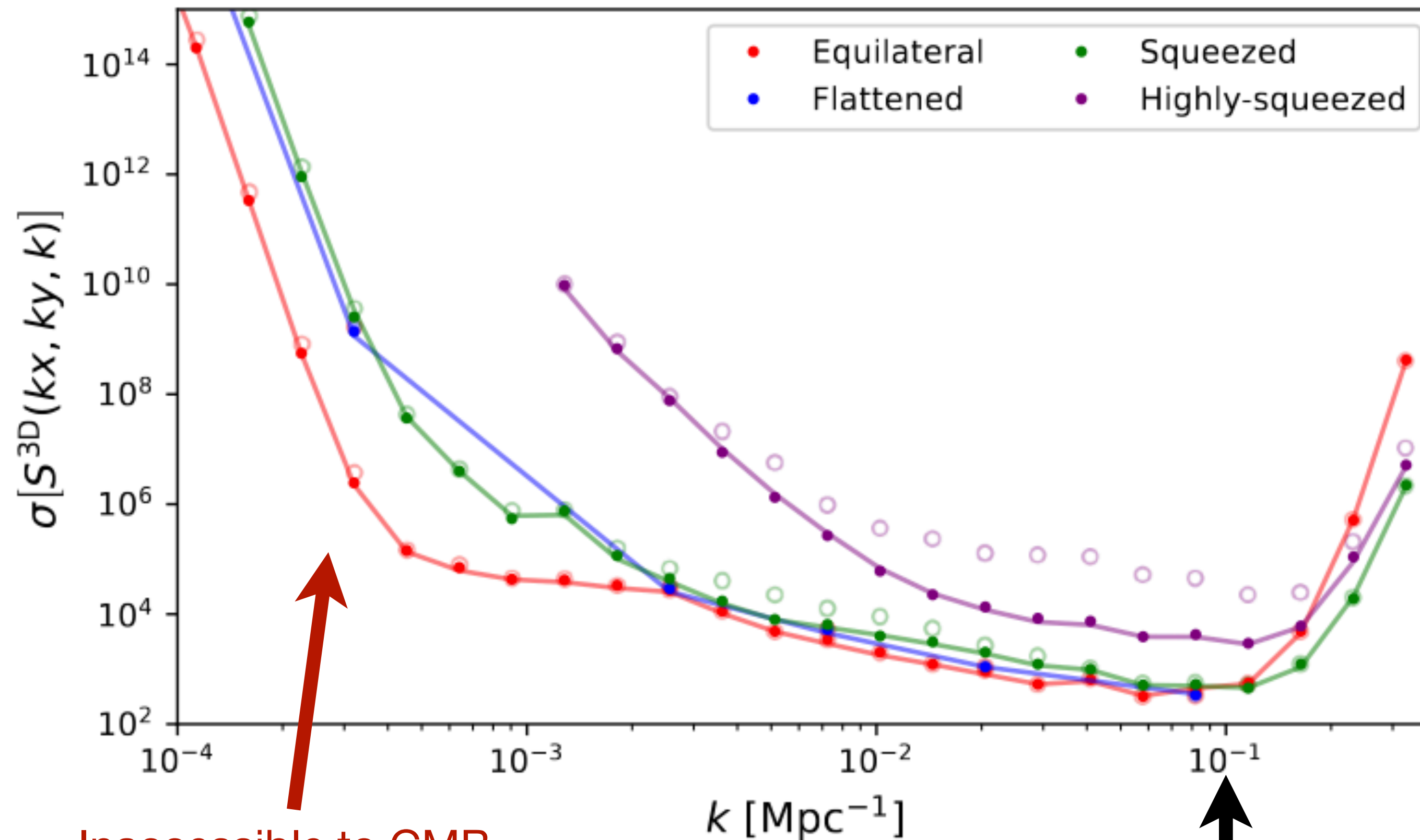
Yes!

We get the f_{NL} amplitudes and errors correct to $\mathcal{O}(10\%)$

Where does the information come from?

Using the binned **three-dimensional** bispectrum $S(k_x, k_y, k)$ we can probe the **information content** with respect to k

~ 6 e-folds accessible to PNG



Inaccessible to CMB

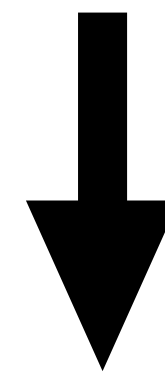
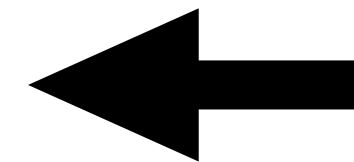
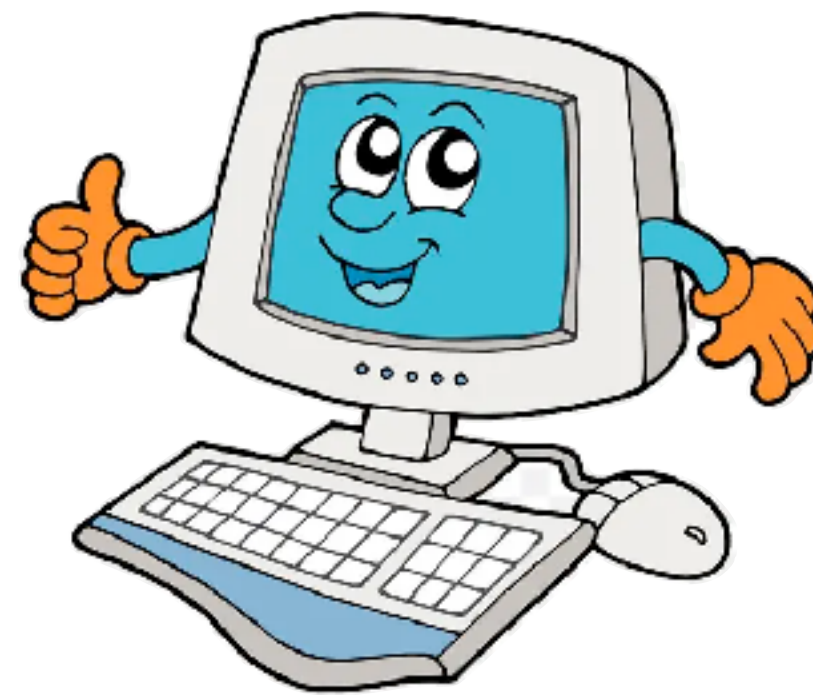
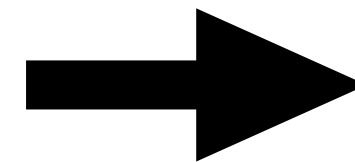
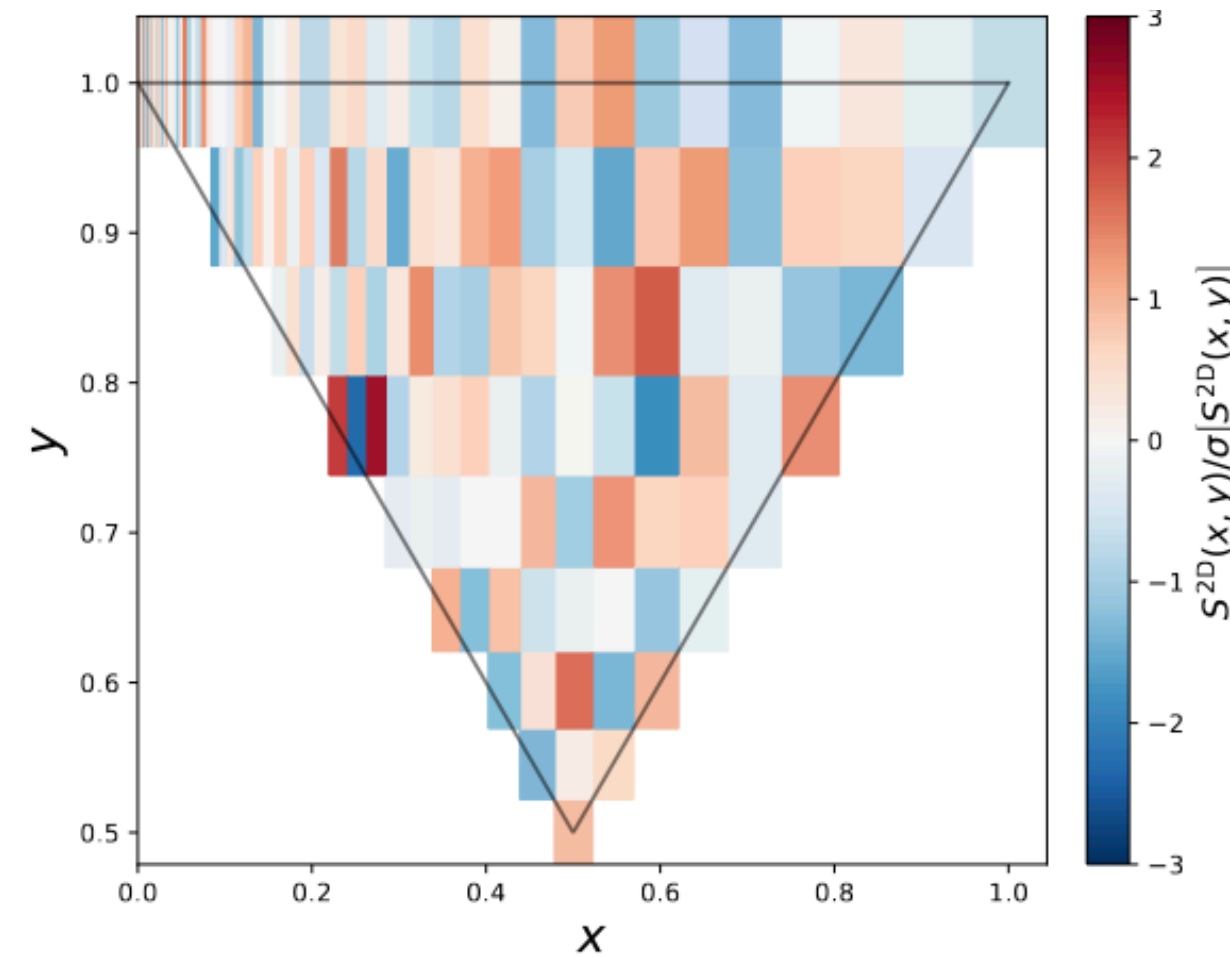
Inaccessible to Planck

Most information from here

How to Constrain Inflation

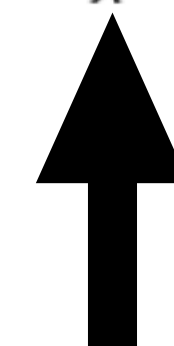
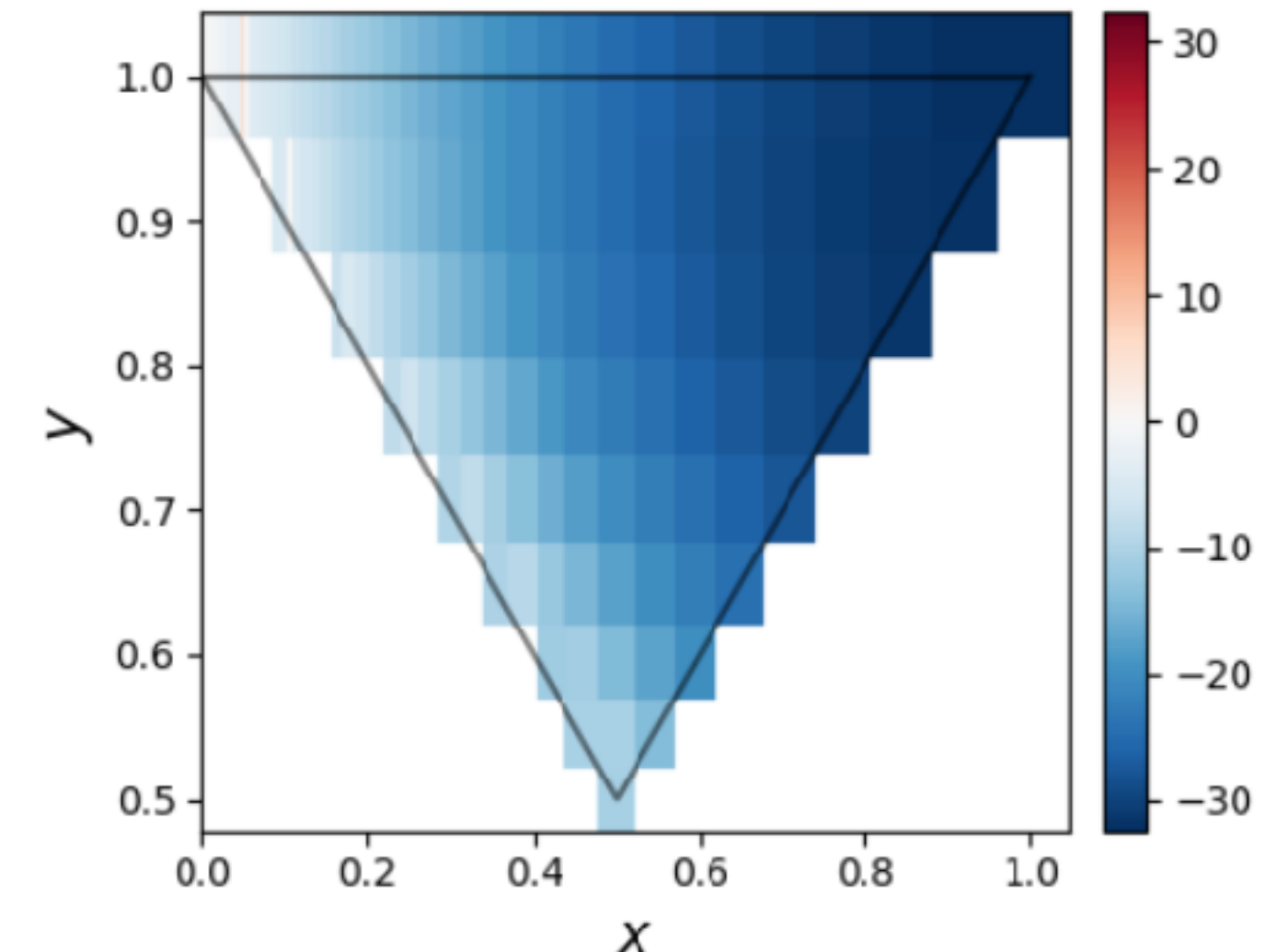
We can analyze (scale-invariant) bispectrum templates much faster ($\mathcal{O}(\text{ms})$ not $\mathcal{O}(\text{lots of hours})$)

Observed Shape



Signal-to-
Noise

Theoretical Shape

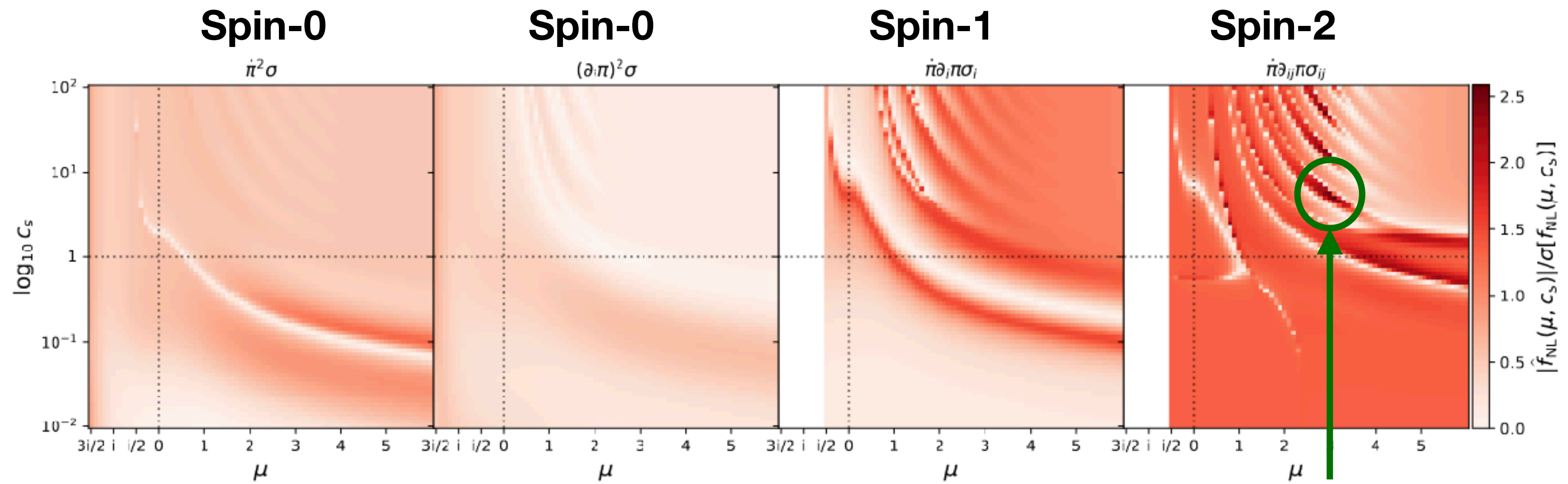


Model Parameters

Searching for the Cosmological Collider

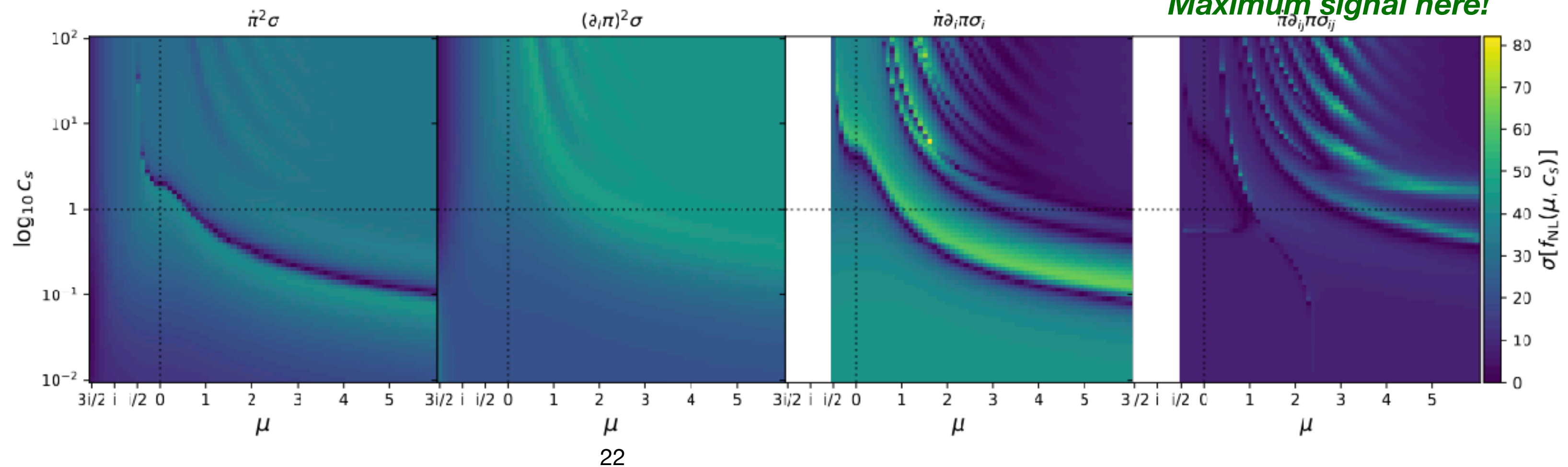
We can use this to constrain **massive** particle (single)-exchange, as a function of **mass**, **spin**, and **sound-speed** by coupling to (exact) **bootstrap** codes

Signal-to-Noise



Maximum signal here!

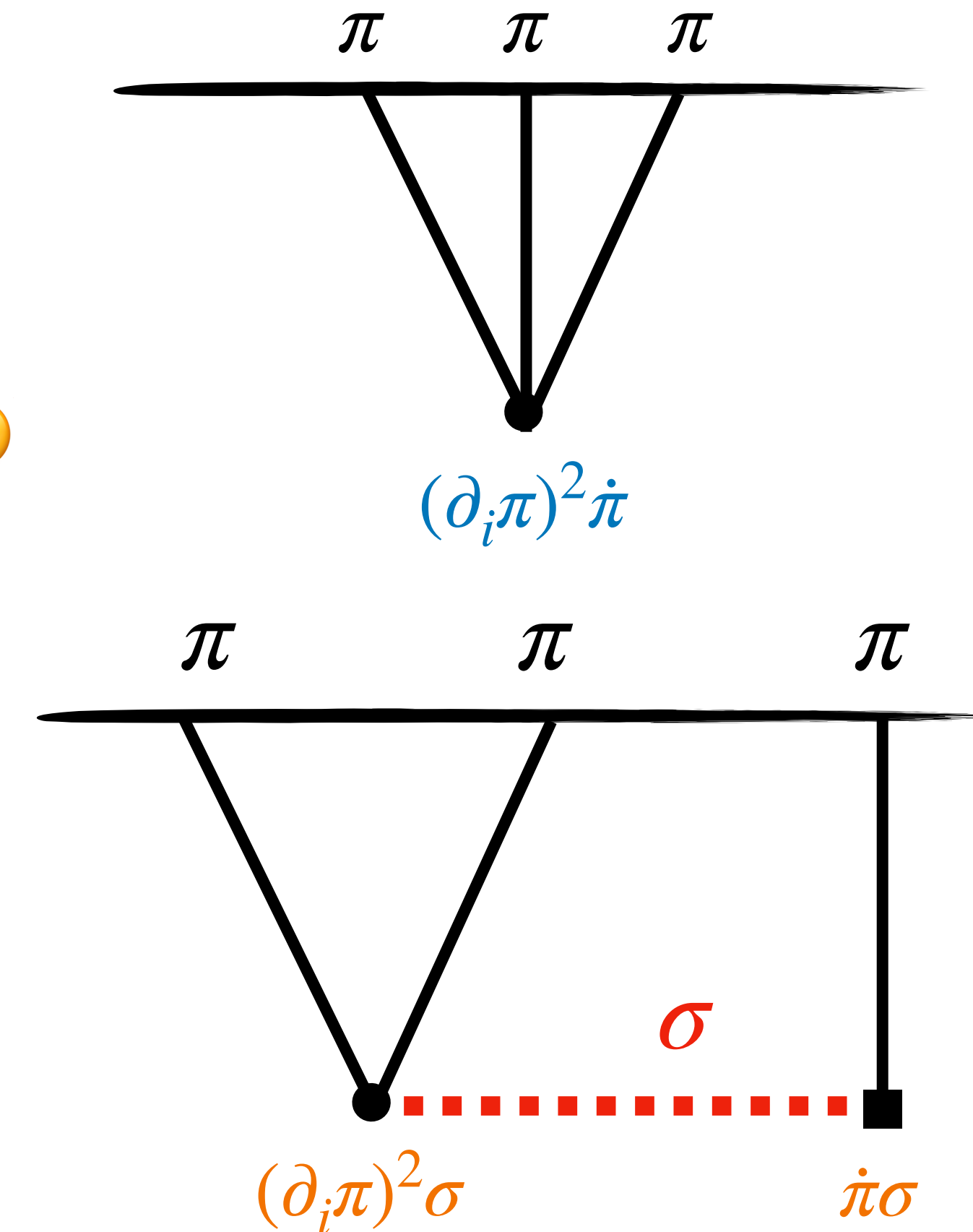
Errorbar



This takes 0.6 seconds!

Beyond Template Analyses

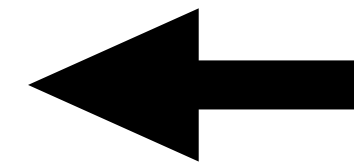
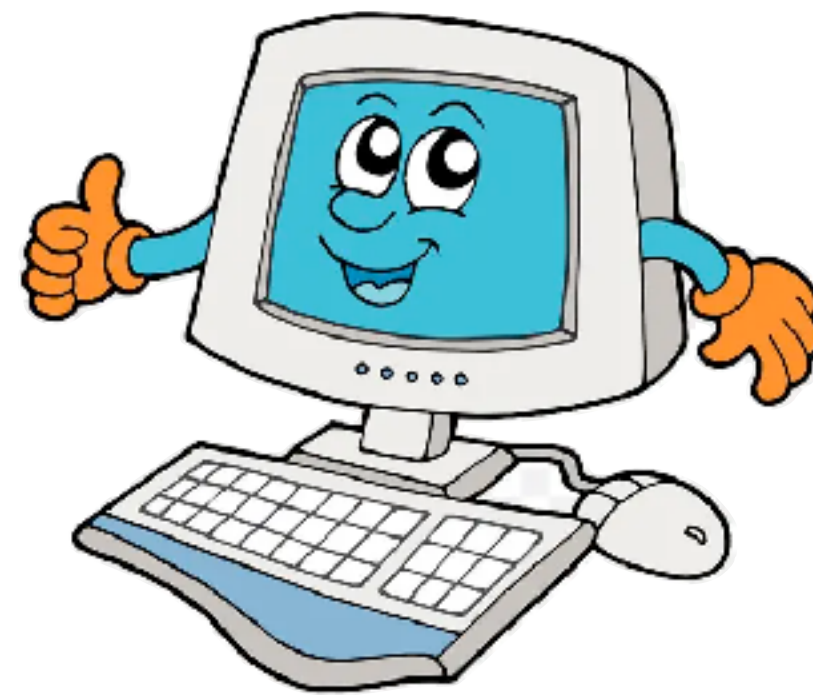
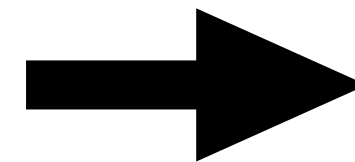
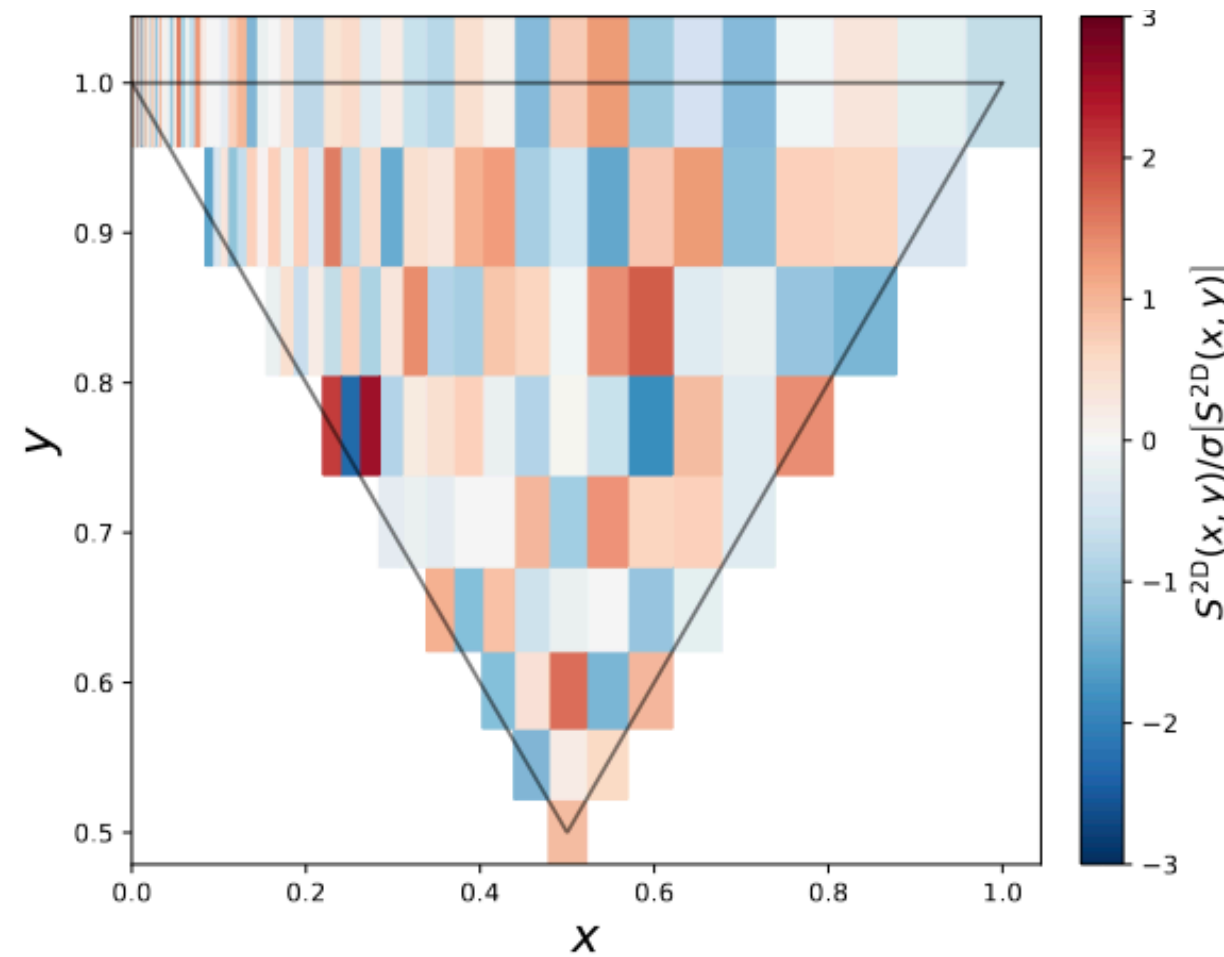
- Various terms in the model are **non-linearly** related, e.g.,
 - low-sound speed \Rightarrow large $(\partial_i \pi)^2 \sigma$ 😊 \Rightarrow large $(\partial_i \pi)^2 \dot{\pi}$ 😭
 - large collider signal \Rightarrow large $(\partial_i \pi)^2 \sigma$ 😊 \Rightarrow large quadratic mixing $\dot{\pi} \sigma$ 🤔
- **Template** analyses ignore this information!
 - (e.g., no correlations imposed between $f_{\text{NL}}^{\text{eq}}$ and $f_{\text{NL}}^{\text{collider}}$)
- **Solution:** analyze the full cubic Lagrangian self-consistently!



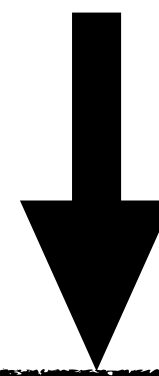
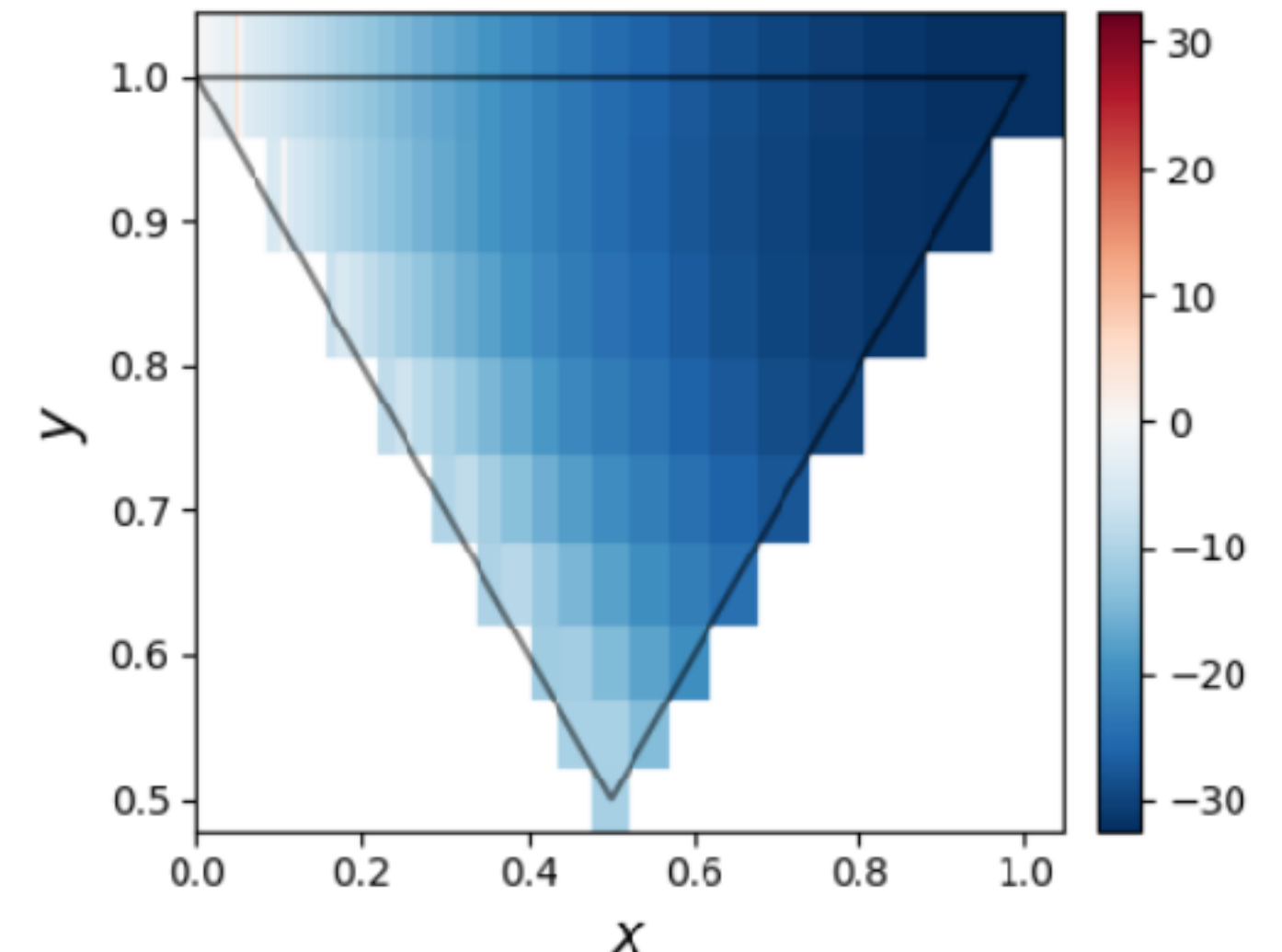
How to Constrain Inflation

We can **consistently** analyze the full tree-level bispectrum!

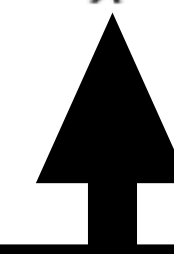
Observed Shape



Theoretical Shape

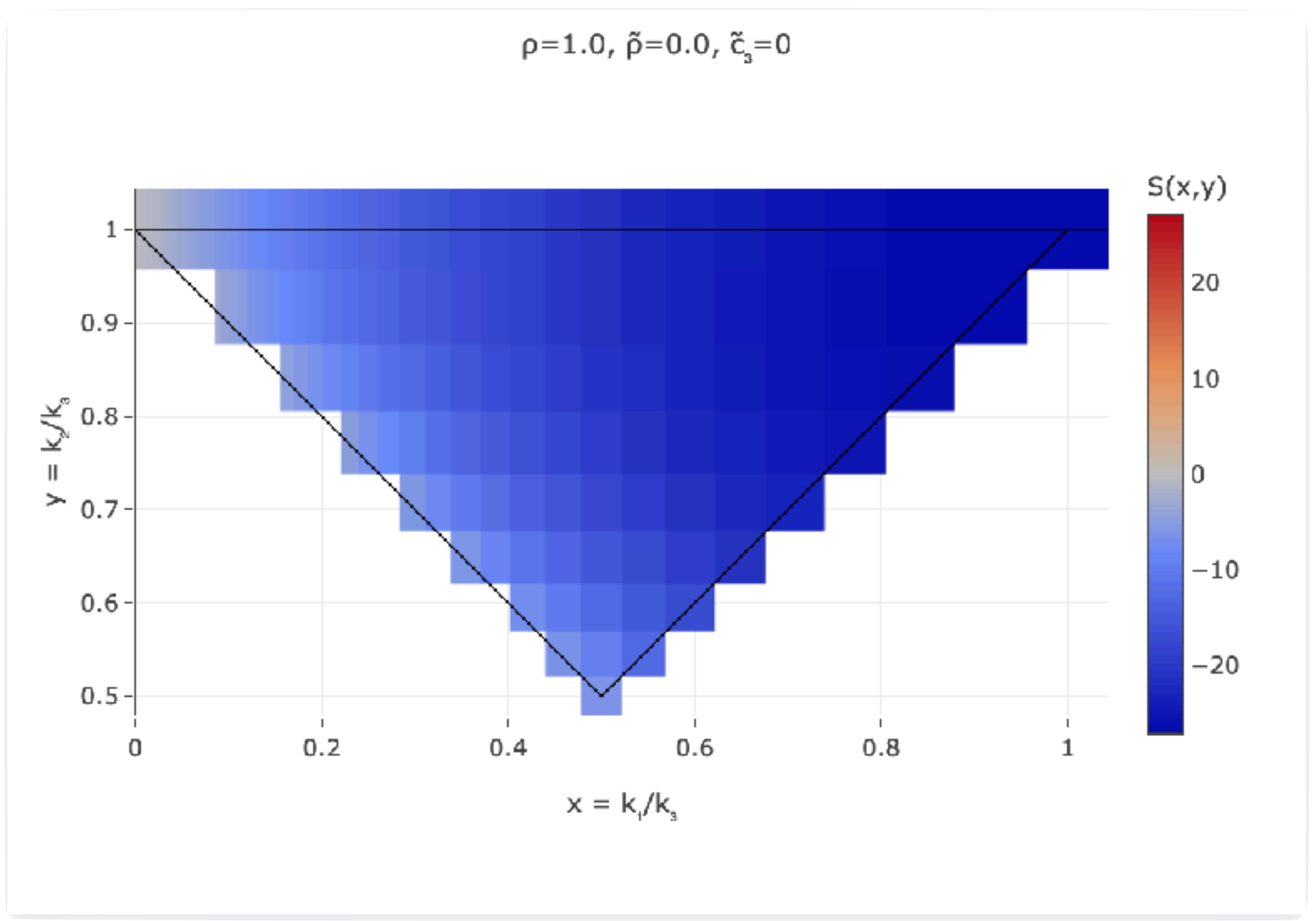
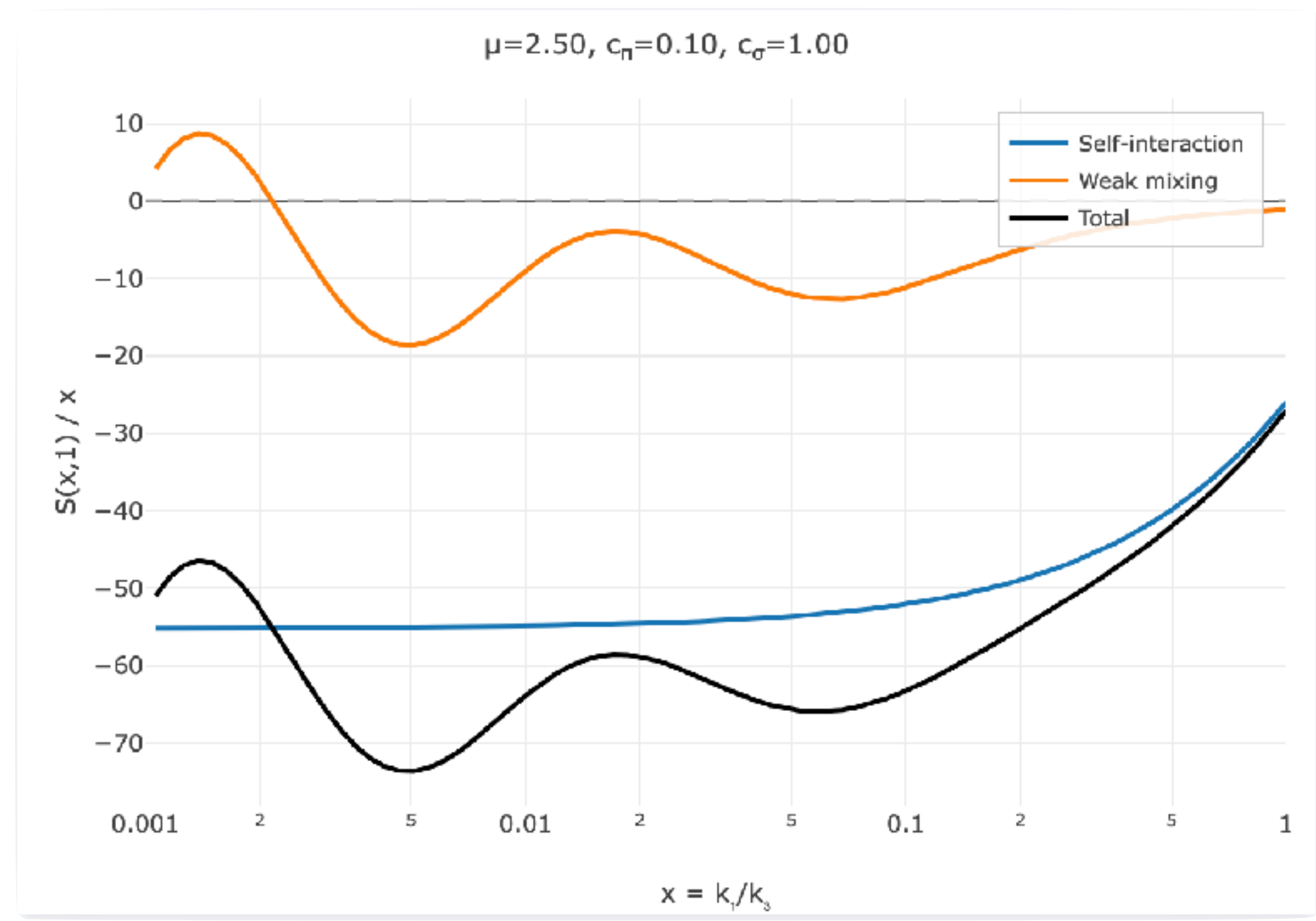


Constraints on Couplings



$$\mathcal{L} = \frac{1}{2} \left[\dot{\pi}_c^2 - c_{\text{rel}}^2 \frac{(\partial_i \pi_c)^2}{a^2} \right] + \frac{1}{2} \left[\dot{\sigma}_c^2 - \frac{(\partial_i \sigma_c)^2}{a^2} - m^2 \sigma_c^2 \right] + \rho \dot{\pi}_c \sigma_c - \lambda_1 \frac{(\partial_i \pi_c)^2}{a^2} \dot{\pi}_c - \lambda_2 \dot{\pi}_c^3 - \frac{\kappa_1}{2} \frac{(\partial_i \pi_c)^2}{a^2} \sigma_c - \frac{\kappa_2}{2} \dot{\pi}_c^2 \sigma_c$$

Weakly-Mixed Spin-Zero Colliders



Interactive Demo!



Log y-axis Divide by x

Self-Interactions

Sound Speed $\log_{10} c_\pi$ -1.00

π^2 Interaction $c_s^2(c_\pi^{-2}-1)$ 0

Weak Mixing

Scalar Mass μ 2.50

Sound Speed $\log_{10} c_\sigma$ 0.00

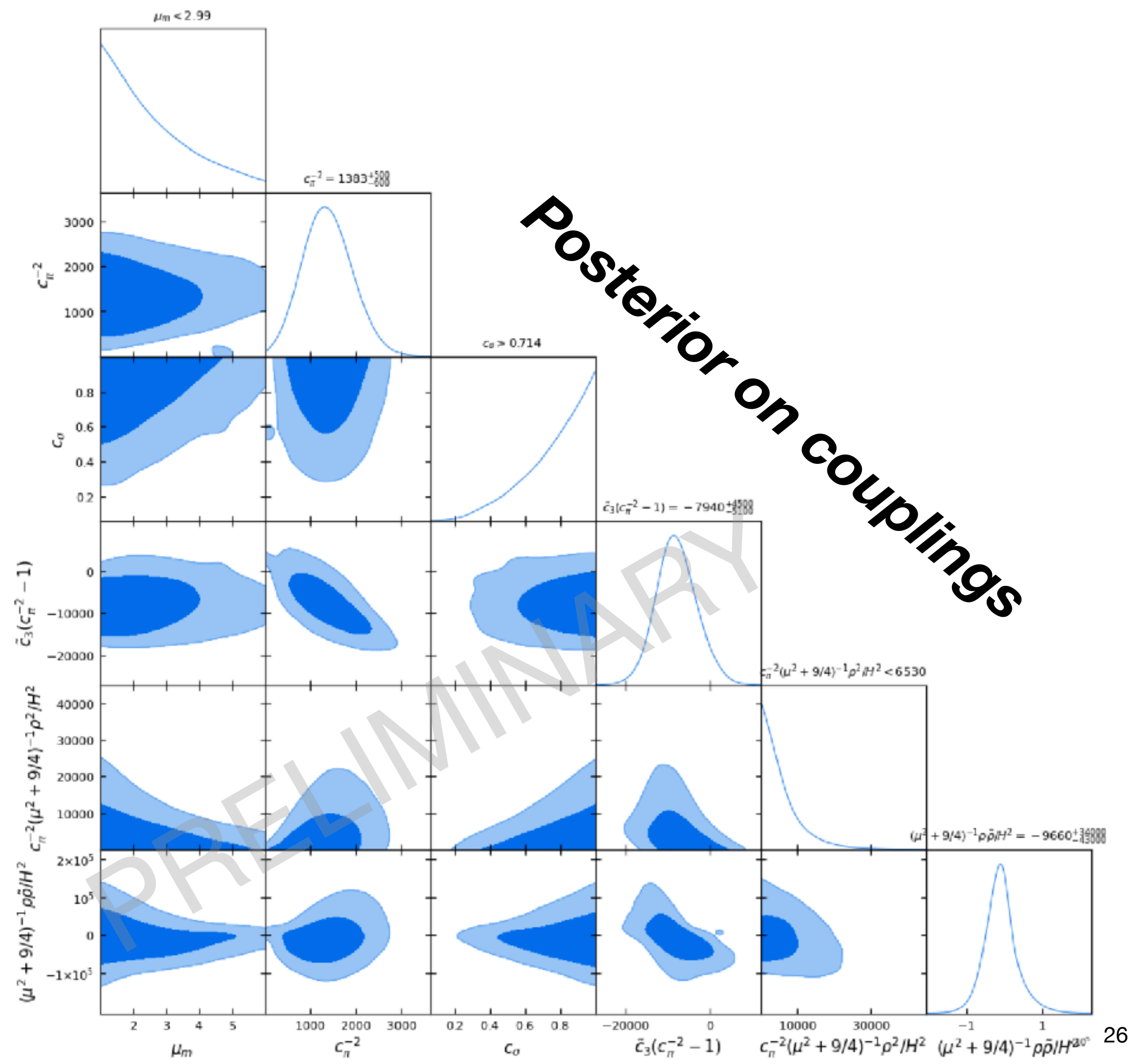
$\pi\sigma$ Mixing ρ 1.00

$\pi^2\sigma$ Interaction $\tilde{\rho}$ 0.0

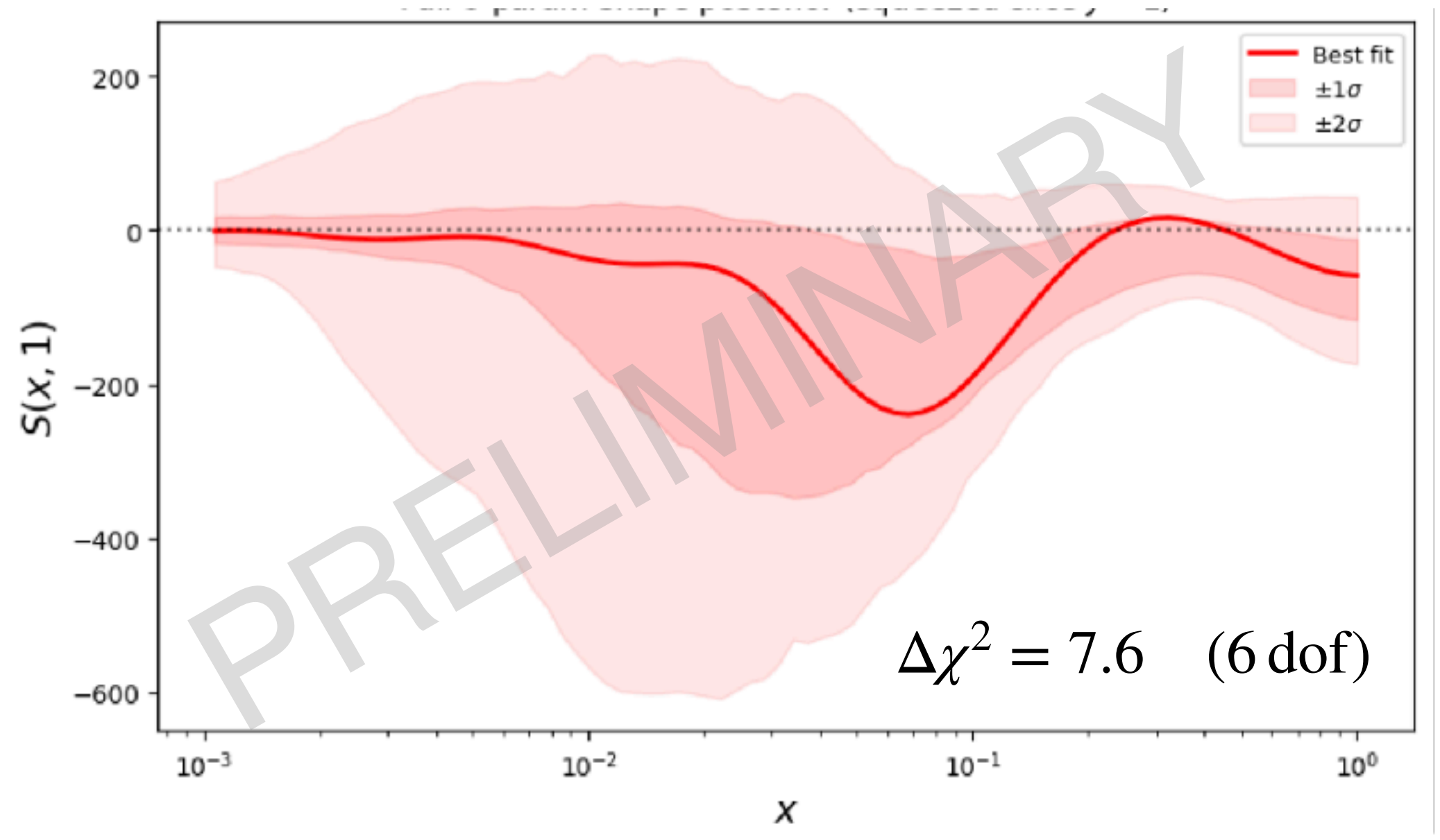
Interactive Demo!



Weakly-Mixed Spin-Zero Colliders



Maximum *a posteriori* solution



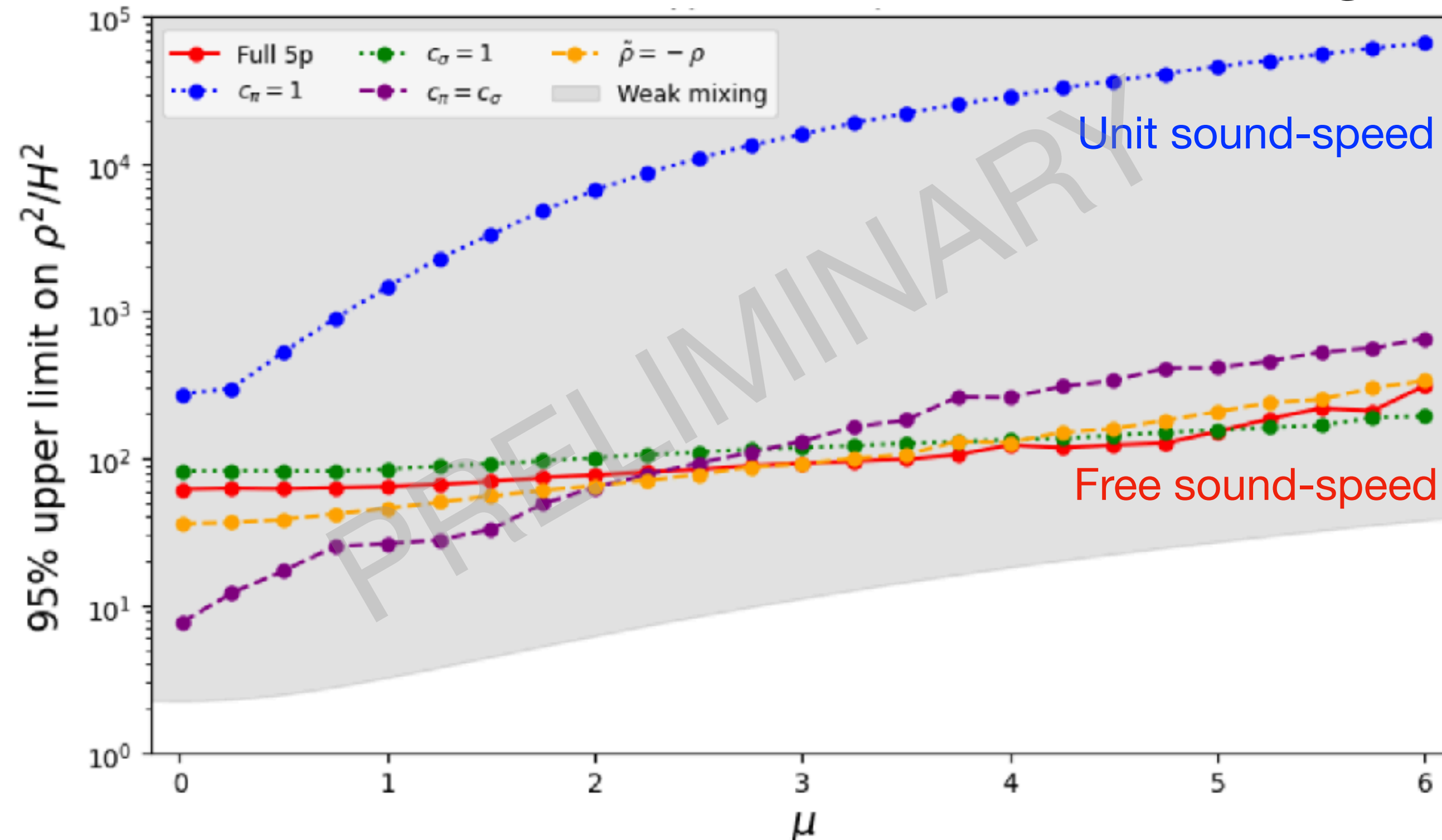
Interactive Demo!



Weakly-Mixed Spin-Zero Colliders

“large collider signal \Rightarrow large $(\partial_i \pi)^2 \sigma$ 😊 \Rightarrow large quadratic mixing $\dot{\pi} \sigma$ 🤔”

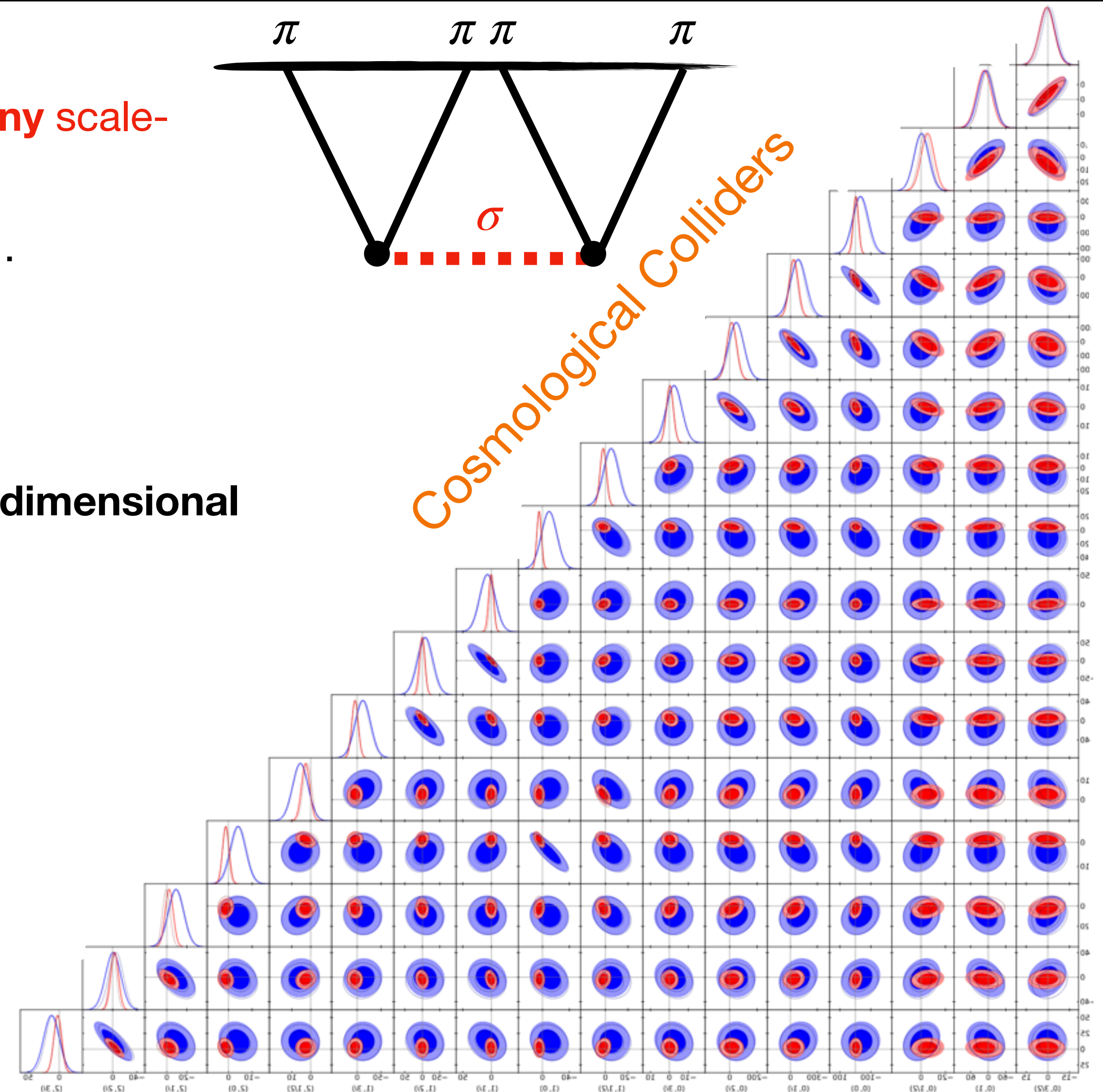
Upper limit on quadratic mixing



Conclusion: detectable single-exchange diagrams must be **strongly-mixed**

What's Next for the CMB?

- With the binned **shape function** estimators, we can constrain **any** scale-invariant primordial bispectrum
 - Bootstraps, Cosmological Flows, Mellin transforms, loops, ...
- What about **trispectra**?
 - This is **hard** — the scale-invariant four-point function is four-dimensional $S^{(3)}(x, y) \rightarrow S^{(4)}(x, y, u, v)$
 - There has been work searching for **factorizable shapes**
 - Self-interactions
 - Oscillatory **cosmological collider** signals
 - Modifications to τ_{NL}
 - **No detections yet!** (except CMB lensing)

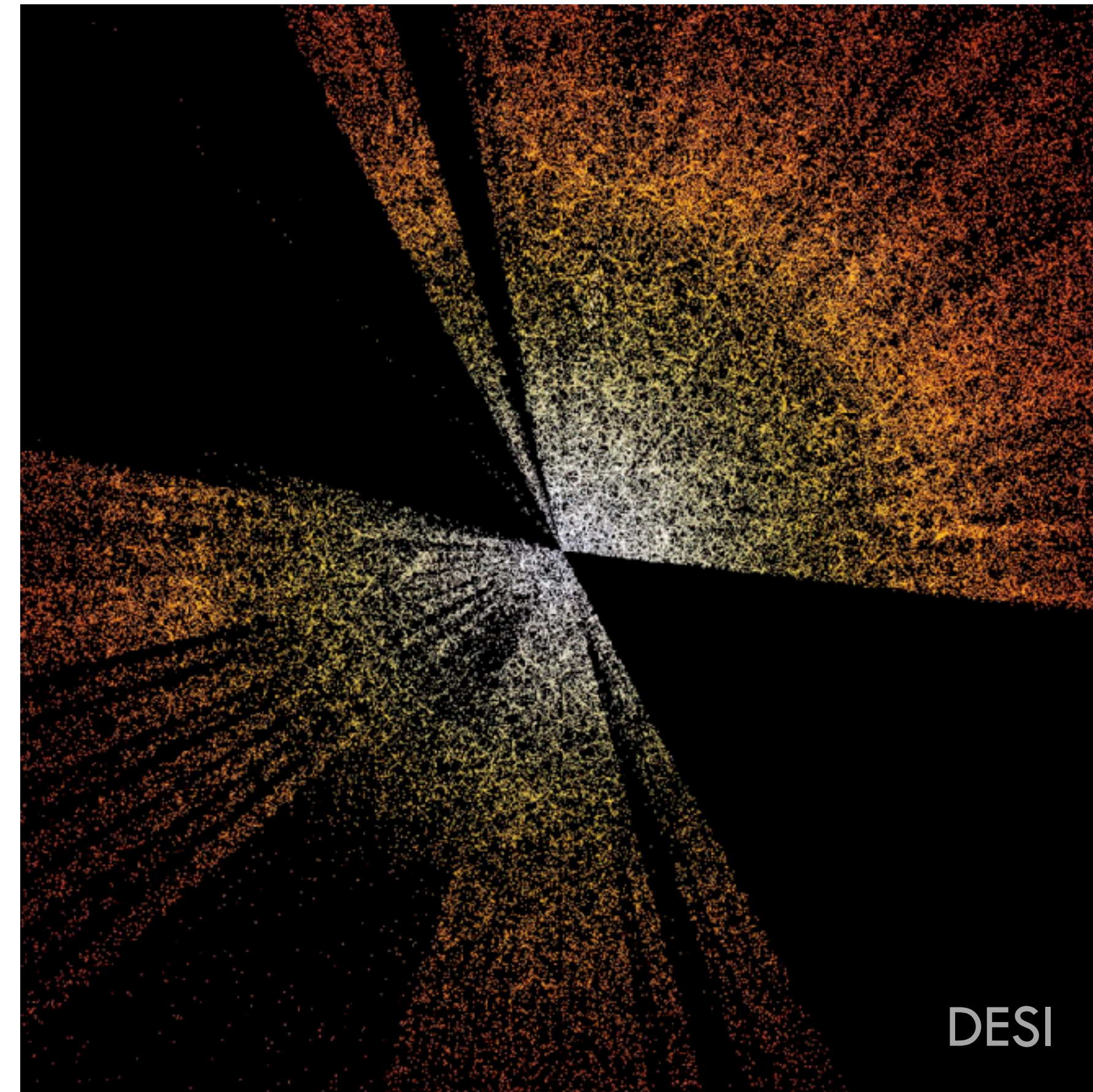


The Future of Non-Gaussianity

- **CMB experiments** have provided a precise **two-dimensional** map of the Universe at age 400,000 yr
 - Their **primordial** constraining power is saturating!
- **Galaxy surveys** will map the Universe in **three-dimensions** at age $\sim 10^9 - 10^{10}$ yr
 - Legacy surveys have mapped a **million** galaxies [BOSS]
 - New surveys map $\sim \mathbf{100} \times$ more! [DESI, Euclid, Rubin, Roman, ...]

see Uros's talk!

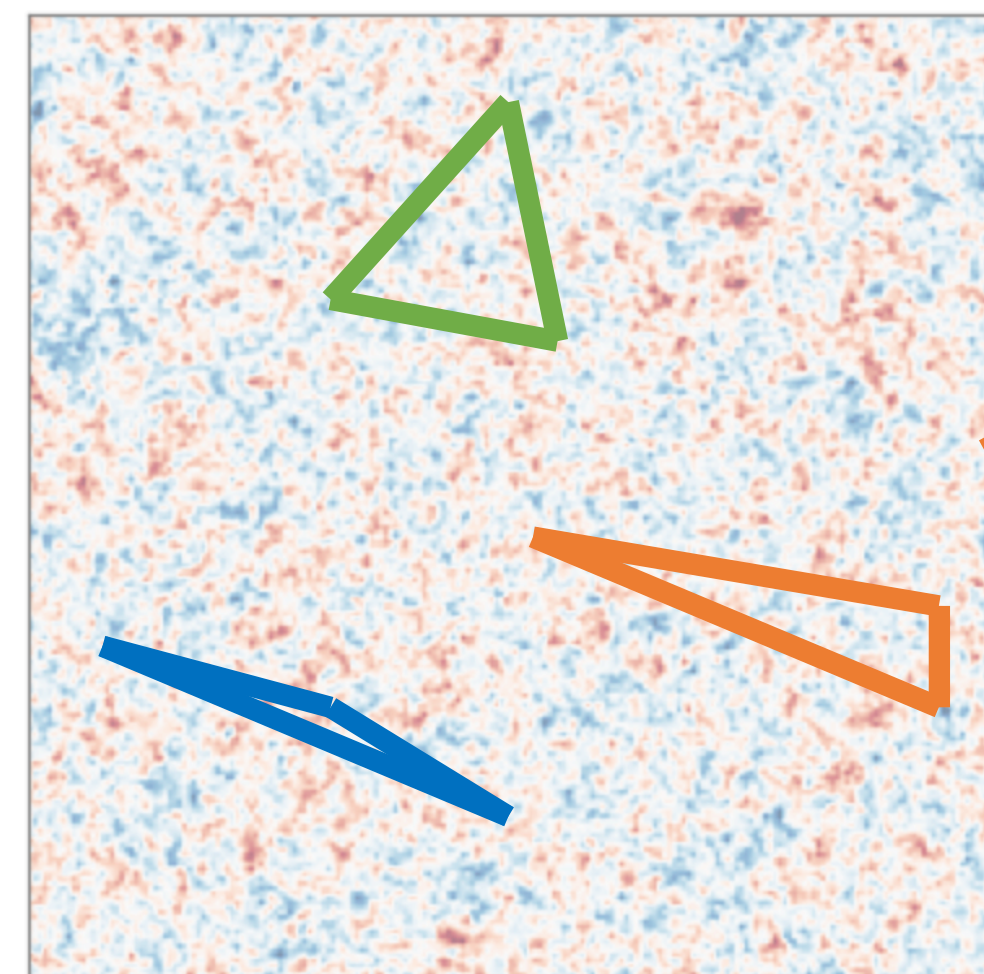
Density of galaxies, n_{gal}



Can we use **galaxy surveys** to learn about **inflation**?

How to Measure Primordial Non-Gaussianity

The curvature perturbation ζ sets the initial conditions for the late Universe!

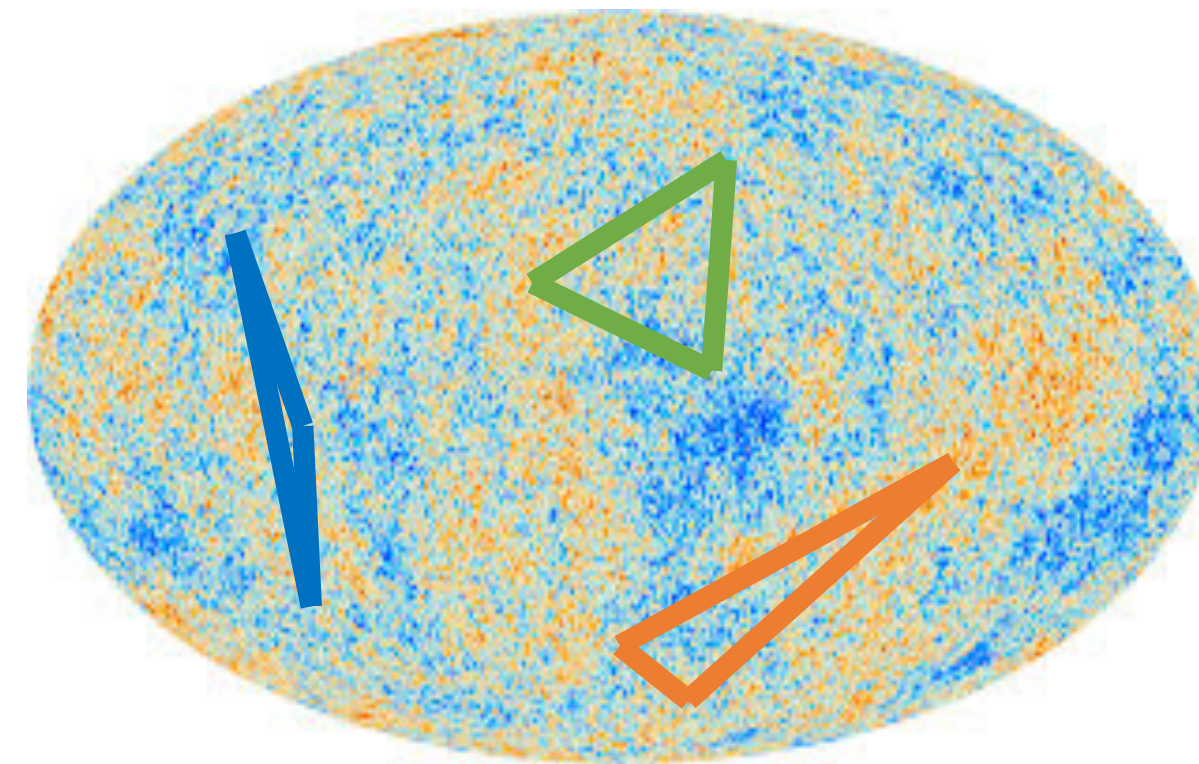


Primordial Curvature

$$\langle \zeta^n \rangle \neq 0?$$

Linear Physics

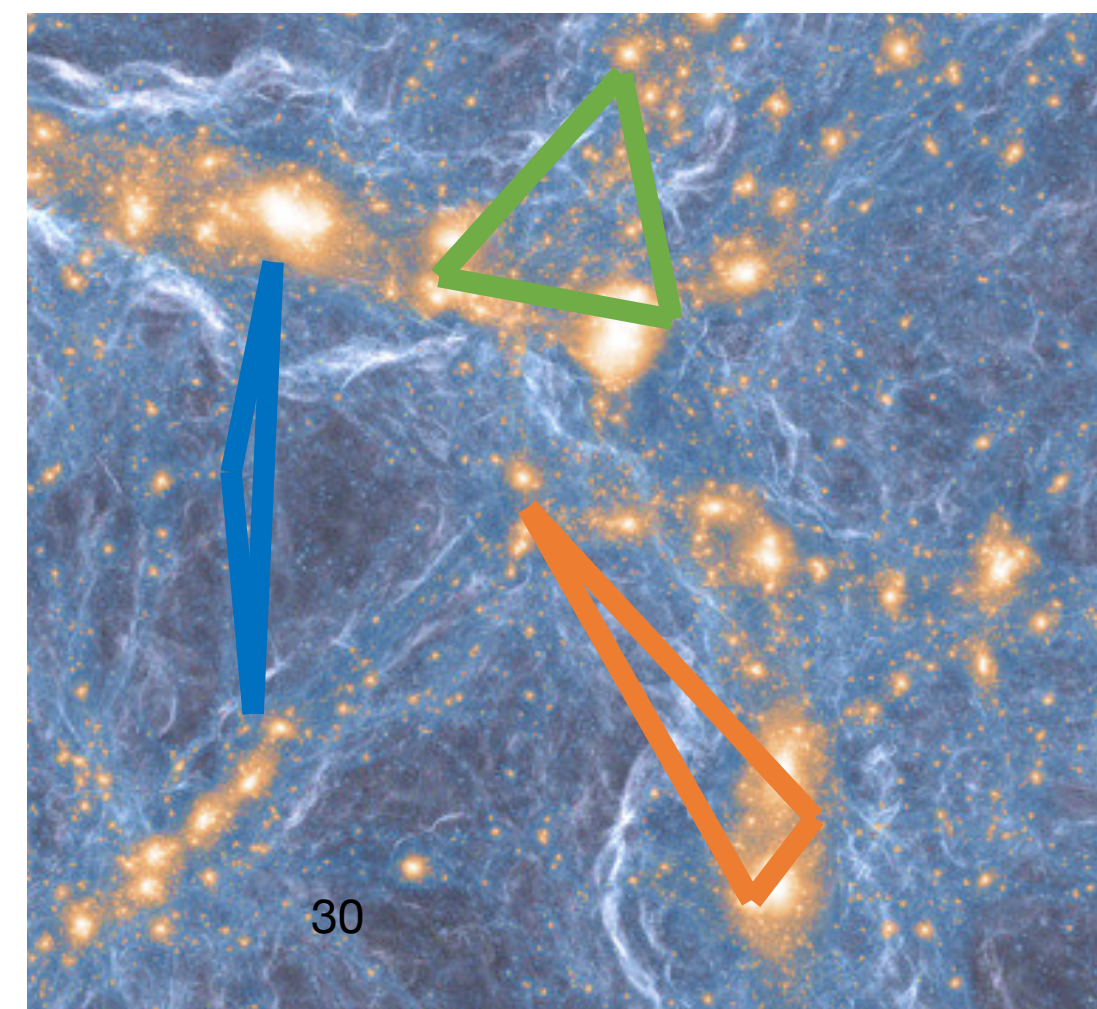
Non-Linear Physics



Cosmic Microwave Background fluctuations

$$\langle \delta T^n \rangle \neq 0?$$

(tracing *photon energies*)



Galaxy Number Density Fluctuations

$$\langle \delta n_{\text{gal}}^n \rangle \neq 0?$$

(tracing *dark matter*)

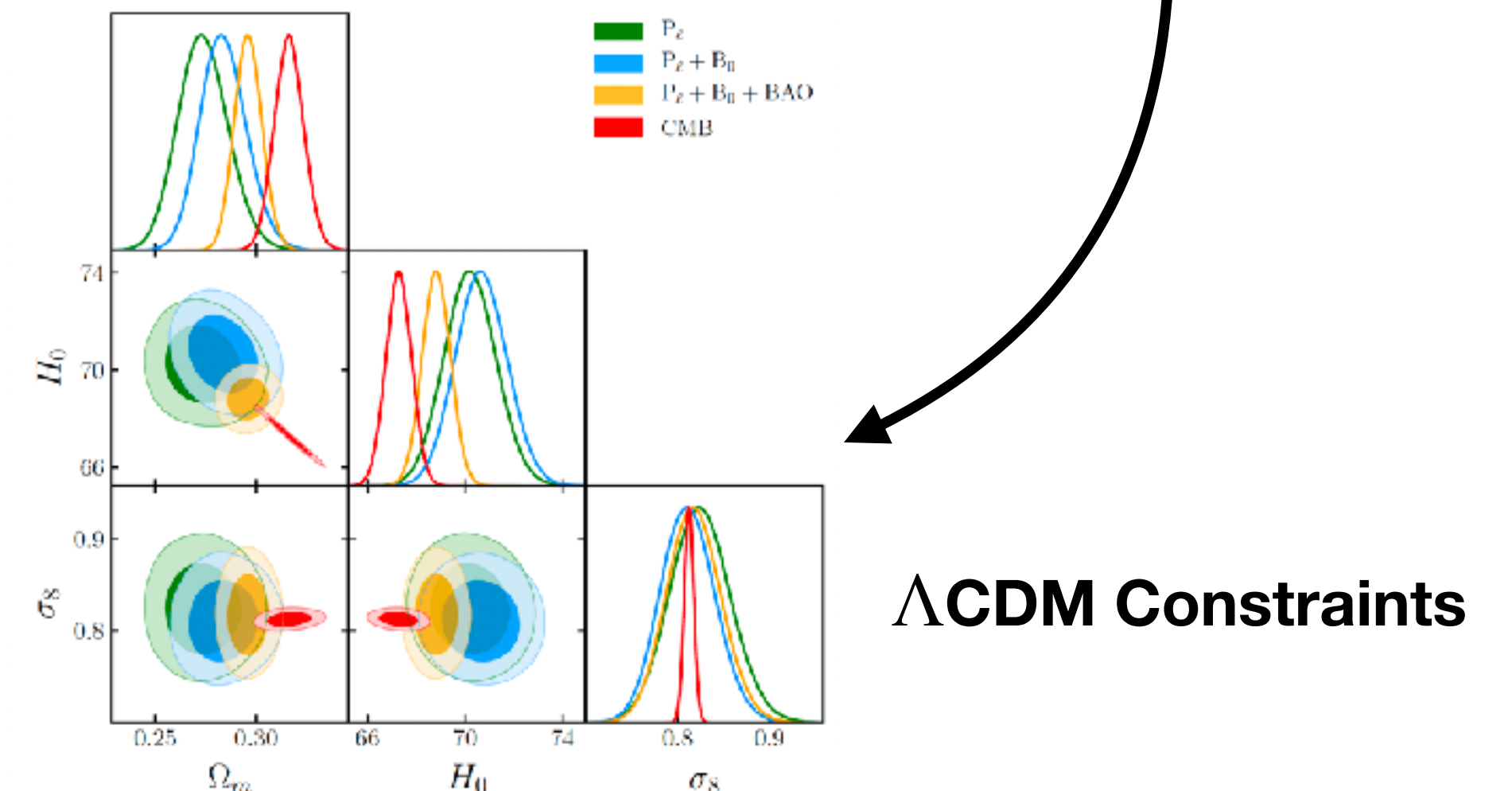
Inflation from DESI

The first year of DESI data is now public!

- We have developed an **independent** pipeline for analyzing the **galaxy two- and three-point** functions based on **Effective Field Theory** methods
- This has been used to constrain: Λ CDM (Ω_m, H_0, σ_8), dark energy ($w_0 w_a$), curvature (Ω_k), neutrino masses ($\sum m_\nu$), both **alone** and in combination with the **CMB** and **gravitational lensing**

Excerpt from DESI Data Release 1

TARGETID int64	Z float64	NTILE int64	RA float64	DEC float64	...
39627540901396844	0.42060841162467566	1	159.30684159361635	-10.155757636765902	...
39627546836338876	0.8668980715716706	1	158.44667596279407	-9.962760066342906	...
39627546840531340	0.9348172077800124	1	158.47992947022238	-9.880343166939232	...
39627546840533707	0.7646678553759423	1	158.65071160360105	-9.900898173028425	...
39627546840534067	0.88129590000311	1	158.67878216902403	-9.91791308567385	...
39627546840534396	0.6646155566176719	1	158.70027052890555	-9.885818986284596	...
39627546844725593	0.7619120932610688	1	158.72751630870823	-10.011383569041937	...
39627546844726132	0.8129116729090922	1	158.75343950179967	-9.912671320450734	...
39627546844726593	0.835471640017949	1	158.79898500886574	-9.952788127324565	...
39627546848921194	0.8148312339778753	1	159.052157885943	-9.992428612452807	...
39627546848922139	0.7200341373651288	1	159.10202657806508	-9.938566366253678	...
39627546848922621	0.7606337242857438	1	159.1309146297404	-10.02377942401391	...
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39627546848923188	0.7210857282186207	1	159.15399100631358	-9.912947332242044	...
39627546848923381	0.569430729151765	1	159.17802210549974	-9.97892860399317	...
39627546848923415	0.8891288789150124	1	159.18008439182032	-10.072752528866118	...
39627546848923493	0.9513285375888253	1	159.1840389390485	-9.910321824120278	...
39627546848923519	0.7212784017696859	1	159.1860701777553	-9.944737378735352	...
39627546853114634	0.8131126675553368	1	159.25137421656687	-10.058275905081351	...
39627546853115304	0.5559672054059013	1	159.23855963426028	-9.955979493106813	...
39627546853115470	0.7147216867384578	1	159.2970230990033	-10.012836906791499	...
39627546853115682	0.9274570688680336	1	159.30835543527493	-10.106935803496164	...



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New constraints on inflation!

- **Multi-field:** $f_{\text{NL}}^{\text{loc}} = -0.1 \pm 7.4$
- **Single-Field:** $f_{\text{NL}}^{\text{eq}} = 200 \pm 230$, $f_{\text{NL}}^{\text{orth}} = -24 \pm 86$

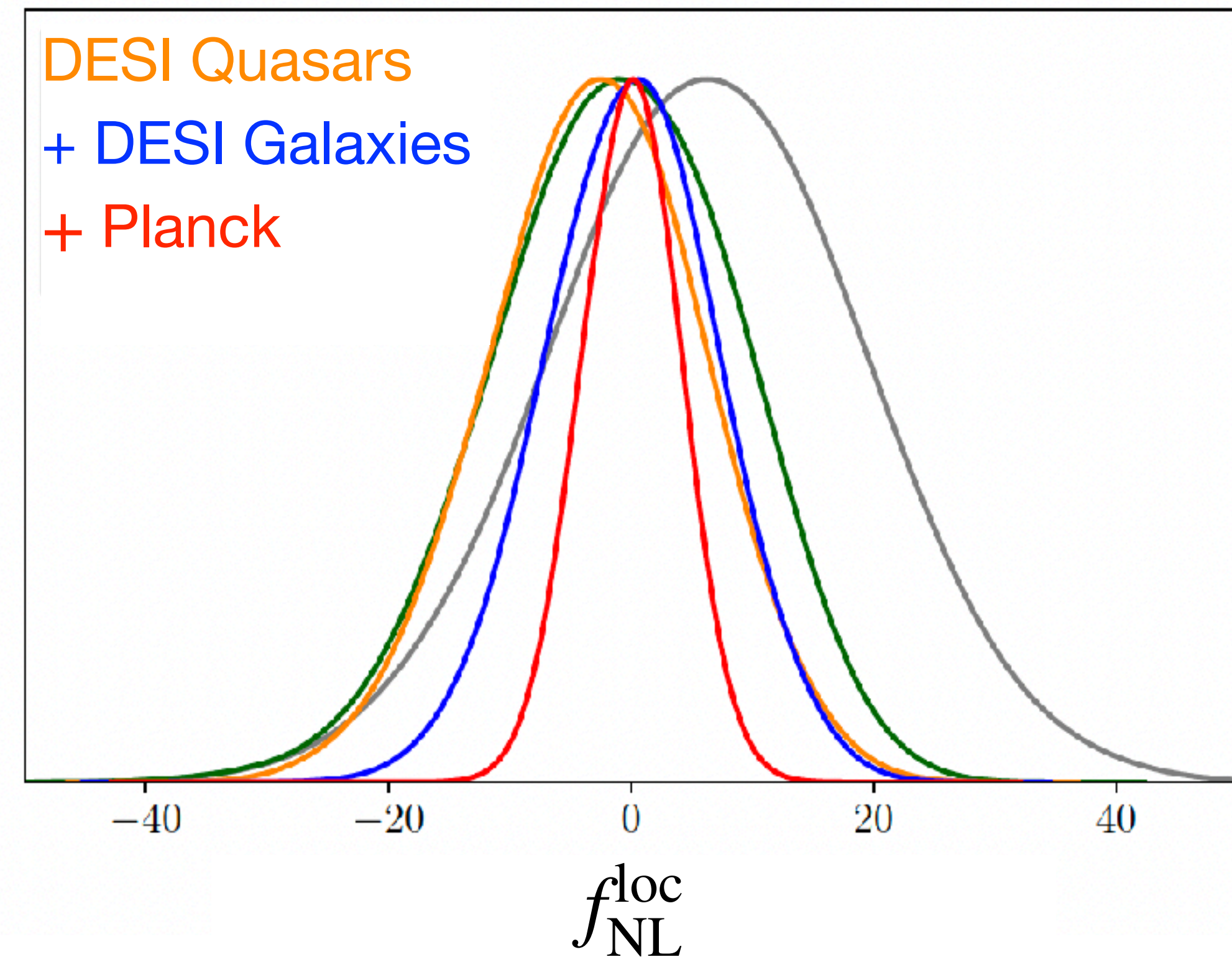
Compare to official DESI results:

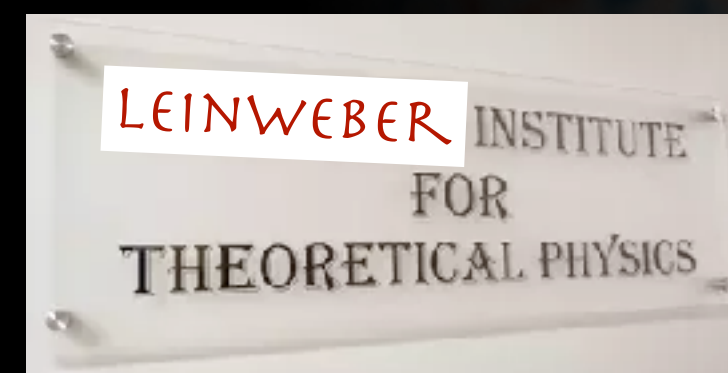
$$f_{\text{NL}}^{\text{loc}} = -2 \pm 10$$

(Many more models to explore!)

- Adding **Planck**, we obtain the **tightest** constraint on local PNG yet!!

$$f_{\text{NL}}^{\text{loc}} = 0.0 \pm 4.1$$





Summary

- Extracting **inflationary physics** from CMB data is challenging
- New methods reconstruct the underlying **inflationary shape** allowing a wide range of **new tests** of inflation
- **Galaxies** are the next frontier for primordial non-Gaussianity

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